SEMBODAI RUKMANI VARATHARAJAN ENGINEERING COLLEGE

SEMBODAI-614 809

DEPARTMENT OF MECHANICAL ENGINEERING

ACADEMIC YEAR 2023-2024/ODD SEMESTER

Subject Code : **ME3351**
Subject Name : **ENGINEERING MECHANICS**
Unit Name : **UNIT II EQUILIBRIUM OF RIGID BODIES**
Year/Semester : **II/III**

Prepared by

Faculty Name : **VEERAPANDIAN.K**
Designation : **Assistant Professor/MECH**

UNIT-II.

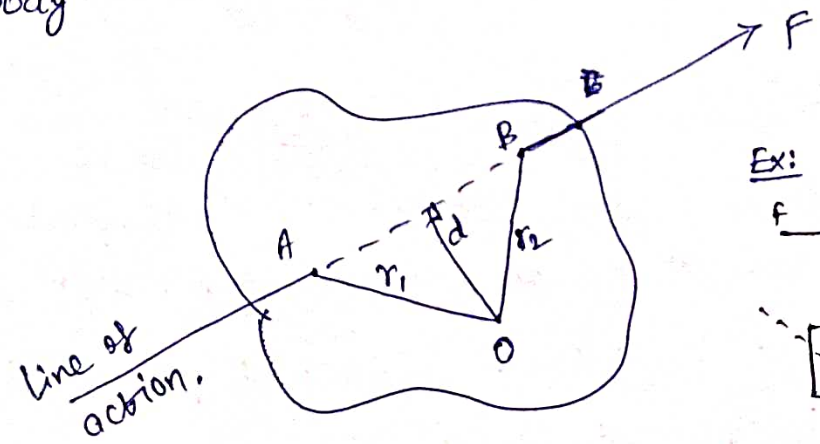
EQUILIBRIUM OF RIGID BODIES.

Equilibrium:

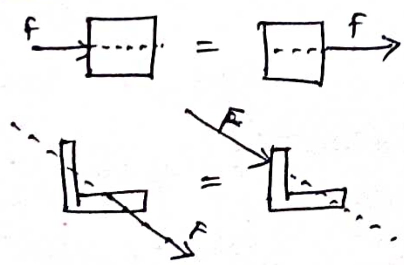
"A particle is in mechanical equilibrium if the net force on that particle is zero".

Principle of Transmissibility:

"The point of application of a force can be moved anywhere along its line of action without changing the external reaction forces on a rigid body"



Ex:



(i) Considering point of application of force F at A, the moment about 'O',

$$M_0 = r_1 \times F = Fd \ (\circlearrowleft) \longrightarrow (1)$$

(ii) Considering point of application of force at B, the moment about 'O',

$$M_0 = r_2 \times F = Fd \ (\circlearrowleft)$$

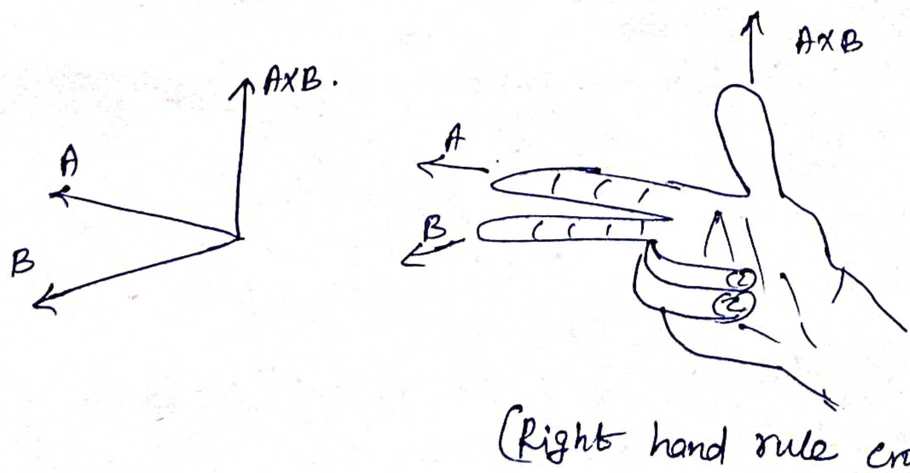
From (1),(2) we get $M_0 = r_1 F = r_2 F$.

