UNIT. I. CONDUCTION

Heat Transfer :

Heat transfer can be defined as the transmission of energy from one region to another region due to temprature difference.

Modes of Heat transfer:

Conduction :

Heat Conduction is mechanism of heat transfer from a region of high temprature to a region of low temprature with in a medium (or) between different medium in direct physical contact.

In Conduction, energy takes place by the kinematic motion or direct impact of molecules, pure conduction is found only in solids.

Convection ;

Convection is a process of heat transfer that will occur between a solid surface and a fluid when they are at different tempratures.

Convection, is possible only in the presence of fluid medium.

Radiation :

The heat transfer from one body to another without any transmitting medium is known as radiation. It is an electromagnetic wave phenomenon.

Fourier Law of Conduction :

The rate of heat conduction is proportional to the area measured normal to the direction of heat flow and to the temprature gradient in that direction.

$$Q \propto -A \frac{dT}{dx}$$

$$Q = -kA \frac{dT}{dx}$$

where,

A - Area in m^2

$\frac{dT}{dx}$ - Temprature gradient K/m

k - Thermal Conductivity in W/mK

(- sign indicates heat flow in that direction decreases)

Thermal Conductivity :

Thermal conductivity is defined as the ability of substance to conduct heat.

General Heat Conduction :

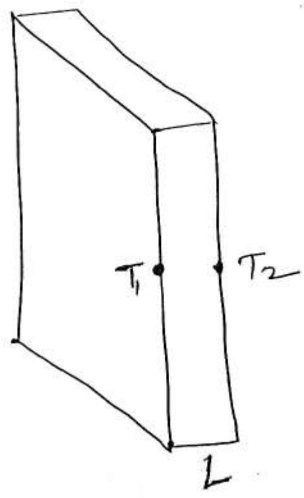
$$\left\{\begin{array}{l}\text{Net heat}\\ \text{Conducted into the}\\ \text{element from}\\ \text{all the}\\ \text{coordinate}\\ \text{directions}\end{array}\right\} + \left\{\begin{array}{l}\text{Heat}\\ \text{generated}\\ \text{with in the}\\ \text{element}\end{array}\right\} = \left\{\begin{array}{l}\text{Heat}\\ \text{Stored}\\ \text{in the}\\ \text{element}\end{array}\right\}$$

Problems on slab (or) plane wall :

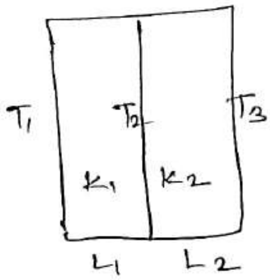
$$Q = \frac{\Delta T_{overall}}{R}$$

$$\Delta T = T_1 - T_2$$

$$R = \frac{L}{KA}$$



L – Slab Thickness
(or)
Wall Thikness



$$Q = \frac{\Delta T_{overall}}{R}$$

$$\Delta T = T_1 - T_3 \quad (or) \quad T_a - T_b$$

$$R = \frac{1}{h_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \frac{1}{h_b A}$$

Ex: A wall of 0.6 m thickness having thermal conductivity of 1.2 W/mK. The wall is to be insulated with a material having an average thermal conductivity of 0.3 W/mK. Inner and outer surface temperatures are 1000°C and 10°C respectively. If heat transfer rate is 1400 W/m². Calculate the thickness of insulation.

Given:

$L_1 = 0.6 m$

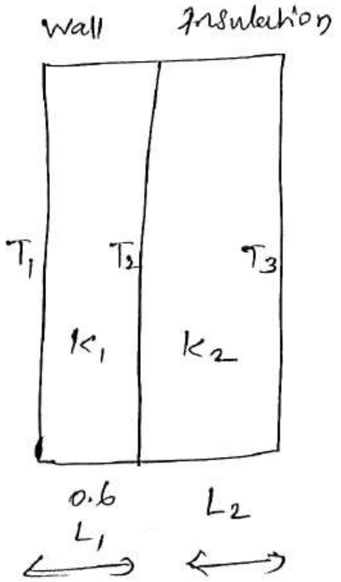
$k_1 = 1.2 \ W/mK$

$k_2 = 0.3 \ W/mK$

$T_1 = 1000°C + 273 = 1273 \ K$

$T_2 = 10°C + 273 = 283 \ K$

$\frac{Q}{A} = 1400 \ W/m^2$



③

To find:

L_2

Solution:

$$Q = \frac{\Delta T}{R} \quad \text{(From HMT 44)}$$

$$\Delta T = T_1 - T_3$$

$$R = \frac{1}{h_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_b A}$$

$$Q = \frac{[T_1 - T_3]}{\frac{1}{h_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_b A}}$$

$$Q/A = \frac{T_1 - T_3}{\frac{1}{h_a} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{1}{h_b}}$$

(neglect h_a, h_b)

$$Q/A = \frac{T_1 - T_3}{\frac{L_1}{k_1} + \frac{L_2}{K_2}}$$

$$1400 = \frac{1273 - 283}{\frac{0.6}{1.2} + \frac{L_2}{0.3}}$$

$$\boxed{L_2 = 0.0621 \text{ m}}$$

Ex: The wall of a Cold room is Composed of th layer. brick is 30 cm thick. The middle layer. Cork 20 cm thick, the inside Layer is cement 15 cm thick. The tempratures of the outside air is 25°c and on the inside air is 20°C. The film co-efficient for outside air and brick is 55.4 W/m²K. Film co-efficient for inside air and cement is 17 W/m²K. Find the heat flow rate. Take k brick 2.5 W/mk, k for cork 0.05 W/mk, k for cement 0.28 W/mk

<u>Given:</u>

$L_3 = 0.3 m$

$L_2 = 0.2 m$

$L_1 = 0.15 m$

$T_a = -20°C + 273 = 253 K$

$T_b = 25°C + 273 = 298 K$

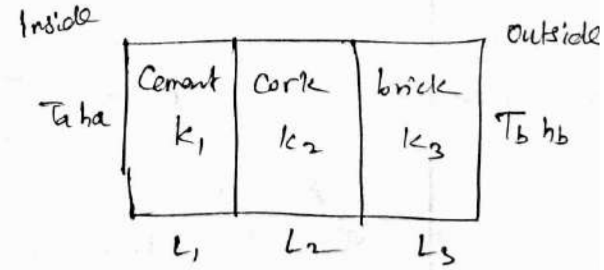
$h_a = 17 W/m^2K$

$h_b = 55.4 W/m^2K$

$k_{brick} = k_3 = 2.5 W/mK$

$k_2 = 0.05 W/mK$

$k_1 = 0.28 W/mK$



(4)

Solution:

$$Q = \frac{\Delta T}{R}$$

$$\Delta T = T_a - T_b$$

$$R = \frac{1}{h_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \frac{1}{h_b A}$$

$$\frac{Q}{A} = \frac{T_a - T_b}{1/h_a + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h_b}}$$

$$Q/A = \frac{253 - 298}{\left(\frac{1}{17}\right) + \frac{0.15}{0.28} + \frac{0.2}{0.05} + \frac{0.3}{2.5} + \frac{1}{55.4}}$$

$$Q/A = -9.5 \ W/m^2$$

Ex: A wall of a cold room is composed of three layers is 20 cm brick thick, middle layer is cork 10 cm thick, the inside layer is cement 5 cm thick; The temprature of the outside air is 25°c and that on the inside air is −20°c. The film co-efficient for outside air is $45.4 \ W/m^2K$ and for inside air and cement is $17 \ W/m^2K$. Find thermal resistance, heat flow rate. Take $k_3 = 3.45 \ W/mk$, $k_2 = 0.043 \ W/mk$, $k_1 = 0.294 \ W/mK$.

