SEMBODAI RUKMANI VARATHARAJAN ENGINEERING COLLEGE

SEMBODAI-614 809

DEPARTMENT OF MECHANICAL ENGINEERING

ACADEMIC YEAR 2023-2024/ODD SEMESTER

Subject Code : ME3351

Subject Name : ENGINEERING MECHANICS

Unit Name : UNIT I STATICS OF PARTICLES

Year/Semester : II/III

Prepared by

Faculty Name : VEERAPANDIAN.K

Designation : Assistant Professor/MECH

ME3351 ENGINEERING MECHANICS.

UNIT. I. STATICS OF PARTICLES.

Syllabus:

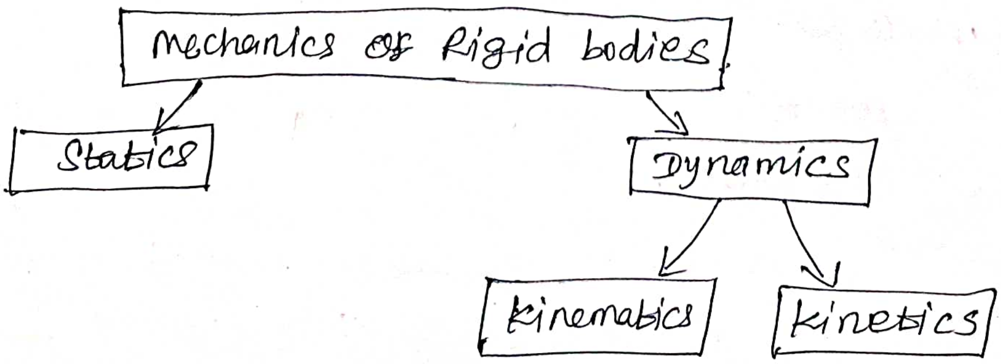
Fundamental Concepts and particles, Systems of units, method of problem Solutions, Statics of particles - Forces in a plane, Resultant of forces, Resolution of a force into components, Rectangular components of force, Unit vectors. Equilibrium of a particles - Newton's first law of motion - Space and free body diagrams, Forces in space, Equilibrium of particle in space.

Engineering mechanics:

Engineering mechanics is defined as the branch of Science which deals with the behaviour of a body with the State of rest or motion, Subjected to the action of forces.

Three branches of EM:

(i) mechanics of Rigid bodies.
(ii) mechanics of deformable bodies.
(iii) Mechanics of fluids.



Statics:

* Study of body at rest.

Ex:

(i) Member of forces in a truss, subjected to some external loads.

(ii) Support reactions of stationary beam subjected to some external load.

Dynamics:

* Study of a body in motion.

Ex:

(i) Force applied on brakes,

- Kinematics:

  * Study of a body in motion without considering the forces, that cause the motion.

- Kinetics:

  * Study of a body in motion, with considering the forces, that cause the motion.

Particle:

A infinitesimal portion of matter is called as a particle.

* Matter is a substance made up of various types of particles that occupies physical space and has inertia.

Mass

The quantity of matter Contained in a body.

Weight:

The force with which the body is attracted towards the centre of earth is called weight.

Weight = mass of body × acceleration due to gravity

$$w = m \times g$$

UNITS OF MEASUREMENT:

→ A physical quantity measured by Comparing the Sample with a known Standard amount. The reference in the measurement of physical quantity Called a unit.

(Or)

The unit is a standard quantity of the Same kind with which a physical quantity is Compared for measuring it.

Two types:

1) Basic units ⇒ ex: mass, length, time
2) Derived units ⇒ Area, speed, velocity, Volume.

System of units:

i) FPS - Foot, Pound, second.
ii) CGS - centimetre, Gram, second.
iii) MKS - Metre, Kilogram, second.
iv) SI - System of International.

Important SI Units:

1) Length – m
2) Area – m^2
3) Density – kg/m^3
4) Velocity – m/s
5) Force – N
6) Accelaration – m/s^2
7) Work – Nm (Joule)
8) power – Nm/sec (Watt)
9) Energy – Nm (Joule)
10) pressure – N/m^2
11) Mass – kg
12) weight – N
13) Time – s

Laws of mechanics:

First law:

A particle remain in its position if the resultant force acting on the particle is zero.

Second law:

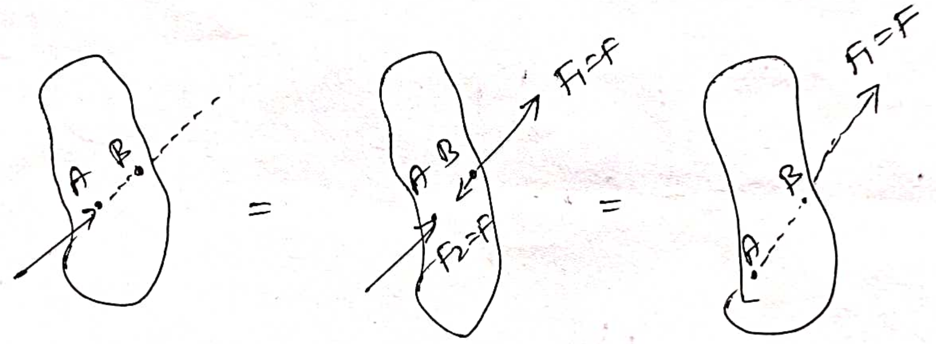
Accelaration of the particle will be proportional to the resultant force and in the same direction, if the resultant force is not zero.

Third Law:

Action and reaction forces between the interacting bodies are in the same line of action, equal in magnitude, but acts in the opposite direction.

Principle of Transmissibility of forces:

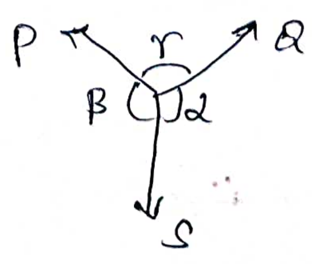
If a force acts at any point on a rigid body it may also be considered to act at any other point on it's line of action.



Lami's theorem;

" If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle b/w the other two "

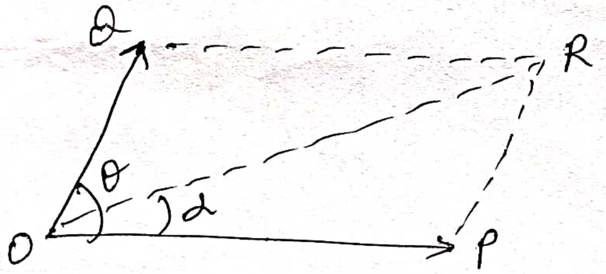
$$\Rightarrow \frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{S}{\sin\gamma}$$



Parallelogram law of forces:

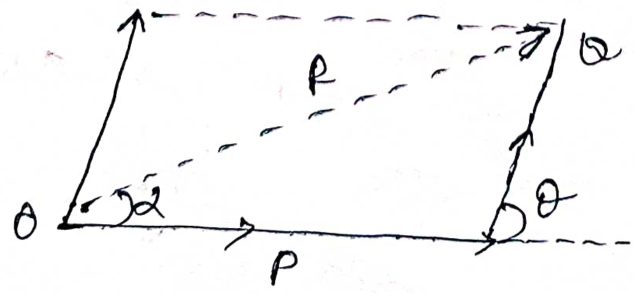
"If two forces acting simultaneously at a point be represented in magnitude and direction by the two adjacent sides of Parallelogram, then the resultant of these two forces is represented in magnitude and direction by the diagonal of that parallogram originating from that point"

$$\Rightarrow \quad R = \sqrt{P^2 + Q^2 + (2PQ\cos\theta)}$$

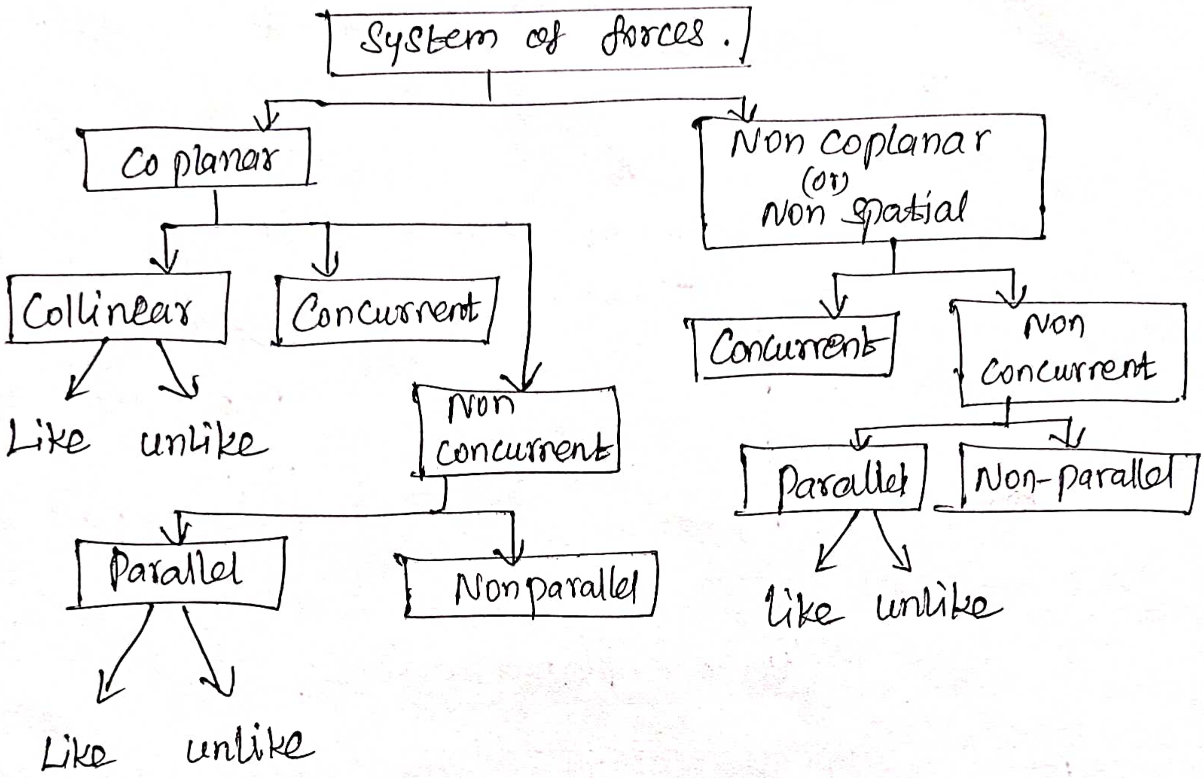


Triangle law of forces:

"If two forces acting at a point are represented by the two sides of triangle taken in order, then their resultant force is represented by the third side taken in opposite order".