Equivalent Forces:

Two forces having the same magnitude, direction and line of action but acting at different points in same line of action, producing same external static effect on the rigid body are said to be equivalent forces.

Vector product of Two vectors:

The vector product (or) cross product of two vectors A and B is denoted by A×B, and it's result vector is vector is perpendicular to the vectors A and B.



(Right hand rule cross product)

We can find the direction of unit vector (resultant vector) with the help of right hand rule.

Cross product formula :

If θ is the angle between the given two vectors A and B, then the formula for the cross product vector is given by,

$$A \times B = |A|\,|B| \sin\theta \quad \text{(or)}$$

$$\vec{A} \times \vec{B} = \|\vec{A}\|\,\|\vec{B}\| \sin\theta\, \hat{n}$$

where,

$\vec{A}, \vec{B}$ - Vectors

$\|\vec{A}\|, \|\vec{B}\|$ - magnitude of given vectors

$\hat{n}$ - unit vector perpendicular to the vectors

Consider two vectors,

$$A = ai + bj + ck$$
$$B = xi + yj + zk$$

Then,

$$\left.\begin{aligned} i \times j &= k, & j \times i &= -k \\ j \times k &= i, & k \times j &= -i \\ k \times i &= j, & i \times k &= -j \end{aligned}\right\} \rightarrow (1)$$

Also, the anti-commutativity of the cross product and the distinct absence of linear independance of these vectors signifies that,

$$i \times i = j \times j = k \times k = 0 \quad \rightarrow (2)$$

Now,

$$A \times B = (ai + bj + ck) \times (xi + yj + zk)$$

$$= ax(i \times i) + ay(i \times j) + az(i \times k) + bx(j \times i) + by(j \times j) + bz(j \times k) + cx(k \times i) + cy(k \times j) + cz(k \times k) \rightarrow ②$$

From ①, Subs ① in ②,

$$A \times B = ax(0) + ay(k) + az(-j) + bx(-k) + by(0) + bz(i) + cx(j) + cz(0) + cy(-i)$$

$$= ay(k) - bx(k) - az(j) + cx(j) + bz(i) - cy(i)$$

$$= (bz - cy)i + (cx - az)j + (ay - bx)k \rightarrow ③$$

Cross product matrix:

$$A \times B = \begin{vmatrix} i & j & k \\ a & b & c \\ x & y & z \end{vmatrix}$$

$$A \times B = (bz - cy)i - (az - cx)j + (ay - bx)k$$

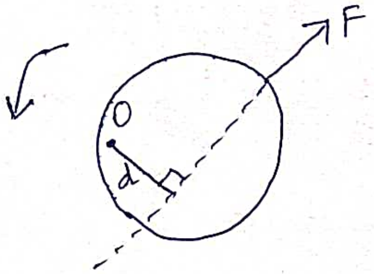
$$= (bz - cy)i + (cx - az)j + (ay - bx)k \rightarrow ④$$

eqns ③, ④ are same.

Moment of Force about an Axis / point :

The turning effect of force is known as moment of force. It's SI unit is Nm.

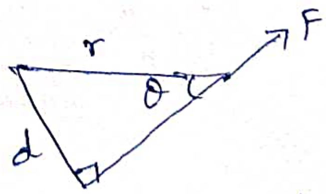
It is the product of the force multiplied by the perpendicular distance from the line of action of the force to the pivot, where the object will turn.



$M = F \times d$

The moment of F about the point O is expressed in the vector approach as.

$$\boxed{\vec{M_O} = \vec{r} \times \vec{F}}$$



$= r.F\sin\theta$

$= F.r\sin\theta$

$\sin\theta = \frac{d}{r}$

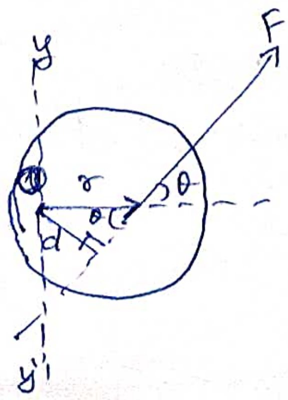
$d = r\sin\theta$.

$$\boxed{M_O = F.d}$$

where,

d - moment arm

O - moment centre.



where,

$F = F\sin\theta$

vertical component.