Ans:

Thermal resistance $R = 2.634 \ K/W$

$Q/A = -17.081 \ W/m^2$

(AU) A furnace wall is made up of three layers thickness 25 cm, 10 cm and 15 cm with thermal Conductivities of 1.65 W/mK, K, and 9.22 W/mK respectively. The inside is exposed to gases at 1250°C with a Convection Co-efficient of 25 W/m² °C and the inside Surface is at 1100°C, the outside Surface is exposed to air at 25°C with convection coefficient of 12 W/m² K. Determine (i) The unknown thermal conductivity (ii) The overall heat transfer Co-efficient, (iii) All the Surface tempratures.

Given:

$L_1 = 0.25 m$

$L_2 = 0.10 m$

$L_3 = 0.15 m$

$k_1 = 1.65 W/mK$

$k_2 = k$

$k_3 = 9.2 W/mK$

$T_a = 1523 K$

$T_b = 298 K$

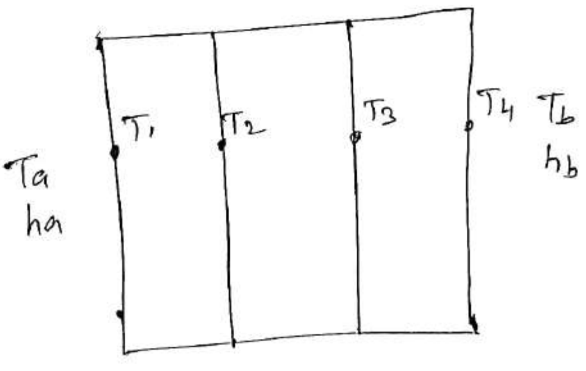
$h_a = 25 W/m^2 K$

$h_b = 12 W/m^2 K$

$T_1 = 1100°C + 273$

$= 1373 K$

(Inner Surface temp)



To find:

k_2,

overall heat tranfer coefficient (U)

T_2, T_3, T_4

(5)

Solution:

$$Q = h_a A\,(T_a - T_1) \quad (\text{Heat transfer})$$

$$\frac{Q}{A} = h_a\,(T_a - T_1)$$

$$Q/A = 3750\ W/m^2$$

WKT,

$$Q = \frac{\Delta T_{overall}}{R} \quad (\text{From HMT DB P.NO 44})$$

$$\Delta T = T_a - T_b$$

$$R = \frac{1}{h_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \frac{1}{h_b A}$$

$$\frac{Q}{A} = \frac{T_a - T_b}{\frac{1}{h_a} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h_b}}$$

$$3750 = \frac{1523 - 298}{\frac{1}{25} + \frac{0.25}{1.65} + \frac{0.10}{k_2} + \frac{0.15}{9.2} + \frac{1}{12}}$$

$$3750 = \frac{1225}{0.2911 + \frac{0.10}{k_2}}$$

$$3750\left[0.2911 + \frac{0.1}{k_2}\right] = 1225$$

$$0.2911 + \frac{0.1}{k_2} = 0.3266$$

$\Rightarrow$ Thermal Conductivity $\boxed{k_2 = 2.816\ W/mK}$

WKT,

Overall heat transfer co-efficient $U = \frac{1}{R_{total}}$.

$$R_{total} = \frac{1}{h_a} + \frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3} + \frac{1}{h_b} \quad (\text{Take } A = 1\,m^2$$

$$\Rightarrow R_{total} = 0.3267\ W/m^2$$

$$\boxed{U = 3.06\ W/m^2 K}$$

WKT,

$$Q = \frac{T_a - T_b}{R} = \frac{T_a - T_1}{R_a} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_4}{R_3} =$$

$$= \frac{T_4 - T_b}{R_b} \longrightarrow (5)$$

$$(1) \Rightarrow \quad Q = \frac{T_1 - T_2}{R_1} \quad \left[\because R_1 = \frac{L_1}{K_1 A}\right]$$

$$Q = \frac{T_1 - T_2}{L_1 / K_1 A}$$

$$Q/A = \frac{T_1 - T_2}{(L_1 / K_1)}$$

$$3750 = \frac{1373 - T_2}{(0.25 / 1.65)}$$

$$\Rightarrow \boxed{T_2 = 804.8\ K}$$

|||ly.