System of forces:



Co-planar forces:

* all forces act in one plane.

Non- Coplanar Forces:

* The forces do not act in one plane.

Collinear Forces:

* Forces acts on a common line of action are called collinear forces.

→ like collinear - Forces act in Same direction

→ unlike collinear - Forces act in opposite direction.

Concurrent forces:

* Forces intersects at a common point.

↓

parallel forces:

* Line of action of forces are parallel to each other.

↑ ↑   ↓ ↑

Like parallel   unlike parallel forces.

Like collinear coplanar forces:

Forces acting in the same direction, lies on a common line of action and acts in a single plane.

Unlike collinear coplanar forces:

forces acting in opposite direction, lies on a common line of action, acts in a single plan.

Coplanar concurrent forces:

forces intersects at a common point and lies in a single plane.

Non coplanar concurrent forces:

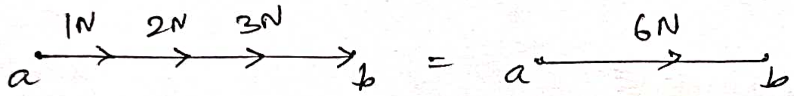
Forces intersects at one point, but their lines of action do not lie on the same plane.

Statics of particles in Two dimensions - Resultant force.

Resultant force:

If a number of forces acting on a particle simultaneously are replaced by a single force, that is called Resultant force.

ex:



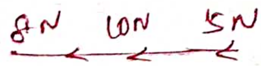
Resultant force systems determined by;

1) Analytical method,
2) Graphical method.

Signs:

Upward vertical force (↑ +)
Downward vertical force (↓ -)
Horizontal left to right (⟶ +)
Horizontal right to left (⟵ -)

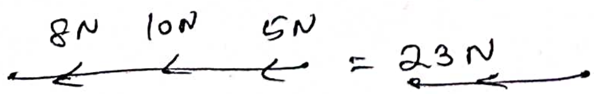
Ex: Find the resultant force of the collinear forces, shown in diagram,



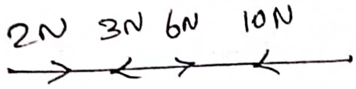
Solution:

Resultant force $R = 8+10+5$

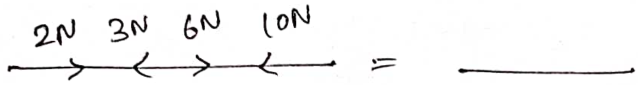
$= 23\,N.$



Ex 2: Find the resultant of collinear forces, shown in the diagram.



Solution:



$\therefore$ Resultant force $= 2 - 3 + 6 - 10$

$= -5\,N$

$\therefore$ Negative sign indicates force towards left.

Ex: 3 Two concurrent forces of 12N and 18N are acting at an angle of 60°. Find the resultant force.

Solution:

Let $P = 12\,N$

$Q = 18\,N$, $\theta = 60^\circ$

$\therefore$ Resultant force $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$= \sqrt{12^2 + 18^2 + (2 \times 12 \times 18 \times \cos 60^\circ)}$

$$\boxed{R = 26.15\,N}$$

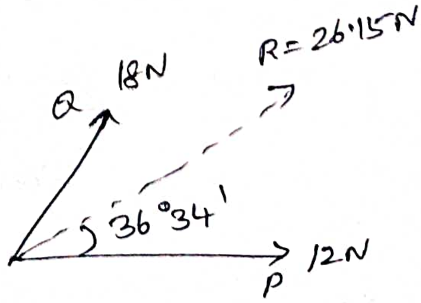
$\therefore$ Inclination of Resultant force, P

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$$= \frac{18\sin 60^\circ}{12 + (18\cos 60^\circ)}$$

$\alpha = \tan^{-1}(0.742)$

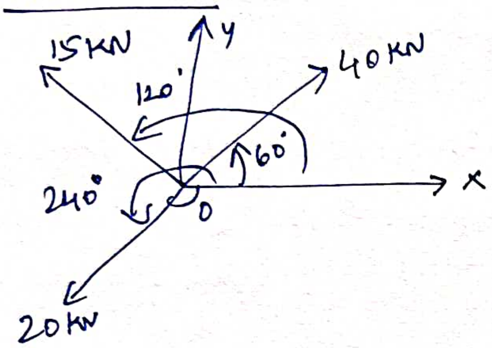
$$\boxed{\alpha = 36^\circ 24'}$$



1.

Ex: Three forces of magnitude 40 KN, 15 KN and 20 KN are acting at a point 'O' as shown in figure. The angles made by 40KN, 15 KN and 20KN forces with x-axis are 60°, 120°, 240° respectively. Determine the magnitude and resultant of force.

Given data:



$F_1 = 40$ KN, $\theta_1 = 60°$

$F_2 = 15$KN, $\theta_2 = 120°$

$F_3 = 20$KN, $\theta_3 = 240°$

Solution:

Apply $\Sigma H = 0$,

$$\Sigma H = 40\cos 60° + 15\cos 120° + 20\cos 240°$$

$$\Sigma H = 2.5 \text{ KN}$$

Apply $\Sigma V = 0$,

$$\Sigma V = 40\sin 60° + 15\sin 120° + 20\sin 240°$$

$$\Sigma V = 30.31 \text{ KN}$$

Resultant $R = \sqrt{\Sigma H^2 + \Sigma V^2}$

$$R = \sqrt{2.5^2 + 30.31^2}$$

$\boxed{R = 30.41 \text{ KN}}$ (magnitude of Resultant)

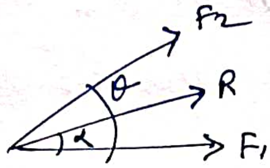
Direction of Resultant:

$$\tan\theta = \frac{\Sigma V}{\Sigma H} \Rightarrow \tan\theta = \left[\frac{30.31}{2.5}\right]$$

$$\theta = \tan^{-1}\left[\frac{30.31}{2.5}\right] \Rightarrow \boxed{\theta = 85.28°}$$

Ex: Two forces of magnitude F_1 and F_2 are acting at a point. They are such that, if the direction of one is reversed than the resultant turns through a right angle, show that $F_1 = F_2$.

Solution:



By Parallelogram law,

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta} \longrightarrow (1)$$

Thus, given condition is, F_2 is reversed the resultant turns through right angle. (ie α is changed to $90+\alpha$ when $F_1 = -F_2$)

$$\tan(90+\alpha) = \frac{-F_2 \sin\theta}{F_1 - F_2 \sin\theta} \qquad (\because F_2 \text{ reversed})$$

$$-\cot\alpha = \frac{-F_2 \sin\theta}{F_1 - F_2 \cos\theta} \qquad \because \tan(90+\alpha) = -\cot\alpha$$

$$\cot\alpha = \frac{F_2 \sin\theta}{F_1 - F_2 \cos\theta}$$

$$\tan\alpha = \frac{F_1 - F_2 \cos\theta}{F_2 \sin\theta} \qquad \left[\because \frac{1}{\cot\alpha} = \tan\alpha\right]$$

$$\longrightarrow (2)$$

equating (1) and (2),

$$\frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta} = \frac{F_1 - F_2 \cos\theta}{F_2 \sin\theta}$$

$$F_2^2 \sin^2\theta = (F_1 + F_2 \cos\theta)(F_1 - F_2 \cos\theta)$$

$$F_2^2 \sin^2\theta = F_1^2 - F_2^2 \cos^2\theta \qquad \because (a+b)(a-b) = a^2 - b^2$$

$$F_2^2 \sin^2\theta + F_2^2 \cos^2\theta = F_1^2$$

$$F_2^2 (\sin^2\theta + \cos^2\theta) = F_1^2$$

$$F_2^2 (1) = F_1^2$$

$$\Rightarrow \boxed{F_2 = F_1} \quad \therefore \text{Hence proved.}$$

Ex: The concurrent forces acts at an angle of $30°$. The resultant force is 15N and one of the force is 10N. Find the other force.

Solution:

Let $R = 15N$

$P = 10N$, $\theta = 30°$, $Q = ?$

$\therefore R = \sqrt{P^2 + Q^2 + (2PQ\cos\theta)}$

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$15^2 = 10^2 + Q^2 + (2 \times 10 \times Q\cos 30°)$

$15^2 = 100 + Q^2 + 17.32Q$

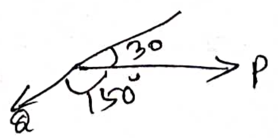
$\Rightarrow Q^2 + 17.32Q - 125 = 0$

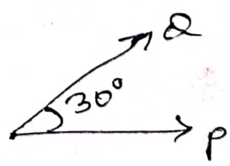
$\Rightarrow Q = \dfrac{-17.32 \pm \sqrt{(17.32)^2 - (4 \times 1(-125))}}{2 \times 1}$

$= \dfrac{-17.32 \pm 28.28}{2}$

$= -22.8N$ (or) $5.48N$

$-22.8N$ is in the reverse direction

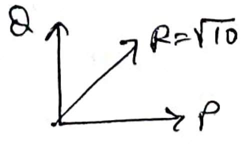




$\therefore$ Hence $\boxed{Q = 5.48N}$

Ex: Find the magnitude of two forces, such that if they act at right angles, their resultant is $\sqrt{10}$ N. But if they acts at $60°$, their resultant is $\sqrt{13}$ N.