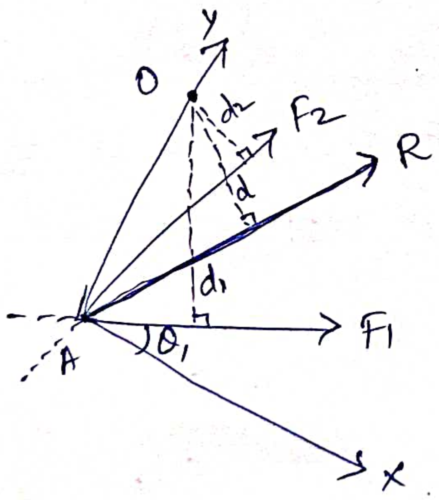
Varignon's Theorem:

"The moment of force about any point is equal to the sum of moments of the components of the force about the same point"

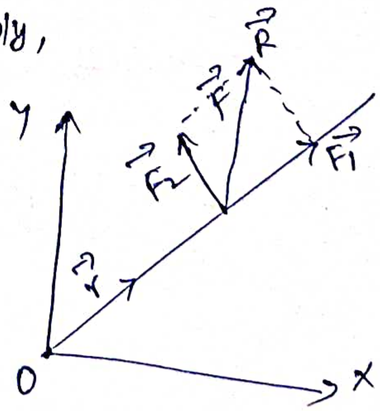
(or).

"If many coplanar forces are acting on a body, then algebric sum of moments of all the forces about a point in the plane of the forces is equal to the moment of their resultant about the same point"

$$\Sigma M_{forces} = M_{resultant}.$$



Simply,



where,

$\vec{r}$ - Position vector

$\vec{R}$ - Resultant of $\vec{F_1}, \vec{F_2}$

$\vec{F_1}, \vec{F_2}$ - Forces

$M_O = \vec{r} \times \vec{R}$ $\quad \because \vec{R} = \vec{F_1} + \vec{F_2}$

$= \vec{r} \times (\vec{F_1} + \vec{F_2})$

$= (\vec{r} \times \vec{F_1}) + (\vec{r} \times \vec{F_2})$

$M_O = \vec{r} \times \vec{R} = (\vec{r} \times \vec{F_1}) + (r \times \vec{F_2})$

$\therefore$ Hence theorem prooved.

Rectangular Components of the moment of force.

The moment of the force $\vec{F}$, passing through the point $P(x, y, z)$ about the point $O(0,0,0)$ is given by $\vec{M_O} = \vec{r}_{op} \times \vec{F}$

$$\vec{r}_{op} = \text{position vector} = (x-0)\vec{i} + (y-0)\vec{j} + (z-0)\vec{k}$$

$$= x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$\therefore \vec{M_O} = (x\vec{i} + y\vec{j} + z\vec{k}) \times (F_x\vec{i} + F_y\vec{j} + F_z\vec{k})$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= \vec{i}\,(yF_z - zF_y) - \vec{j}\,(xF_z - zF_x) + \vec{k}\,(xF_y - yF_x)$$

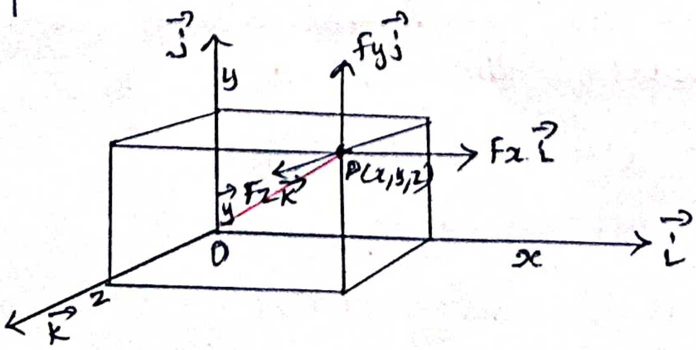
$$\vec{M_O} = (yF_z - zF_y)\vec{i} + (-xF_z + zF_x)\vec{j} + (xF_y - yF_x)\vec{k}$$

$$\vec{M_O} = M_x\vec{i} + M_y\vec{j} + M_z\vec{k}$$

Where, $M_x = yF_z - zF_y$

$M_y = zF_x - xF_z$

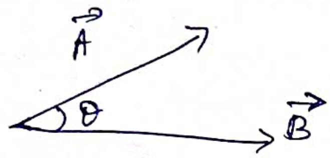
$M_z = xF_y - yF_x$ are the rectangular components of moment.