$$Q = \frac{T_2 - T_3}{R_2} \quad \left[\because R_2 = \frac{L_2}{K_2 A}\right]$$

$$\Rightarrow \boxed{T_3 = 671.45\ K}$$

|||ly

$$Q = \frac{T_3 - T_4}{R_3} \quad \left[\because R_3 = \frac{L_3}{K_3 A}\right]$$

$$\Rightarrow \boxed{T_4 = 610.30\ K}$$

Ex) A mild steel tank of wall thickness 20 mm contains water at 100° c. Estimate the loss of heat per square meter area of the tank surface, if the tank is exposed to an atmosphere at 15°c. Thermal conductivity of steel is is 50 W/mK, while heat transfer co.efficient for the outside and inside tank are 10 W/m²K and 2850 W/m²K respectively. What will be the temprature on the outside of the wall.

Given:

$L_1 = 0.02\,m$

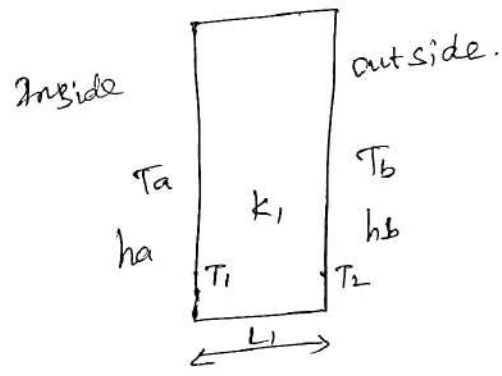
$T_a = 373\,K$

$T_b = 288\,K$

$k_1 = 50\,W/mK$

$h_a = 2850\,W/m^2K$

$h_b = 10\,W/m^2K$



To find:

(i) Heat loss per square meter area (Q/A)

(ii) outside temp (T_2)

Solution:

$$\text{Heat loss } Q = \frac{\Delta T_{overall}}{R} \quad (\text{HMT } 44)$$

$$\Delta T = T_a - T_b$$

$$R = \frac{1}{h_a A} + \frac{L_1}{k_1 A} + \frac{1}{h_b A}$$

$$\frac{Q}{A} = \frac{T_a - T_b}{1/h_a + \frac{L_1}{k_1} + \frac{1}{h_b}}$$

$$\frac{Q}{A} = \frac{373 - 288}{\frac{1}{2850} + \frac{0.02}{50} + \frac{1}{10}}$$

$$\boxed{\frac{Q}{A} = 843.66 \ W/m^2}$$

WKT,

$$Q = \frac{T_a - T_b}{R} = \frac{T_a - T_1}{R_a} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_b}{R_b} \rightarrow (1)$$

$$(1) \Rightarrow \quad Q = \frac{T_a - T_1}{R_a} \quad \left(\because R_a = \frac{L}{h_a A}\right)$$

$$Q = \frac{T_a - T_1}{1/h_a A}$$

$$\frac{Q}{A} = \frac{T_a - T_1}{1/h_a}$$

$$843.66 = \frac{373 - T_1}{1/2850}$$

$$\Rightarrow \boxed{T_1 = 372.7K}$$

(7)

III[ly]

$$Q = \frac{T_1 - T_2}{R_1}$$

$$Q = \frac{T_1 - T_2}{L_1 / K_1 A}$$

$$\frac{Q}{A} = \frac{T_1 - T_2}{L_1 / K_1}$$

$$\Rightarrow \boxed{T_2 = 372.4K}$$

<u>Result:</u>

Heat loss per m^2 $Q/A = 843.66\ W/m^2$

$T_2 = 372.4K$ //

Ex:

A furnace wall is madeup of three layers, inside layer with thermal conductivity 8.5 W/mK, the middle layer with conductivity 0.25 W/mK, the outer layer with conductivity 0.08 W/mK. The respective thickness of the inner, middle, outside layers are 25 cm, 5 cm, 3 cm respectively. The inside and outside wall tempratures are 600°C and 50°C respectively. Draw the equivalent electrical circuit for conduction of heat through the wall. Find thermal resistance, heat flow/m^2 and the inner side tempratures.