Solution: Case (i) $\theta = 90°$

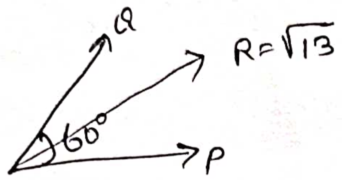


$$R^2 = P^2 + Q^2 + 2PQ\cos 90^\circ$$

$$R^2 = P^2 + Q^2.$$

$$\therefore \; 10 = P^2 + Q^2 \rightarrow (1)$$

Case (ii) ($\theta = 60^\circ$)



$$R^2 = P^2 + Q^2 + 2PQ\cos 60^\circ$$

$$R^2 = P^2 + Q^2 + PQ$$

$$13 = P^2 + Q^2 + PQ \rightarrow (2)$$

Subs eqn (1) in (2)

$$13 = P^2 + Q^2 + PQ$$

$$13 = 10 + PQ$$

$$\therefore \; PQ = 3$$

Using $(P+Q)^2 = P^2 + Q^2 + 2PQ$

$$= 10 + (2 \times 3)$$

$$P+Q = \sqrt{16}$$

$$P+Q = 4 \longrightarrow (3)$$

$$(P-Q)^2 = P^2 + Q^2 + 2PQ$$

$$= 10 - (2 \times 3)$$

$$= 4$$

$$\therefore \; P - Q = 2 \longrightarrow (4)$$

Solving the equations, (iii) and (iv)

$$P+Q = 4$$

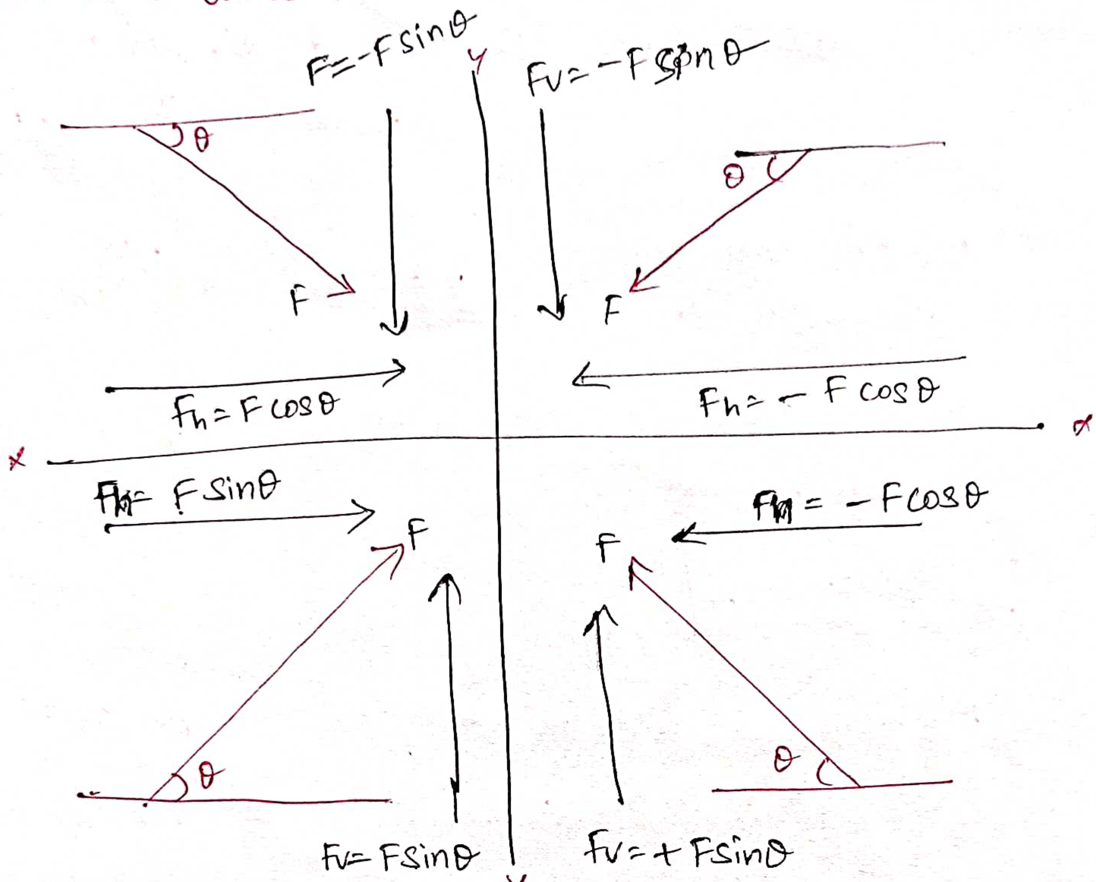
$$P-Q = 2$$

$$P =$$

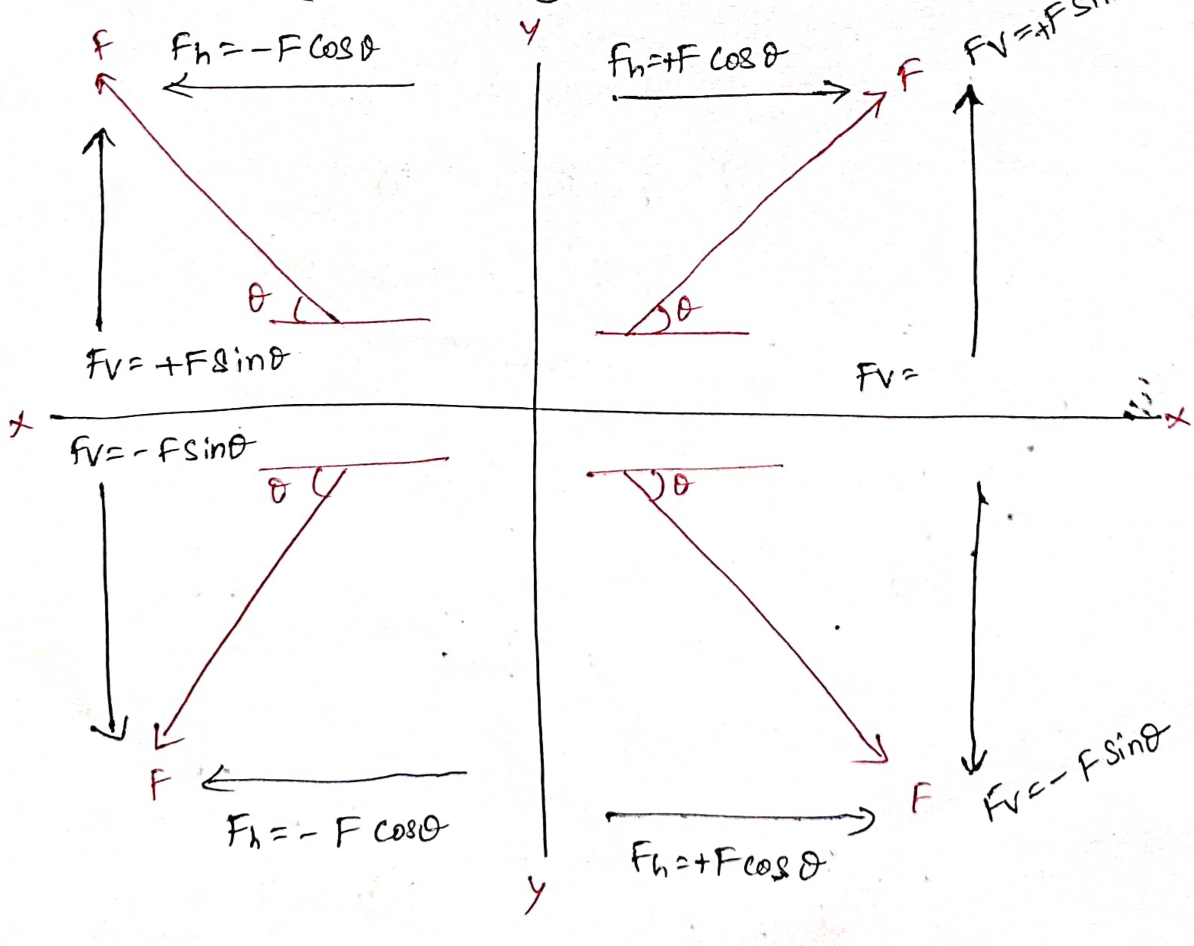
Ans: $P =$

$Q =$

Resultant force more than 2 concurrent forces:

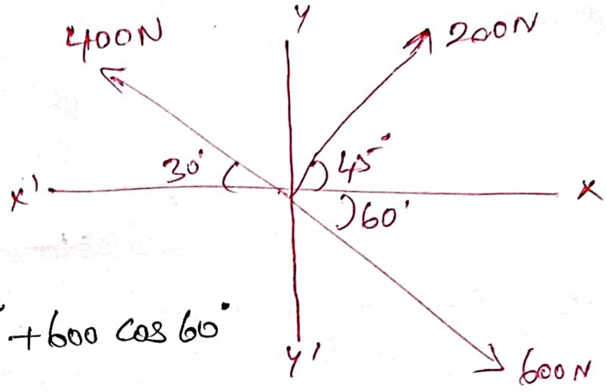


(forces acting towards origin)



Ex. Three coplanar concurrent forces are acting at a point as given diagram. Determine the resultant in magnitude and direction.

Solution:



Algebric sum of Horizantal forces,

$$\Sigma H = 200\cos 45^\circ - 400\cos 30^\circ + 600\cos 60^\circ$$

$$= 141.42 - 346.41 + 300$$

$$\boxed{\Sigma H = 95.01\,N}$$

Algebric sum of ~~Horizontal~~ vertical forces,

$$\Sigma V = 200\sin 45^\circ + 400\sin 30^\circ - 600\sin 60^\circ$$

$$\boxed{\Sigma V = -178.2\,N}$$

∴ Magnitude of Resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{95.01^2 + (-178.2)^2}$$

$$\boxed{R = 201.95\,N}$$

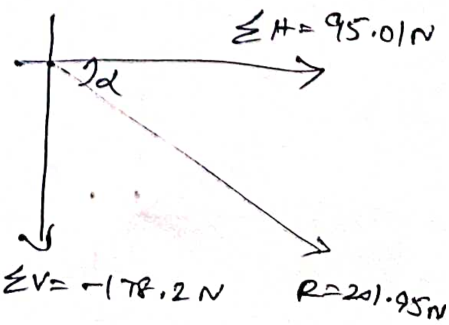
∴ Direction of Resultant force,

$$\tan\alpha = \frac{\Sigma V}{\Sigma H}$$

$$\alpha = \tan^{-1}\left[\frac{\Sigma V}{\Sigma H}\right]$$

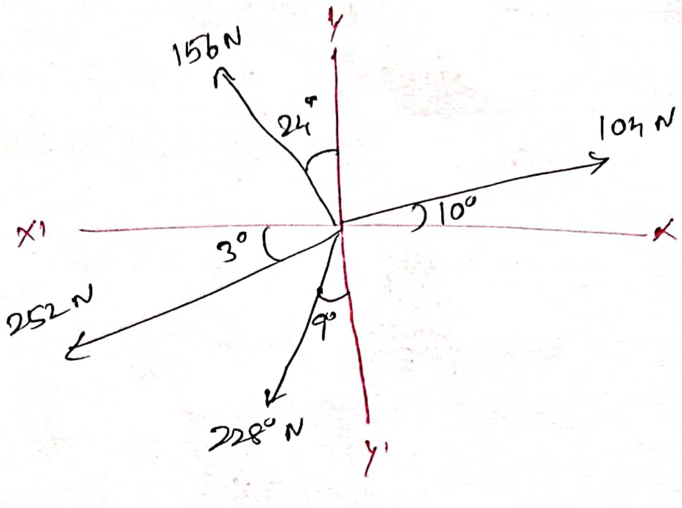
$$= \tan^{-1}\left[\frac{+178.2}{95.01}\right]$$

$$\boxed{\alpha = 61.93^\circ}$$



(∵ Don't put sign, It mention direction only)

Ex: The four Coplanar forces are acting at point in given diagram, Determine the resultant in magnitude and direction.