Scalar product of Two Vectors:

" The product of the magnitudes of the two vectors and the cosine of the angles ~~of the~~ between them".

Note:
(Scalar product other name dot product (or) Inner product)



Scalar product of $\vec{A}\cdot\vec{B} = AB\cos\theta$

Properties of Scalar product:

1. The Scalar product of two vector is commutative.
   (ie) $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$ for any vectors $\vec{a}$ and $\vec{b}$

2. Scalar product of collinear vectors.

   (i) When the vectors $\vec{a}$ and $\vec{b}$ are collinear and in the same direction, $\theta = 0$.

   (ie) $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$ $\quad [\theta = 0^\circ]$

   $= |\vec{a}||\vec{b}|(1)$

   $= ab$

   (ii) When the vectors $\vec{a}$ and $\vec{b}$ are collinear and are in the opposite direction, then $\theta = \pi$

   (ie) $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cdot\cos\theta$

   $= |\vec{a}||\vec{b}|\cos\pi$

   $= |\vec{a}||\vec{b}|(-1)$

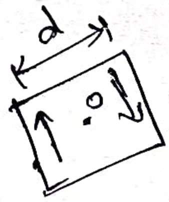
   $= -ab$

COUPLE:

A Couple is a pair of forces, equal in magnitude, oppositely directed and displaced by perpendiclalar distance (or) moment.

Moment of Couple:

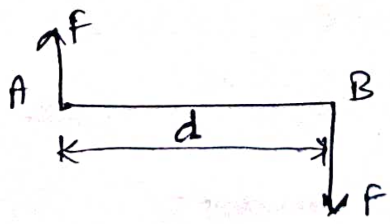
The moment of a couple M is defined as having magnitude of $M = F \times d$, where F is the magnitude of one the forces and d is perpendicular distance.



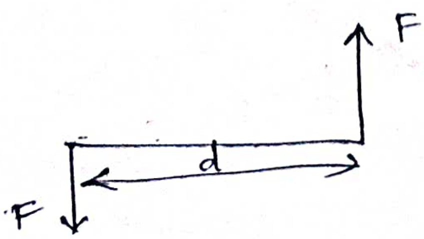
Types of Couple;

Couple is classified into two types,

(i) Clockwise couple:



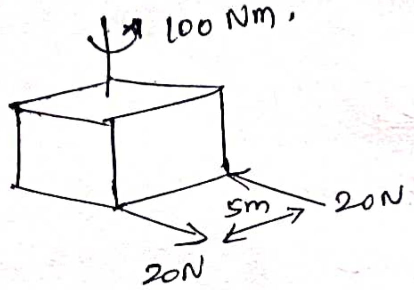
(ii) Anticlockwise Couple:



K. Veerapandian,
AP/mech.

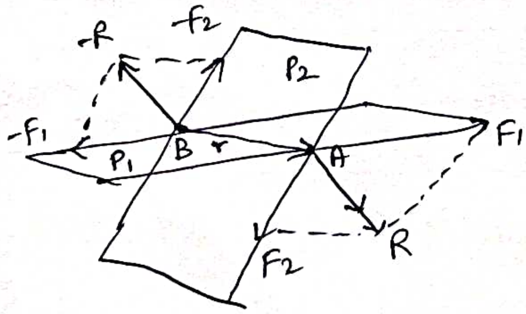
EQUIVELENT COUPLE:

If two couples have same magnitude, then they are called equivalent couple.



ADDITION OF COUPLES:

Consider two interacting ~~forces~~ planes, P_1 and P_2, and two couples acting respectively.