Ex: A Steam boiler furnace is made of fire cla The hot gas temprature inside the boiler furnace is 2100°C, room air temprature is 50°C, heat flow by radiation from gases to inside surface of the wall is 25.2 kW/m², convection heat transfer co-efficient at the interior surface is 12.2 W/m²K, thermal conductance of the wall is 58 W/mK, heat flow by radition from external surface to sorrounding is 8.2 kW/m² and interior wall surface temprature is 1080°C, Calculate the external surface, 1) surface temprature 2) Convective conductance.

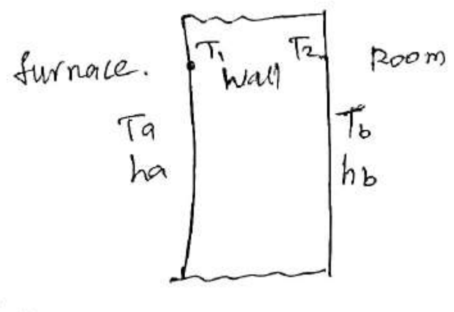
Given :

$T_a = 2373\ K$

$T_b = 323K$

$Q_{R_1} = 25.2 \times 10^3\ W/m^2$

$h_a = 12.2\ W/m^2 K$



Thermal conductance (wall) $= 58\ W/mK$

$Q_{R_2} = 8.2 \times 10^3\ W/m^2$

$T_1 = 1353\ K$

To find :

External surface temp (T_2)

" Convective conductance (h_b)

(8).

Solution:

$$\left.\begin{array}{l}\text{Total heat}\\ \text{entering the wall}\end{array}\right\} Q = \begin{bmatrix}\text{Heat}\\ \text{transfer by}\\ \text{Convection}\\ \text{at interior}\end{bmatrix} + \begin{bmatrix}\text{Heat transfer}\\ \text{by}\\ \text{radiation}\\ \text{at interior}\end{bmatrix}$$

$$Q = Q_c + Q_{R_1}$$

$$Q = h_a A \Delta T + 25.2 \times 10^3$$

$$Q = \{h_a \times A \times (T_a - T_1)\} + 25.2 \times 10^3$$

$$\boxed{Q = 37,644 \ W/m^2} \qquad (\because A = 1m^2)$$

WKT,

$$Q = \frac{T_a - T_b}{R} = \frac{T_a - T_1}{R_a} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_b}{R_b} \rightarrow (1)$$

$$(1) \Rightarrow Q = \frac{T_1 - T_2}{R_1} \qquad \left(R_1 \ \text{Resistance} = \frac{1}{\text{Conductance}}\right)$$

$$37,644 = \frac{1353 - T_2}{R_1}$$

$$37,644 = \frac{1353 - T_2}{1/58}$$

$$\Rightarrow \boxed{T_2 = 703.9 \ K}$$

Heat loss due to radiation at exterior,

$$Q_{R_2} = 8.2 \times 10^3 \ W/m^2$$

Heat loss by convection at exterior,

= total heat entering − Heat loss by radiation at exterior.

$$= 37,644 - 8.2 \times 10^3$$

$$\boxed{Q_c = 29,444 \ W/m^2}$$

$$\Rightarrow \quad Q_c = h_b \, A \, \Delta T$$

$$29,444 = h_b \times 1 \times (T_2 - T_b)$$

$$= h_b \, (703.9 - 323)$$

$\Rightarrow$ External Convection

Co-efficient $h_b = 77.3 \ W/m^2K$.