Solution:

$F_1 = 104N$, $\theta_1 = 10'$

$F_2 = 156N$, $\theta_2 = 66°$

$F_3 = 252N$, $\theta_3 = 3°$

$F_4 = 228N$, $\theta_4 = 81°$

∴ Resolving the horizontal forces,

$$\Sigma H = 104 \cos 10° - 156 \cos 66' - 252 \cos 3° - 228 \cos 81°$$

$$= 102.4 - 63.44 - 251.64 - 35.66$$

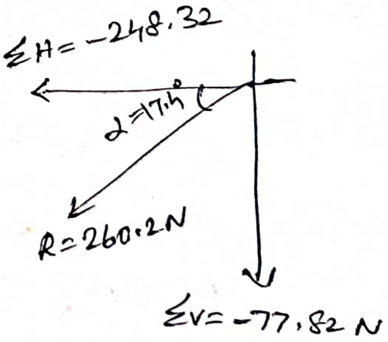
$$\boxed{\Sigma H = -248.32 \ N}$$

∴ Resolving the forces vertically,

$$\Sigma V = 104 \sin 10° + 156 \sin 66° - 252 \sin 3° - 228 \sin 81°$$

$$= 18.06 + 142.5 - 13.18 - 225.2$$

$$\boxed{\Sigma V = -77.82 \ N}$$



∴ Magnitude of Resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(-248.32)^2 + (-77.82)^2}$$

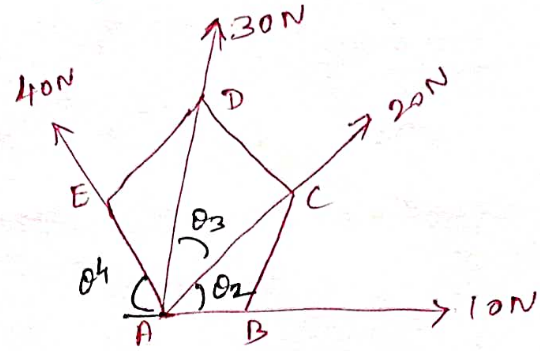
$$\boxed{R = 260.2 \ N}$$

∴ Direction of Resultant force,

$$\tan \alpha = \frac{\Sigma V}{\Sigma H}$$

$$\alpha = \tan^{-1}\left[\frac{\Sigma V}{\Sigma H}\right] \Rightarrow \alpha = \tan^{-1}\left[\frac{77.82}{248.32}\right] \Rightarrow \boxed{\alpha = 17.4°}$$

Ex: The forces 10N, 20N, 30N and 40N are acting on one of the vertices of a regular pentagon, towards the other ~~follow~~ vertices taken in order. Find the magnitude and direction of the resultant force R.



Solution:

Interior angle of polygon is $(2n-4)\times 90^\circ$

$\Rightarrow$ For polygon,

$(2n-4)\times 90 \quad \because n=5$

$(2\times 5-4)\times 90$

$\Rightarrow 540^\circ$

$\therefore$ Each included angle, $\frac{540}{5} = 108^\circ$

$\Rightarrow$ where, we get three angles, by B, C, D, E with A.

$\therefore$ angle $\theta = \frac{108^\circ}{3} = 36^\circ$

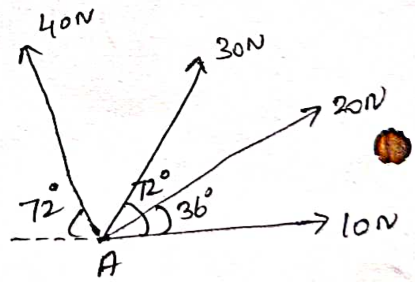
$\Rightarrow$ $F_1 = 10N \quad \theta_1 = 0^\circ$

$F_2 = 20N \quad \theta_2 = 36^\circ$

$F_3 = 30N \quad \theta_3 = 2\times 36^\circ = 72^\circ$

$F_4 = 40N \quad \theta_4 = 180 - (3\times 36)$

$= 72^\circ$



$\therefore$ Resolving the forces in Horizantal direction, $(\because \cos 0^\circ = 1)$

$$\Sigma H = 10\cos 0^\circ + 20\cos 36^\circ + 30\cos 72^\circ - 40\cos 72^\circ$$

$$\boxed{\Sigma H = 23.09\ N}$$

$(\because \sin 0^\circ = 0)$

$\therefore$ Resolving the forces in vertical direction,

$$\Sigma V = 10\sin 0^\circ + 20\sin 36^\circ + 30\sin 72^\circ + 40\sin 72^\circ$$

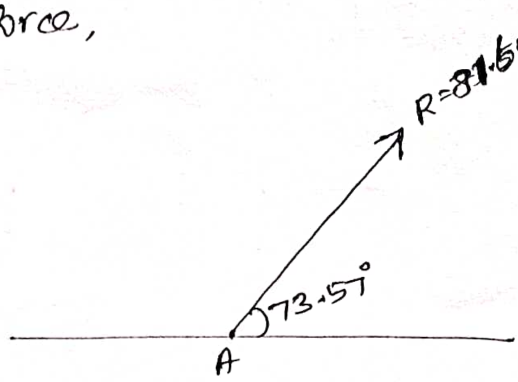
$$\boxed{\Sigma V = 78.33\ N}$$

$\therefore$ Direction of Resultant force,

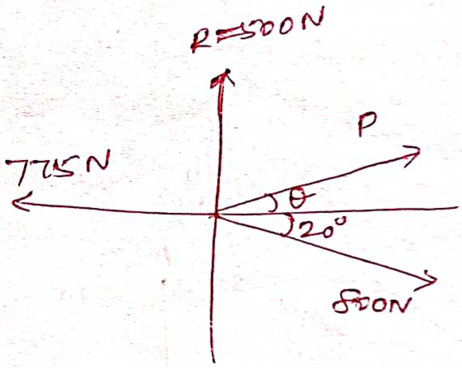
$$\alpha = \tan^{-1}\left[\frac{\Sigma V}{\Sigma H}\right]$$

$$\alpha = \tan^{-1}\left[\frac{78.33}{23.09}\right]$$

$$\boxed{\alpha = 73.57^\circ}$$



Ex: The three coplanar forces are acting at a point as given diagram. One of the forces is unknown and its magnitude is shown by P. The resultant is having a magnitude 500N and is acting along y axis (positive direction). Determine the unknown forces P and its inclination with X axis.



Solution:

Let

$F_1 = P \quad , \theta_1 = \theta$

$F_2 = 775N \quad , \theta_2 = 0$

$F_3 = 800N, \quad \theta_3 = 20^\circ$

Where, $R = 500\ N$ in y direction.

So, $\Sigma V = 500N$

$\Sigma H = 0,$

$\therefore$ Resolving the forces horizantally,

$$0 = \Sigma H = P\cos\theta - 775\cos 0^\circ + 800\cos 20^\circ$$

$$P\cos\theta = 23.25N \longrightarrow (1)$$

$\therefore$ Resolving the forces vertically,

$$500 = \Sigma V = P\sin\theta + 775\sin 0^\circ - 800\sin 20^\circ$$

$$500 = P\sin\theta - 273.61$$

$$P\sin\theta = 773.61N \longrightarrow (2)$$

Solving the equations ① & ②,

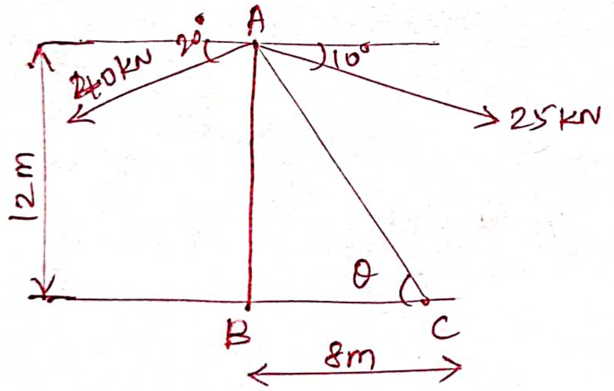
$$\frac{P\sin\theta}{P\cos\theta} = \frac{773.61}{23.25}$$

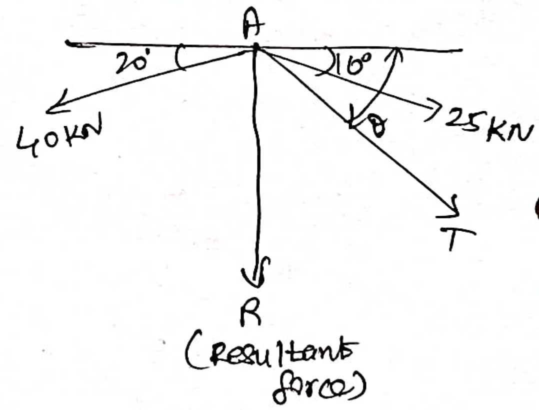
$$\tan\theta = 33.27$$

$$\boxed{\theta = 88.27^\circ}$$ Subs in eqn ①

$$\Rightarrow \boxed{P = 770.13\,N}$$

Ex: Two cables which have known tensions are attached to the top of a tower AB. A third cable AC is used as guy wire as given in diagram. Determine the tension in the AC, if the resultant of the forces exerted at A by the three cables acts vertically downwards.





Solution:

$\therefore$ $\angle ABC$, $\tan\theta = \frac{AB}{BC} = \frac{12}{8}$

$$\Rightarrow \boxed{\theta = 56.30^\circ}$$

$\therefore$ Resultant Force is in y direction.

$$\Rightarrow \Sigma V = R$$

$$\Sigma H = 0,$$

$\therefore$ $\Sigma H = 0$, Resolving the forces horizontally,

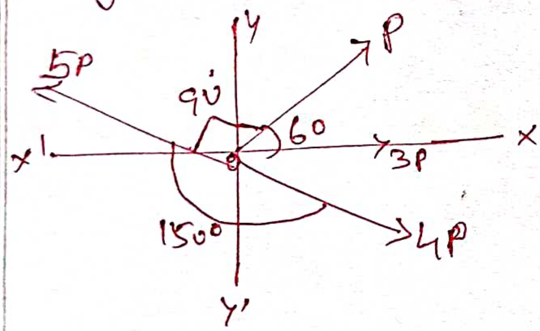
$\Sigma H = 25 \cos 10^\circ - 40 \cos 20^\circ + T \cos 56.3^\circ$

$0 = 24.62 - 37.58 + 0.555T$

$\Rightarrow \boxed{T = 23.35N}$

$\therefore$ Tension in the guy wire is <u>23.35 N</u>

Ex: (AU) Find the magnitude and direction of the resultant 'R' of four concurrent forces acting as given in diagram below.