P_1 - plane-1

P_2 - plane-2

r - Joining vector B to A.

$$\vec{M} = \vec{r} \times \vec{R}$$

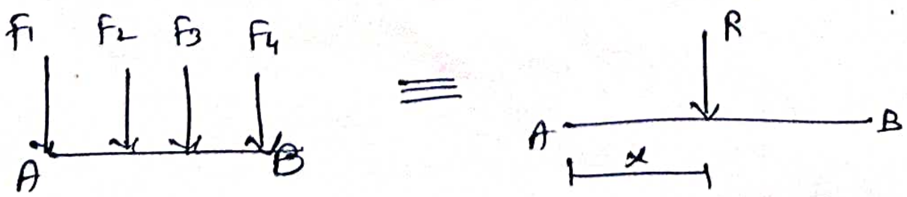
$$= \vec{r} \times (\vec{F_1} + \vec{F_2})$$

$$\vec{M} = (\vec{r} \times \vec{F_1}) + (\vec{r} \times \vec{F_2})$$

$$\Rightarrow \vec{M} = M_1 + M_2$$

FORCE COUPLE SYSTEM:

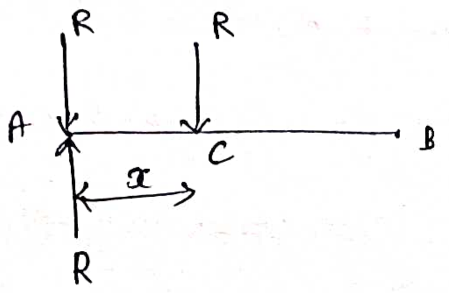
Procedure to replace forces into single force (R).



Step 1:

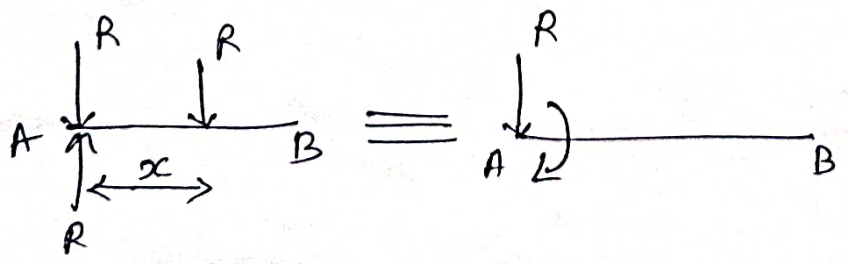
Let the given force system has a resultant 'R' which is acting at a distance of 'x' from a particular point.

Step 2:

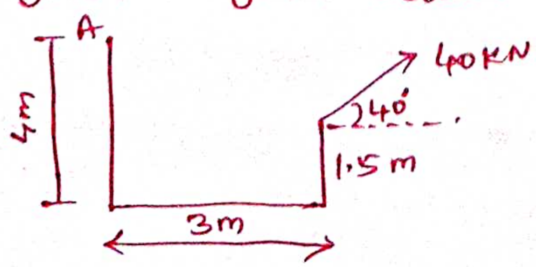


Apply for 'R' at point A both in the upward and downward directions. (There was no change in System by this apply).

Step 3: Now, the downward resultant R and at C and R at A forms couple in Counter clockwise direction.

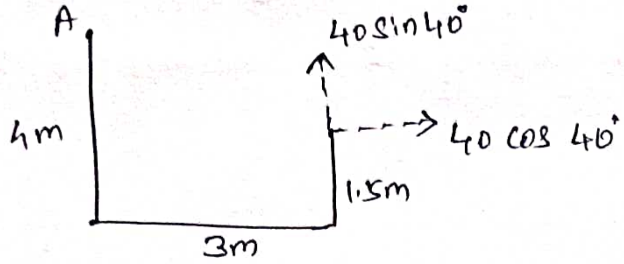


Ex: Find the moment about 'A' for the force shown in given diagram below.



Solution:

First Resolve the forces,

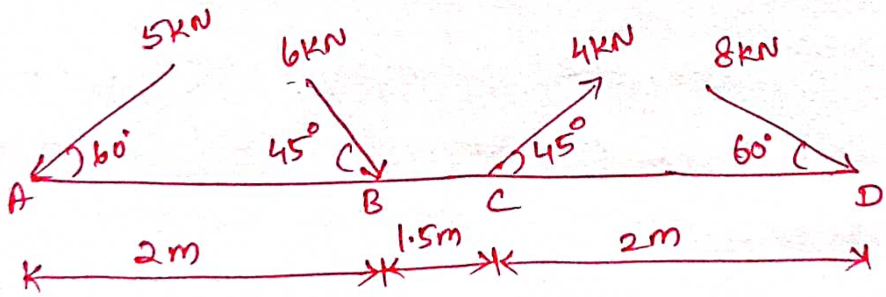


Taking moment about A,

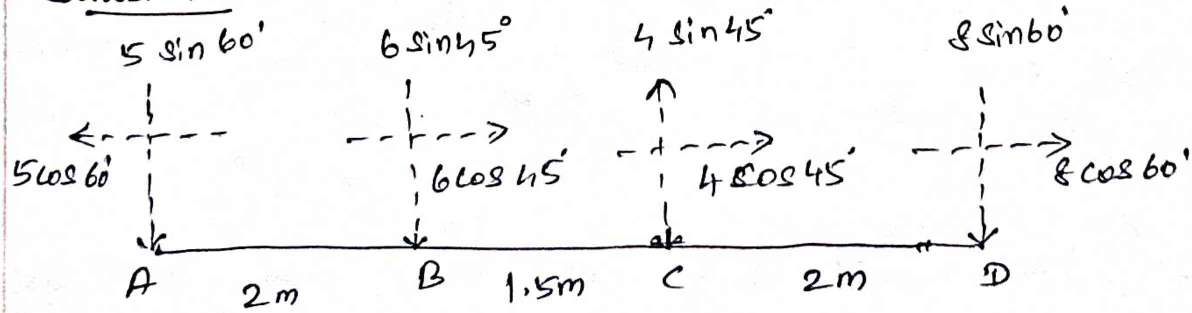
$$\Sigma M_A (\circlearrowright +) = (40 \sin 40^\circ \times 3) + [40 \cos 40^\circ \times (4-1.5)]$$

$$= 153.74 \text{ KNm} \circlearrowright$$

Ex: A system of forces acts on a weightless beam as shown in figure below. Find the magnitude of the resultant and the location of the point where the resultant meets the beam.



Solution:



∴ Resolving the forces horizontally,

$$\Sigma H = -5 \cos 60^\circ + 6 \cos 45^\circ + 4 \cos 45^\circ + 8 \cos 60^\circ$$

$$\Sigma H = 8.57 \text{ KN} (\rightarrow)$$

Resolving the forces vertically.

$\Sigma V = -5\sin 60^\circ - 6\sin 45^\circ + 4\sin 45^\circ - 8\sin 60^\circ$

$\Sigma V = -12.673 (\downarrow)$ kN

∴ magnitude of Resultant,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

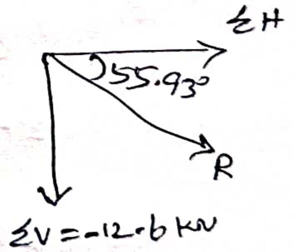
$$\boxed{R = 15.3\,kN}$$

∴ Direction of Resultant,

$$\tan\alpha = \frac{\Sigma V}{\Sigma H}$$

$$\alpha = \tan^{-1}\left[\frac{12.673}{8.57}\right]$$

$$\boxed{\alpha = 55.93^\circ}$$



∴ Location of Resultant (R).

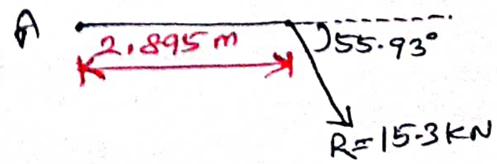
Taking moment about (A),

$\Sigma M_A (\circlearrowright +) = -(8\sin 60 \times 5.5) + (4\sin 45 \times 3.5) - (6\sin 45 \times 2)$

$\Sigma M_A = -36.69$ kNm

$\Sigma M_A = 36.69 (\circlearrowleft)$ (← direction notice the anticlockwise)

By Varignon's theorem,



$\Sigma M_A = \Sigma V \times x$

$$\Rightarrow \quad x = \frac{\Sigma M_A}{\Sigma V} = \frac{36.69}{12.673}$$

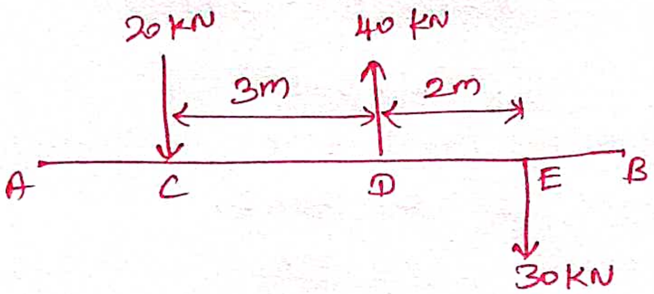
$$\boxed{x = 2.895\,m \text{ from } A}$$

K. Veerapandian, AP/mech.

Ex: A coplanar parallel force system consisting of three forces act on a rigid bar AB as given diagram.