Ex: A wall is constructed of several layers. The first layer consists of masonary brick 20 cm thick of thermal conductivity 0.66 W/mK, the second layer consist of 3 cm thick mortar of thermal conductivity 0.6 W/mK. The third layer consists of 8 cm thick lime stone of thermal conductivity 0.58 W/mK and the outer layer consists of 1.2 cm thick plaster of thermal conductivity 0.6 W/mK. The heat transfer co-efficient on the interior and exterior of the wall are 5.6 W/m² K and 11 W/m² k respectively. Interior room temprature is 22°c and outside air temprature is −5°c. (9).

Calculate,

a) overall heat transfer co-efficient.

b) overall thermal resistance.

c) The rate of heat transfer.

d) The temprature at the junction b/w the mortar and Limestone.

Given:

$L_1 = 0.2 m$

$k_1 = 0.66 W/mK$

$L_2 = 0.03 m$

$K_2 = 0.6 W/mK$

$L_3 = 0.08 m$

$k_3 = 0.58 W/mK$

$L_4 = 0.012 m$

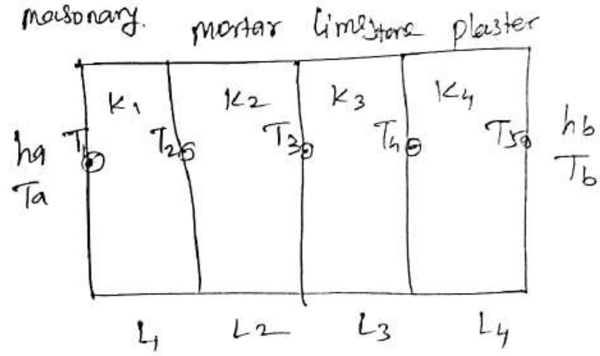
$k_4 = 0.6 W/mK$

$h_a = 5.6 W/m^2K$

$h_b = 11 W/m^2K$

$T_a = 295 K$

$T_b = 268 K$



To find:

$U, R, Q/A, T_3$

Solution:

$$Q = \frac{\Delta T_{overall}}{R} \qquad (HMT\ DB\ 44)$$

$$\Delta T = T_a - T_b$$

$$R = \frac{1}{h_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \frac{L_4}{k_4 A} + \frac{1}{h_b A}$$

$$\Rightarrow \boxed{Q/A = 34.56\ W/m^2}$$

WKT,

$$Q = UA\,(T_a - T_b)$$

$$U = \frac{Q}{A \times (T_a - T_b)}$$

$$U = \frac{34.56}{295 - 268}$$

$$\boxed{U = 1.28\ W/m^2 K}$$

WKT,

$$R = \frac{1}{h_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \frac{L_4}{k_4 A} + \frac{1}{h_b A}$$

$$\boxed{R = 0.778\ K/W} \qquad (\because \text{For } A = 1m^2)$$

WKT,

$$Q = \frac{T_a - T_b}{R_a} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_4}{R_3} = \frac{T_4 - T_5}{R_4} = \frac{T_5 - T_b}{R_b} \longrightarrow (1)$$

(10).

$$Q = \frac{T_a - T_1}{R_a}$$

$$\frac{Q}{A} = \frac{295 - T_1}{1/h_a}$$

$$\Rightarrow \boxed{T_1 = 288.8\,K}$$

(1) $\Rightarrow$ $$Q = \frac{T_1 - T_2}{R_1}$$

$$Q/A = \frac{288.8 - T_2}{L_1/K_1}$$

$$\Rightarrow \boxed{T_2 = 278.3\,K}$$

III^{ly},

$$\boxed{T_3 = 276.5\,K}$$

Ex: The inside temprature of the refrigerator is $-10^\circ C$ and outside surface temprature is $30^\circ C$ and area is $30\ m^2$. This refrigirator consists of 2.2mm of steel at the inner surface, 15mm plywood at the outer surface and 10cm of glass wool in between steel and plywood. Calculate the heat loss and the capacity of the refgirator in tons of refrgeration. Assume k (steel) = 20 W/mK. k for plywood = 0.05 W/mK, glass k = 0.06 W/mK.