<u>Solution:</u>

where,

Angle of 3P $= 0^\circ$

Angle of P $= 60^\circ$ with horizontal

Angle of 5P $= 180 - 150$

$= 30^\circ$ with horizont

Angle of 4P $= 360 - 90 - 60 - 150$

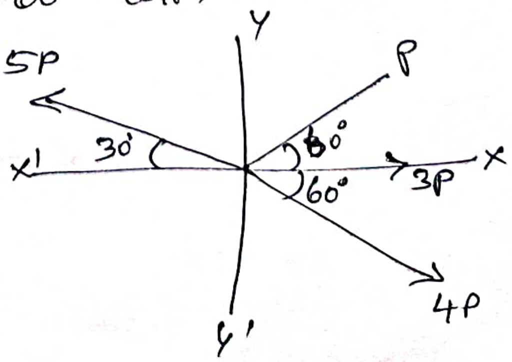
$= 60^\circ$ with horizontal

Let, $F_1 = 3P \quad \theta_1 = 0$

$F_2 = P \quad \theta_2 = 60^\circ$

$F_3 = 5P \quad \theta_3 = 30^\circ$

$F_4 = 4P \quad \theta_4 = 60^\circ$



Algebric Sum of horizantal forces,

$$\Sigma H = 3P\cos 0^\circ + P\cos 60^\circ + 4P\cos 60^\circ - 5P\cos 30^\circ$$

$$\Sigma H = 1.17P$$

Algebric sum of vertical forces,

$$\Sigma V = 3P\sin 0^\circ + P\sin 60^\circ - 4P\sin 60^\circ + 5P\sin 30^\circ$$

$$\Sigma V = -0.098P$$

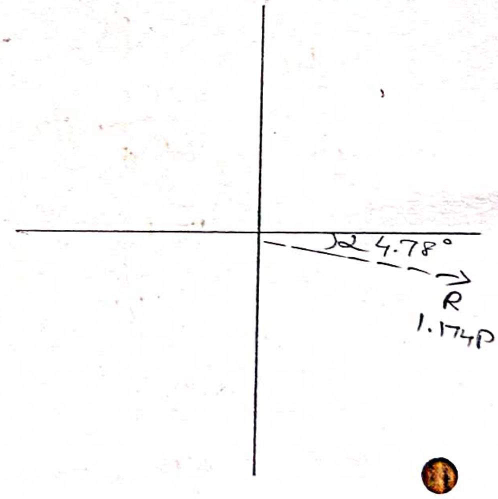
∴ Magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(1.17P)^2 + (0.098P)^2}$$

$$R = 1.174P$$

∴ Direction of Resultant force,

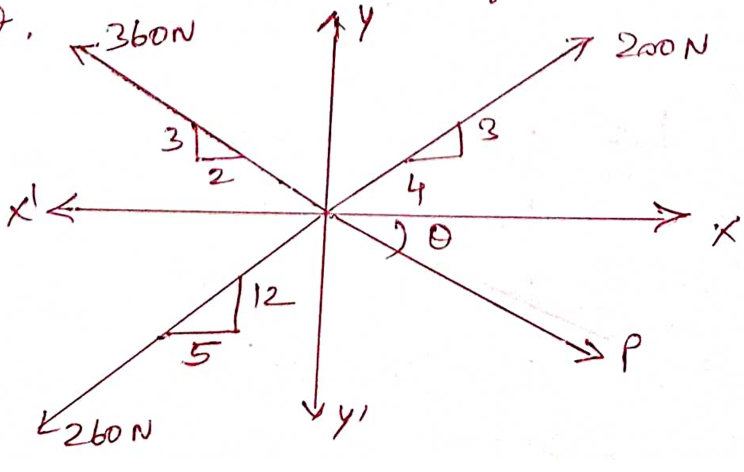


$$\alpha = \tan^{-1}\left[\frac{\Sigma V}{\Sigma H}\right]$$

$$\alpha = \tan^{-1}\left[\frac{0.098P}{1.17P}\right]$$

$$\alpha = 4.78^\circ$$

Ex: The resultant of the force system shown below, is 520N along the negative direction of y axis. Determine P and θ.



Resultant force is 520N, in y direction,

So, $\Sigma V = R = -520\,N$

$\Sigma H = 0$

Let $F_1 = 200N$ $\quad \theta_1 = \tan^{-1}\left[\frac{3}{4}\right] = 36.87^\circ$

$F_2 = P$ $\quad \theta_2 = \theta$

$F_3 = 260N$ $\quad \theta_3 = \tan^{-1}\left[\frac{12}{5}\right] = 67.38^\circ$

$F_4 = 360N$ $\quad \theta_4 = \tan^{-1}(3/2) = 56.31^\circ$

Algebric Sum of Horizontal forces,

$$\Sigma H = 200 \cos 36.87^\circ + P\cos\theta - 260 \cos 67.38^\circ - 360 \cos 56.31^\circ$$

$$0 = \Sigma H = P \cos\theta - 139.69$$

$$\therefore P \cos\theta = 136.69\,N \longrightarrow (1)$$

Algebric Sum of vertical forces,

$$\Sigma V = 200 \sin 36.87 - P\sin\theta - 260 \sin 67.38 + 360 \sin 56.31$$

$$-520 = 120 - P\sin\theta - 240 + 299.53$$

$$\therefore P \sin\theta = 699.53 \longrightarrow (2)$$

eqn (2) $\div$ (1),

$$\Rightarrow \frac{P\sin\theta}{P\cos\theta} = \frac{699.53}{139.69}$$

$$\tan\theta = 5.007$$

$$\Rightarrow \theta = \tan^{-1}(5.007)$$

$$\boxed{\theta = 78.7^\circ}$$

Subs $\theta = 78.7^\circ$ in eqn (1)

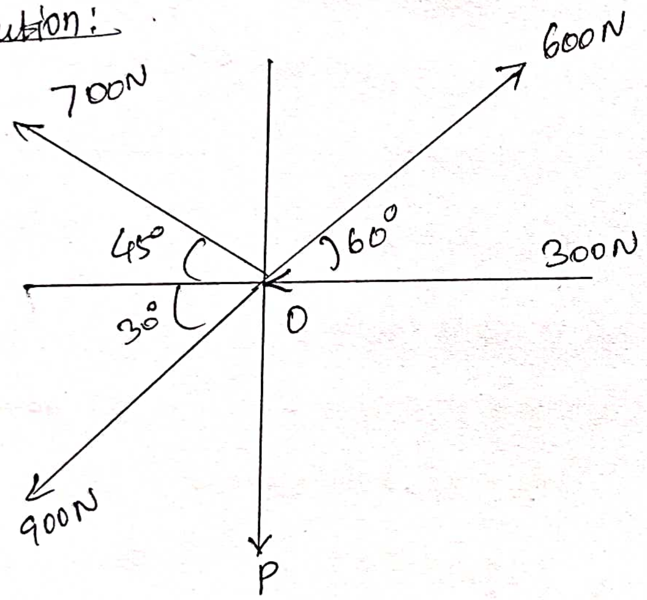
$$P \cos 78.7 = 139.69$$

$$\Rightarrow P = \frac{139.69}{\cos 78.7}$$

$$\boxed{P = 712.9\,N}$$

Ex: (AU) Five forces are acting on a particle. The magnitude of the forces are 300 N, 600 N, 700 N, 900 N and P and their respective angles with horizontal are $0°, 60°, 135°, 210°$ and $270°$. If the vertical component of the all forces is -1000 N, find the value of P. Also calculate the magnitude and the direction of the resultant, assuming that the first force acts towards the point, while all the remaining forces acts away from the point.

Solution:



Solution:

Vertical component of all force is -1000 N.

ie $\Sigma V = -1000$ N

$\therefore$ Algebric sum of vertical components,

$$\Sigma V = 700 \sin 45° + 600 \sin 60° - 900 \sin 30° - P$$

$$-1000 = 494.97 + 519.61 - 450 - P$$

$$\Longrightarrow \boxed{P = 1564.58\ N}$$

$\therefore$ Resultant force,

Algebric sum of horizontal components,

$$\Sigma H = -700 \cos 45° - 900 \cos 30 + 600 \cos 60 - 300$$

$\Sigma H = -1274.94\ N$

$\therefore$ Magnitude of Resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

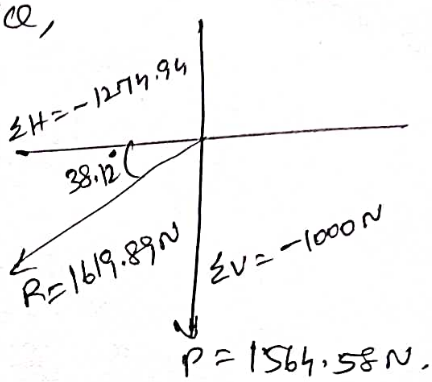
$$\therefore \boxed{R = 1619.89\ N}$$

$\therefore$ Direction of Resultant force,

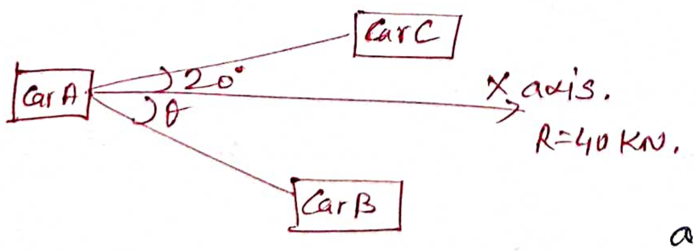
$$\alpha = \tan^{-1}\left[\frac{\Sigma V}{\Sigma H}\right]$$

$$\alpha = \tan^{-1}\left[\frac{1000}{1274.39}\right]$$

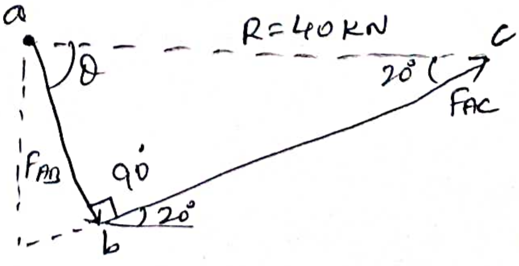
$$\boxed{\alpha = 38.12^\circ}$$



Ex: (AU). A car is pulled by means of two cars as given diagram below. If the resultant of two forces acting on the car A is 40 kN being directed along the positive direction of x-axis, determine the angle θ of the cable attached to the car at B, such that the force in cable AB is minimum. What is the magnitude of force in each cable when this occurs.