(a) Determine the simplest equivalent action for the force system.

(b) If an additional force of 10 kN acts along the bar from A to B, what would be the simplest equivalent action.



Solution:

(a) Simplest equivalent force;

Resolving force horizontally,

$\Sigma H = 0$ (∴ There was no horizontal forces)

Resolving forces vertically,

$\Sigma V = -30 + 40 - 20$

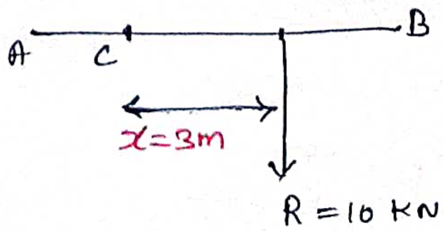
$\Sigma V = -10$ kN (↓)

$\Sigma V = 10$ (downward force)

∴ Resultant force (R),

$R = \sqrt{\Sigma H^2 + \Sigma V^2}$

$\boxed{R = 10 \text{ kN}}$



Location of Resultant forces,

Taking moment about 'C'

$\Sigma M_c = (40 \times 3) - (30 \times 5)$

$\Sigma M_c = -30$ kNm [↻](CW)

The direction of 'R' is known(↓)

$\therefore \Sigma M_c = 30$ kNm (CW)

∴ To have clockwise moment,

R should act at right side of 'c'.

Let x is distance of R from C.

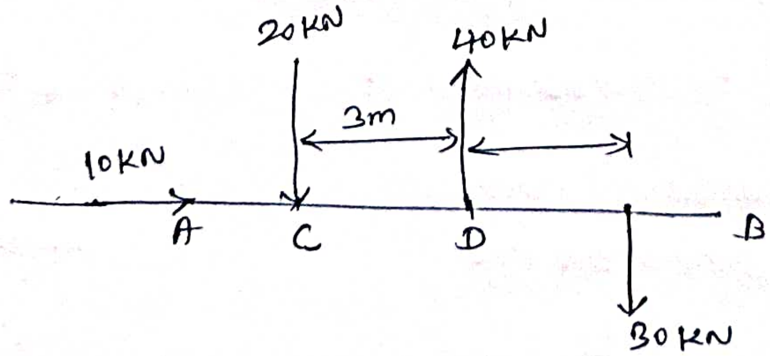
∴ By using Varignon's theorem,

$\Sigma M_c = R \times x$

$\Rightarrow x = \frac{\Sigma M_c}{R} = \frac{30}{10}$

$\boxed{x = 3 \text{ m from C}}$

(b) Simplest Equivalent force after the additional force. (1 kN acts from A to B)



∴ Resolving force horizontally,

$\Sigma H = 10$ kN

Resolving the forces vertically,

$$\Sigma V = -20 + 40 - 30$$

$$\Sigma V = -10\ KN\ (\downarrow)$$

∴ Magnitude of Resultant force,

$$R = \sqrt{\Sigma H^2 + \Sigma V^2}$$

$$= \sqrt{10^2 + (-10)^2}$$

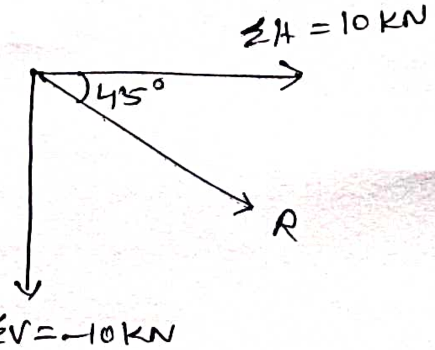
$$\boxed{R = 14.14\ kN}$$

∴ Direction of Resultant force,

$$\tan\alpha = \frac{\Sigma V}{\Sigma H}$$

$$\alpha = \tan^{-1}\left(\frac{10}{10}\right)$$

$$\boxed{\alpha = 45°}$$



∴ Taking moment about 'C'

$$\Sigma M_C = (-20 \times 0) + (40 \times 3) - (30 \times 5)$$

$$\Sigma M_C = -30\ KNm\ [\circlearrowright]\ (CW)$$

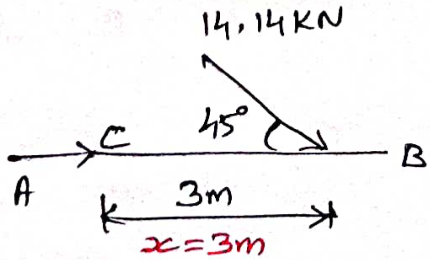
To have clockwise moment 'R' should be at right side of 'C'.