Solution:



To find θ or F_{AB} minimum:

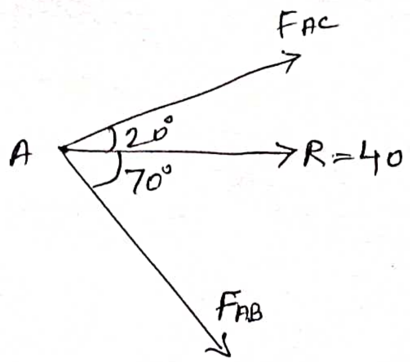
$\therefore$ The angle $\theta = 180 - 90 - 20$

$$\boxed{\theta = 70^\circ}$$

Consider

Forces in Cables, (abc triangle)

apply sine rule,



$$\frac{40}{\sin 90} = \frac{F_{AB}}{\sin 20} = \frac{F_{AC}}{\sin 70}$$

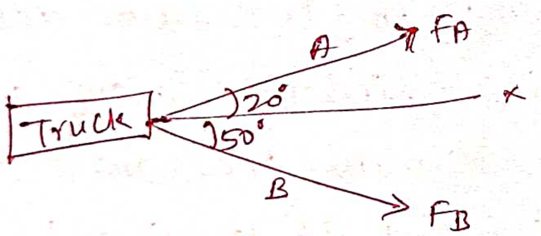
$$\therefore F_{AB} = \frac{40 \sin 20}{\sin 90}$$

$$\boxed{F_{AB} = 13.68 \text{ kN}}$$

$$\therefore F_{AC} = \frac{40 \sin 70}{\sin 90^\circ}$$

$$\boxed{F_{AC} = 37.587 \text{ kN}}$$

Ex: The truck is to be towed using two ropes. Determine the magnitudes of forces F_A and F_B acting on each rope in order to develop a resultant force of 950N directed along the positive x-axis.



Solution:

$$\therefore \Sigma H = 950 \text{ N},$$
$$\Sigma V = 0$$

$\therefore$ Resolving the forces on horizontal,

$$950 = F_A \cos 20^\circ + F_B \cos 50^\circ \longrightarrow (1)$$

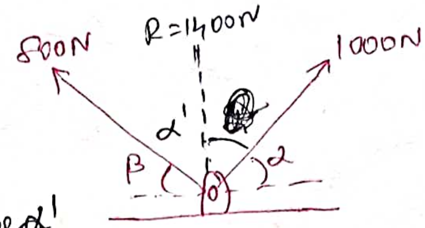
$\therefore$ Resolving the forces on vertical,

$$\Sigma V = 0 = F_A \sin 20^\circ - F_B \sin 50^\circ \longrightarrow (2)$$

$$F_A = \frac{F_B \sin 50^\circ}{\sin 20^\circ} \Rightarrow F_A = 2.23 F_B \text{ Subs in } (1)$$

$$\Rightarrow \boxed{F_B = 346.3 \text{ N}}, \quad \boxed{F_A = 774.5 \text{ N}}$$

Ex: Forces are transmitted by two members as shown in diagram given below. If the resultant of these forces is 1400N directed vertically upward, find angles α and β.



Solution:

$$R^2 = P^2 + Q^2 + 2PQ\cos\alpha'$$

$$1400^2 = 1000^2 + 800^2 + 2(1000)(800)\cos\alpha'$$

$$\Rightarrow \boxed{\alpha' = 78.46^\circ}$$

$$\tan\theta = \frac{Q\sin\alpha'}{P + Q\cos\alpha'}$$

$$\boxed{\theta = 34.05^\circ}$$

$$\alpha = 90 - \theta$$

$$\boxed{\alpha = 55.95^\circ}$$

$$\beta = 180 - \alpha - \alpha'$$
$$= 180 - 55.95 - 78.46$$

$$\boxed{\beta = 45.59^\circ}$$

EQUILIBRIUM OF PARTICLES IN TWO DIMENSIONS.

Equilibrant:

Equilibrant (E) is equal to the resultant force (R) in magnitude and direction, collinear but opposite in nature.

Conditions of equilibrium:

For equilibrium condition of force system, the resultant is zero. ie $R = 0$.

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} \qquad \left[\because \begin{array}{l} \Sigma H = 0 \\ \Sigma V = 0 \end{array}\right]$$

equations of equilibrium in two dimensions:

$\Sigma H = 0$ for horizontal collinear forces,

$\Sigma V = 0$ for vertical collinear forces.

$\Sigma H = 0$ & $\Sigma V = 0$ for concurrent forces.

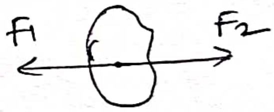
Principle of Equilibrium:

Equilibrium principles are developed from the force law of equilibrium.

ie ($\Sigma F = 0$)

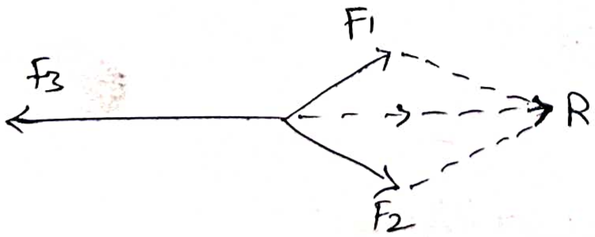
Two force principle:

"If a body subjected to two forces, then the body will be in equilibrium if the two forces are collinear, equal and opposite."



Three force principle:

"If a body subjected to three forces, then the body will be in equilibrium if the resultant of any two forces is equal, opposite and collinear with the third force."



R- Resultant force

F_1-F_2 Force.

$R = F_3$

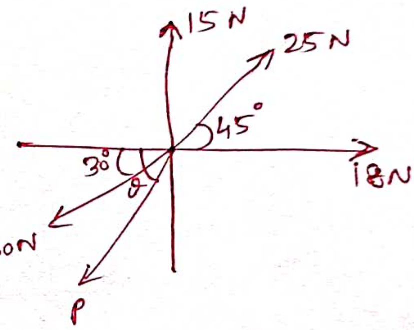
Lami's theorem:

"If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two".

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{S}{\sin\gamma}$$

Ex: The forces shown in diagram. Find the magnitude and direction θ of the unknown force P.

Solution:



∴ Algebric sum of horizontal forces,

$\Sigma H = 0$

$\Sigma H = 25\cos 45° + 18\cos 0 - 30\cos 30 - P\cos\theta$

$P\cos\theta = 9.7 \longrightarrow (1)$

∴ Algebric sum of vertical forces,

$\Sigma V = 0 = 15\sin 90° + 25\sin 45° + 18\sin 0 - 30\sin 30 - P\sin\theta$

$P\sin\theta = 17.68 \longrightarrow (2)$

Divide eqn (2) by (1), we get,

$$\frac{P\sin\theta}{P\cos\theta} = \frac{17.68}{9.7}$$

$\tan\theta = 1.822 \Rightarrow \boxed{\theta = 61.24°}$

Subs θ in (1) we get.

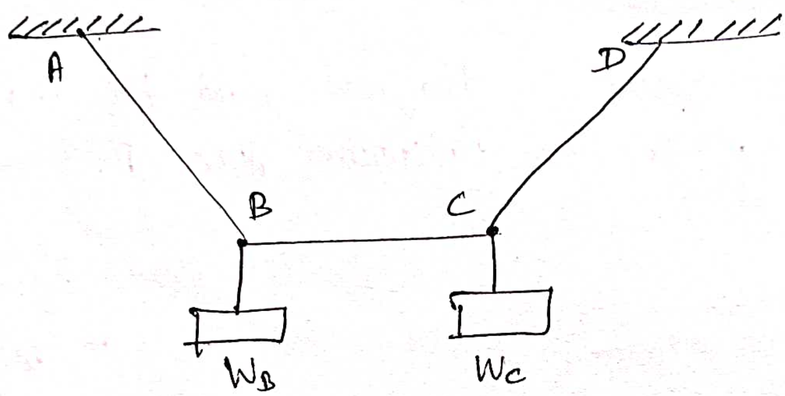
$P\cos(61.24°) = 9.7$

$\Rightarrow \boxed{P = 20.16\,N}$

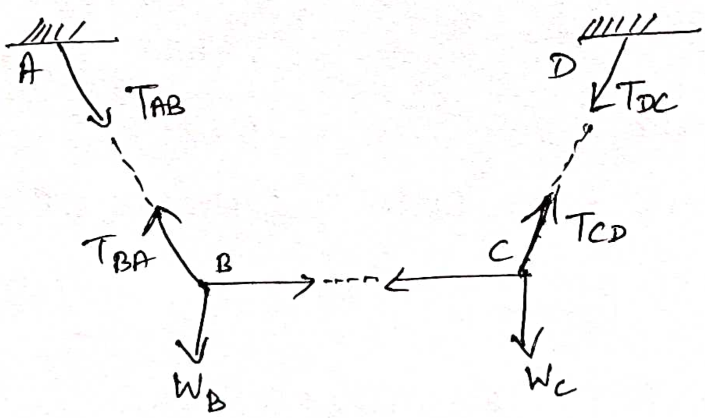
FREE BODY DIAGRAM:

"A body which has been so seperated (or) Isolated from the Sorrounding bodies is called free body".

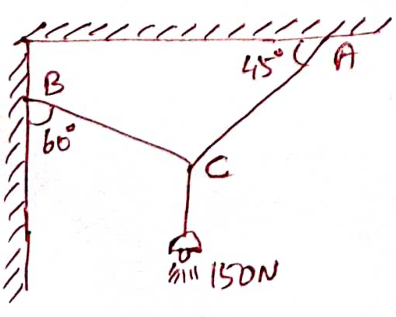
Ex:



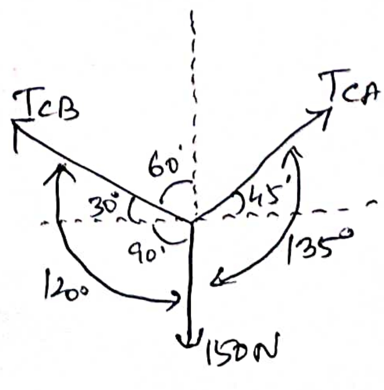
Free body diagram:



Ex: A electric light fixture weighing 150 N hangs from a point C, by two strings AC and BC shown in diagram. Determine the forces in the strings AC and BC.



Solution:



19/8/23

T_{CB} = Tension in the string CB, from C to B,

T_{CA} = Tension in the String CA, from C to A.

by applying lami's equation at C,

$$\frac{T_{CB}}{\sin 135^\circ} = \frac{T_{CA}}{\sin 120^\circ} = \frac{150}{\sin 105^\circ}$$

$$\therefore \quad T_{CB} = \frac{150 \sin 135^\circ}{\sin 105^\circ}$$

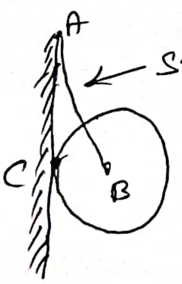
$$\Rightarrow \boxed{T_{CB} = 109.81\ N}$$

$$T_{CA} = \frac{150 \sin 120^\circ}{\sin 105^\circ}$$

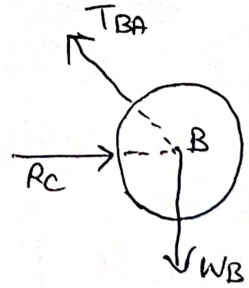
$$\Rightarrow \boxed{T_{CA} = 134.49\ N}$$

Free body diagram (samples)

1.

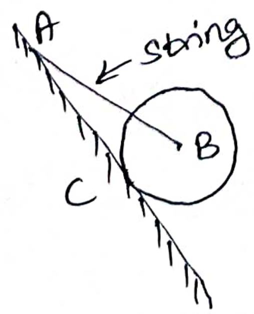


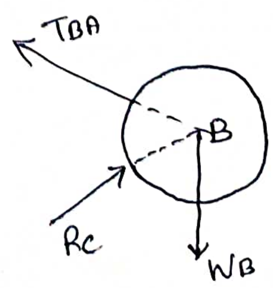
Body in equilibrium



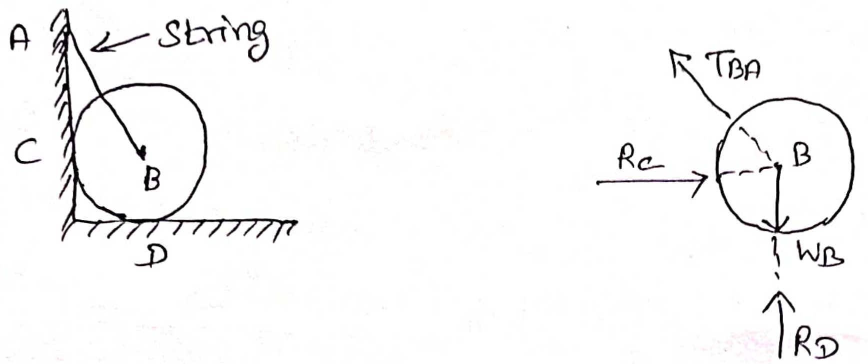
Free body diagram.

2.

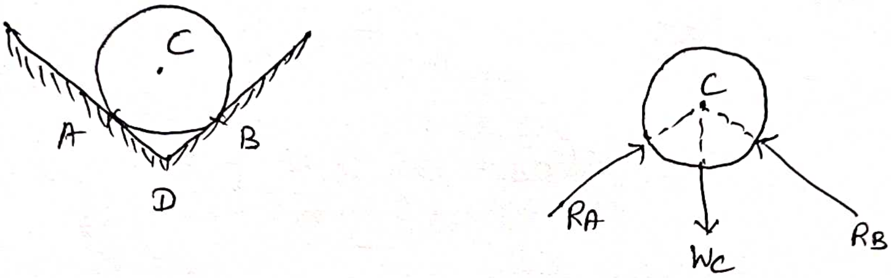




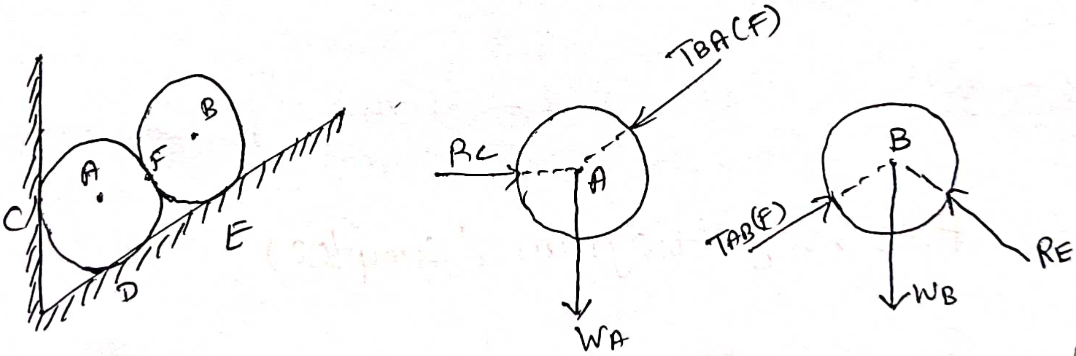
(3)



(4)

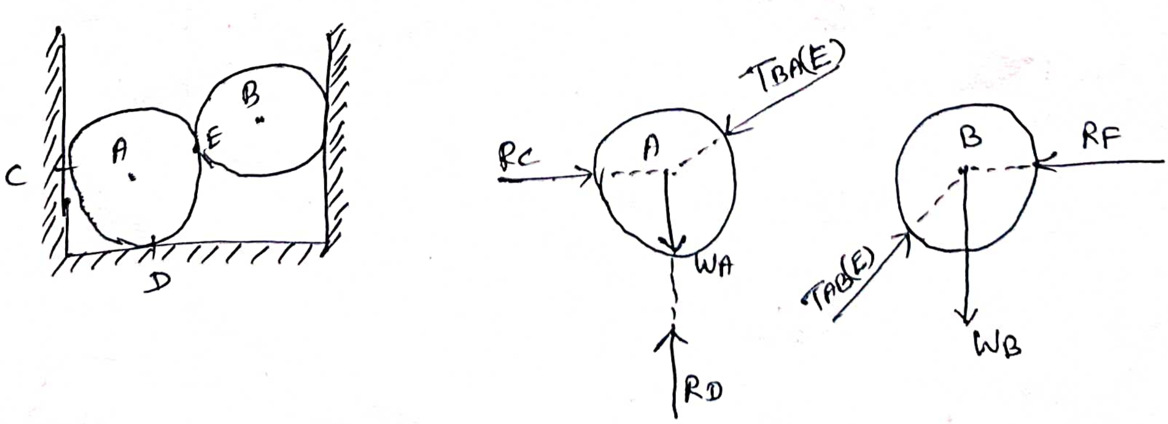


(5)



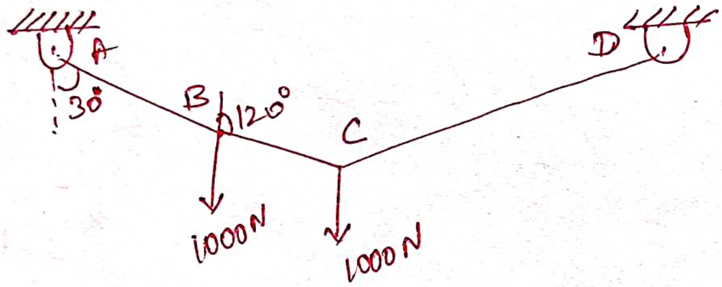
$T_{BA}(F) = T_{AB}(F)$

(6)

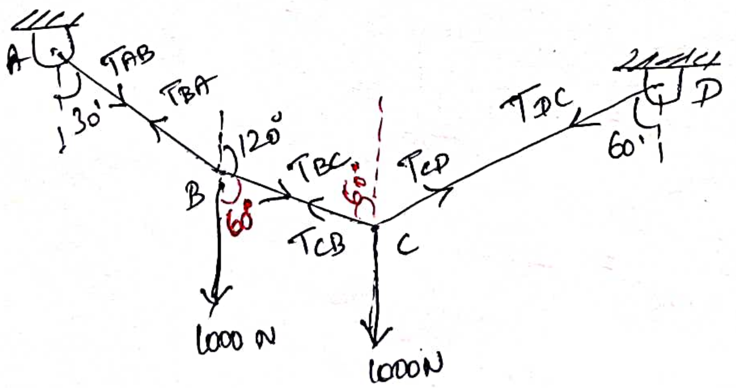


$T_{BA}(E) = T_{AB}(E)$.

Ex: A string ABCD, attached to two fixed points A and D has two equal weights of 1000 N attached to it at B and C. The weight rest with the portions AB and CD inclined at angles of 30° and 60° respectively, to the vertical as given in diagram. Find the tensions in the portions AB, BC, CD of the string, if the inclination of the portion BC with the vertical is 120°.



<u>Solution:</u>

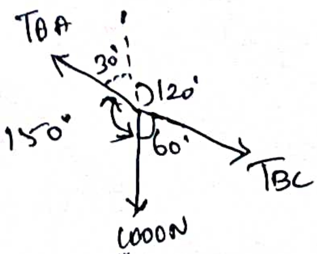


where,

$T_{AB} = T_{BA}$

$T_{BC} = T_{CB}$, $T_{CD} = T_{DC}$.

Apply lami's equation at joint 'B',



$$\frac{T_{BA}}{\sin 60^\circ} = \frac{T_{BC}}{\sin 150^\circ} = \frac{1000}{\sin 150^\circ}$$

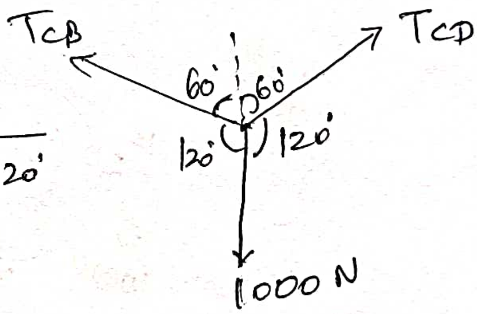
$$\Rightarrow T_{BA} = \frac{1000 \sin 60^\circ}{\sin 150}$$

$$\Rightarrow \boxed{T_{BA} = 1732\,N}$$

$$\Rightarrow T_{BC} = \frac{1000 \sin 150^\circ}{\sin 150^\circ}$$

$$\Rightarrow \boxed{T_{BC} = 1000\,N}$$

Apply lami's Equation at joint 'C'



$$\therefore \frac{T_{CB}}{\sin 120^\circ} = \frac{T_{CD}}{\sin 120^\circ} = \frac{1000}{\sin 120^\circ}$$

$$\Rightarrow \frac{T_{CB}}{\sin 120} = \frac{1000}{\sin 120^\circ}$$

$$T_{CB} = \frac{1000 \sin 120^\circ}{\sin 120^\circ}$$

$$\boxed{T_{CB} = 1000\,N}$$

$$\Rightarrow \frac{T_{CD}}{\sin 120^\circ} = \frac{1000}{\sin 120^\circ}$$

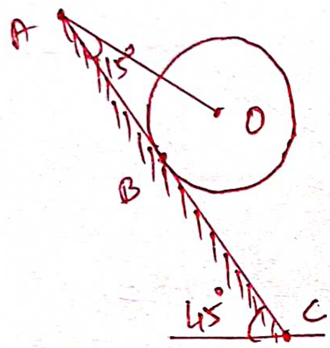
$$T_{CD} = \frac{1000 \sin 120^\circ}{\sin 120^\circ}$$

$$\boxed{T_{CD} = 1000\,N}$$

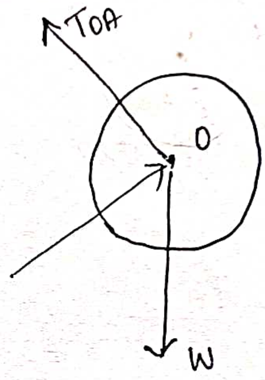
Result:

$$\boxed{\begin{aligned} T_{AB} &= 1732\,N \\ T_{BC} &= 1000\,N \\ T_{CD} &= 1000\,N \end{aligned}}$$

Ex: A string AO holds a smooth sphere on an inclined plane ABC, as given below. The weight of the sphere is 1000 N and the plane is smooth. Calculate the tension in the string and the reaction at the point of contact B.