By Varignon's theorem,

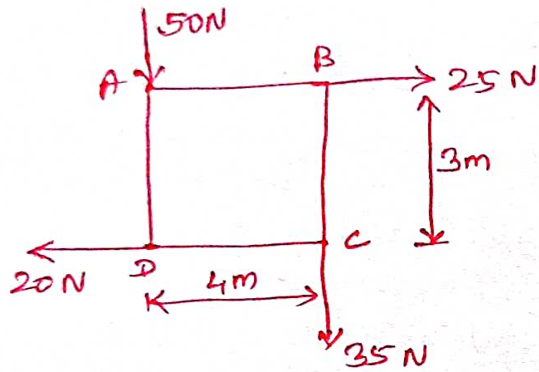
$$\Sigma M_C = \Sigma V \times x$$

$$x = \frac{\Sigma M_C}{\Sigma V} = \frac{30}{10}$$

$$\boxed{x = 3\ m \text{ from } C}$$



Ex:- Determine the magnitude, direction and position of a single force 'P' which keeps in equilibrium the system of forces acting at the corners of a rectangular block as given diagram below. The position of force P may be stated by references to axes with origin O and coinciding with the edges of the block.



Solution:

Lets 'E' be the equilibrant,

which is equal in magnitude but opposite in direction.

∴ Algebric sum of Horizontal forces,

$$\Sigma H = 25 - 20 = 5\,N\ (\rightarrow)$$

∴ Algebric sum of vertical forces,

$$\Sigma V = -50 - 35$$

$$= -85\,N\ (\downarrow)$$

∴ Magnitude of Resultant force,

$$R = \sqrt{\Sigma H^2 + \Sigma V^2}$$

$$= \sqrt{5^2 + 85^2}$$

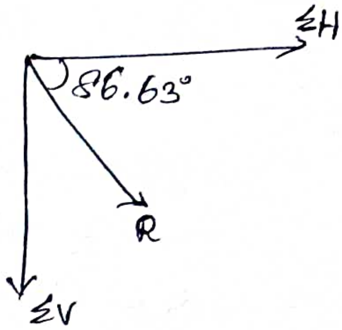
$$\boxed{R = 85.15\,N}$$

Direction of Resultant force,

$$\tan \alpha = \frac{\Sigma V}{\Sigma H} = \frac{85}{5}$$

$$\alpha = \tan^{-1}\left[\frac{85}{5}\right]$$

$$\boxed{\alpha = 86.63^\circ}$$



Location of Resultant force,
(Taking moment about D)

$$\Sigma M_D = -(35\times4) - (25\times3)$$

$$= -215 \text{ Nm} \quad (\text{cw})$$

Now, $R = 85.15$ N

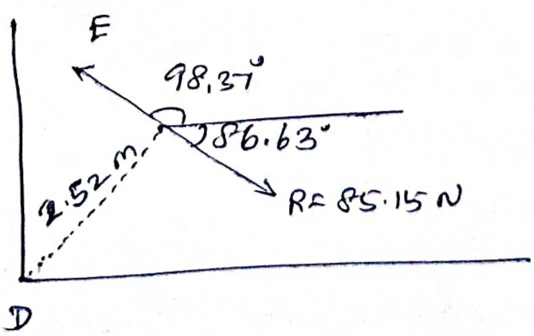
To have cw moment, R should act above 'D'

Now, using Varigon's theorem,

$$\Sigma M_D = R \times d$$

$$d = \frac{\Sigma M_D}{R} = \frac{215}{85.15}$$

$$\boxed{d = 2.52 \text{ m}}$$



$180^\circ - 86.63^\circ = 98.37^\circ$

The equilibrium E = 85.15 N acting at 'O'
with angle 98.37°