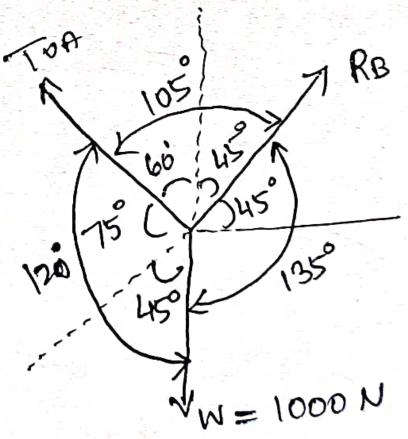


Solution:



Apply lami's theorem.

$$\frac{T_{OA}}{\sin 135^\circ} = \frac{W}{\sin 105^\circ} = \frac{R_B}{\sin 120^\circ}$$



(Free body diagram).

$$\Rightarrow \frac{T_{OA}}{\sin 135^\circ} = \frac{1000}{\sin 105^\circ}$$

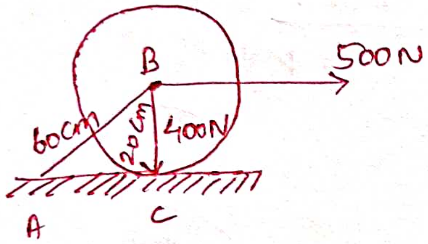
$$T_{OA} = \frac{1000 \sin 135^\circ}{\sin 105^\circ}$$

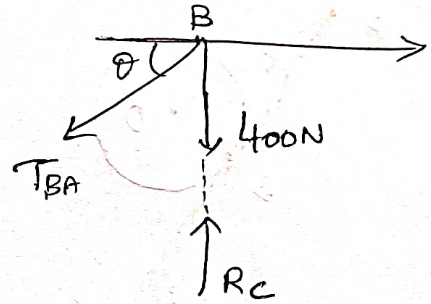
$$\boxed{T_{OA} = 732\,N}$$ Ans

$$\Rightarrow \frac{R_B}{\sin 120^\circ} = \frac{1000}{\sin 105^\circ}$$

$$\Rightarrow \boxed{R_B = 896.57\,N}$$ Ans

Ex: A circular roller of radius of 20 cm and of weight 400 N rests on a smooth horizontal surface and is held in position by an inclined bar AB of length 60 cm as shown in diagram below. A horizontal force of 500 N is acting at B. Find the tension in the bar AB and reaction at C.

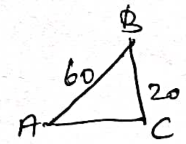




Solution:

From Δ ABC

Let ∠BAC



$$\therefore \quad \sin\theta = \frac{BC}{AB} = \frac{20}{60}$$

$$\boxed{\theta = 19.47^\circ}$$

Applying equations of equilibrium at B,

$$\Sigma H = 0$$

$$500 - T_{BA} \cos 19.47 = 0$$

$$\therefore \quad T_{BA} = \frac{500}{\cos 19.47} \implies \boxed{T_{BA} = 530.32\,N}$$

$$\Sigma V = 0$$

$$R_c - 400 - T_{BA} \sin 19.47 = 0$$

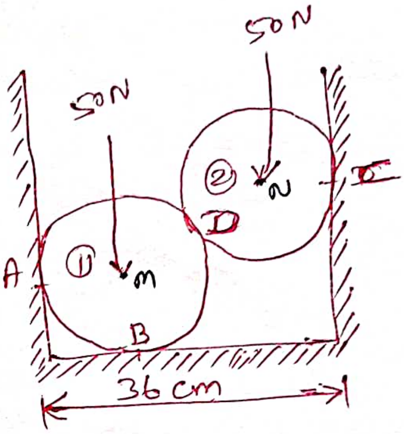
$$R_c = 400 + T_{BA} \sin 19.47$$

$$\boxed{R_c = 576.76\,N}$$

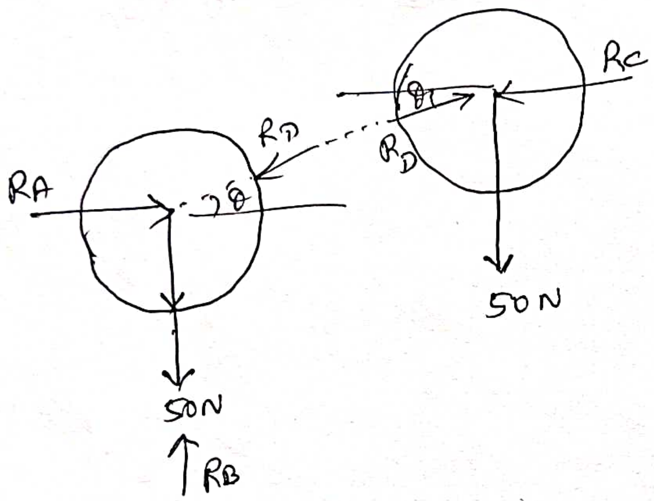
Answers: Tension in the bar = 530.32 N

Reactions at C = 576.76 N //

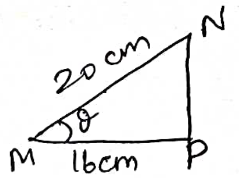
Ex: Two rollers, each of weight 50 N and of radius 10 cm rest in a horizontal channel of width 36 cm as shown in diagram. Find the reaction on the point of contacts A, B and C.



Solution:



Let, centre of balls ①, ② are M and N.



$MP = -(2 \times \text{radius}) + \text{Base}$

$= -20 + 36$

$\boxed{MP = 16}$

$MN = 2 \times \text{radius} = 20 \text{ cm}.$

$\boxed{MN = 20 \text{ cm}}$

$\therefore \cos\theta = \frac{16}{20}$

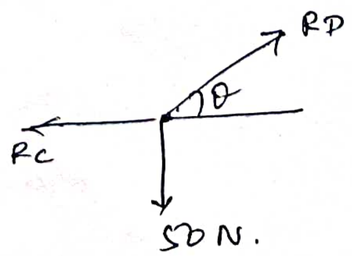
$\boxed{\theta = 36.87^\circ}$

Considering, Roller ②,

$\Sigma H = 0$

$R_D \cos\theta - R_C = 0$

$R_D \cos 36.87 = R_C \rightarrow (1)$



Roller ②
Free body diagram.

$\Sigma V = 0.$

$R_D \sin\theta - 50 = 0$

$R_D \sin\theta = 50 \rightarrow (2)$

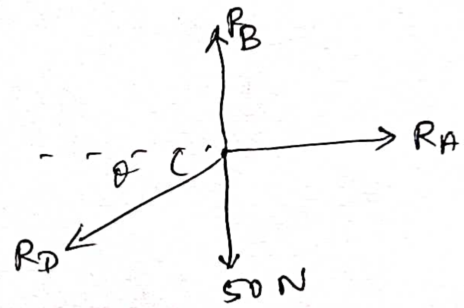
$\Rightarrow R_D \sin 37.87^\circ = 50$

$\boxed{R_D = 83.33\,N}$ Subs in (1).

we get,

$\boxed{R_C = 66.66\,N}$

Considering, the roller (1),



Apply, $\Sigma H = 0$,

$R_A - R_D \cos\theta = 0$

$R_A - R_D \cos 36.87 = 0$

$\Rightarrow \boxed{R_A = 66.66\,N}$

Apply,

$\Sigma V = 0.$

$R_B - 50 - R_D \sin\theta = 0$

$R_B - 50 - R_D \sin 36.87^\circ = 0$

$\Rightarrow \therefore \boxed{R_B = 100\,N}$.

Result:

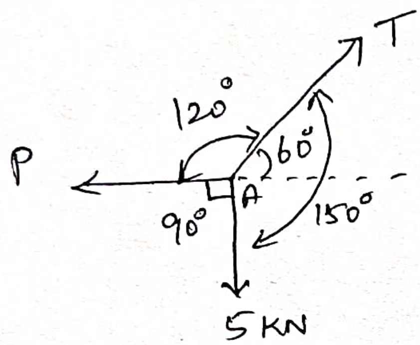
Reaction at A $= R_A = 66.68\,N\ (\rightarrow)$

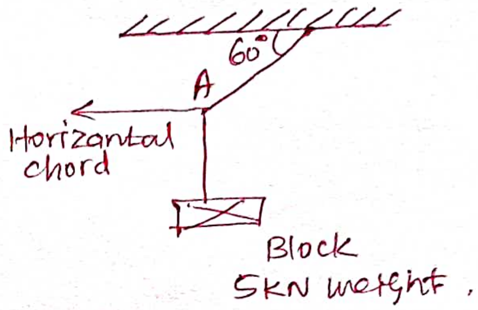
Reaction at B $= R_B = 100\,N\ (\uparrow)$

Reaction at C $= R_C = 66.69\,N\ (\leftarrow)$

Ex: A Block weighing 5KN is Suspended from the ceiling by a Chain. It is pulled aside by a Horizantal chord untill the chain makes 60° with the ceiling as Shown in figure. Find the tension in the chain and in the chord by applying lami's theorem.

Solution:





Let

P = Force in the Horizantal Chord.

T = Tension in the chain.

Applying lami's theorem,

$$\frac{P}{\sin 150} = \frac{5}{\sin 120} = \frac{T}{\sin 90^{\circ}}$$

$$\Rightarrow \frac{P}{\sin 150} = \frac{5}{\sin 120}$$

$$P = \frac{5 \sin 150}{\sin 120}$$

$$\boxed{P = 2.89\,KN}$$

$$\Rightarrow \frac{T}{\sin 90} = \frac{5}{\sin 120}$$

$$\Rightarrow \boxed{T = 5.77\ KN}$$