Given:

$T_1 = 263\,K$

$T_4 = 303\,K$

$A = 30\ m^2$

$L_1 = 0.0022\ m$

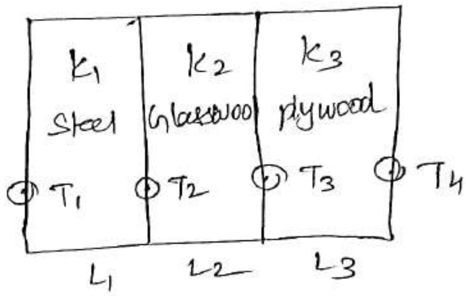
$L_3 = 0.015\ m$

$L_2 = 0.10\ m$

$k_1 = 20\ W/mK$

$k_3 = 0.05\ W/mK$

$k_2 = 0.06\ W/mK.$



To find :

Heat loss (Q)

Capacity of the refrigirator.

Solution :

$$Q = \frac{\Delta T\ overall}{R}$$

$$Q = \frac{T_1 - T_4}{\frac{1}{h_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \frac{1}{h_b A}}$$

(neglect h_a, h_b)

$$\boxed{Q = -0.610\ kW}$$

-ve sign indicates, heat flows from outside into refrigirator.

$3.5\ kW = 1\ ton$

$$\Rightarrow 610\ kW = \frac{0.610}{3.5}\ ton$$

$$= 0.174\ ton.$$

$$\boxed{\text{Capacity of refgerator} = 0.174\ ton}$$

(11).

Ex: A furnace wall made up of 7.5 cm of fire plate and 0.65 cm of mild steel plate. Inside surface exposed to hot gas at 650 °C, and outside air temprature 27°C. The convective heat transfer co-efficient for inner side is 60 W/m²K. The convective heat transfer co-efficient for outer side is 8 W/m²K. Calculate the heat lost per square meter area of the furnace wall and also find outside surface temprature.

Given data:

$L_1 = 0.075$ m

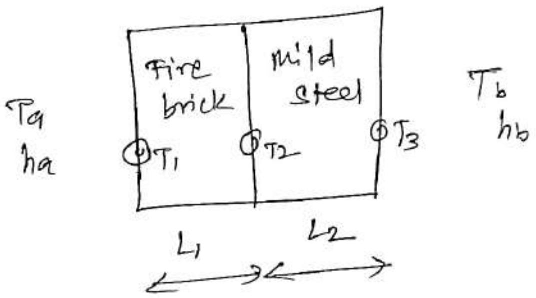
$L_2 = 0.0065$ m

$T_a = 923$ K

$T_b = 300$ K

$h_a = 60$ W/m²K

$h_b = 8$ W/m²K



To find:

(i) Heat loss per square meter Area (Q/A)

(ii) outside Surface temprature (T_3).

Solution:

From (HMT DB 01, 13)

fire plate $k_1 = 1.0$ W/mK , $k_2 = 53.6$ W/mK

WKT, $$Q/A = \frac{\Delta T}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{1}{h_a} + \frac{1}{h_b}} \Rightarrow \boxed{Q/A = 2907.79\ W/m^2}$$

WKT, $$Q = \frac{T_a - T_b}{R} = \frac{T_a - T_1}{R_a} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_b}{R_b} \rightarrow (b)$$

$$\Rightarrow \boxed{T_3 = 663.473\ K}$$

Ex: A wall of a furnace made up of 250 mm thi. clay of thermal conductivity 1.05 W/mK, 120mm thi. of insulation brick of conductivity 0.15 W/mK and 200 mm thick red brick of conductivity 0.85 W/mK. The inner and outer surface temprature of wall are 850°c and 65°c respectively. Calculate the tempratures at the contacts surfaces.

Ans: $Q/A = 616.46\ W/m^2$, $T_2 = 976.2\ K$, $T_3 = 483\ K$.

Ex: A surface wall is made up of 3 layers one of fire brick, one of insulating brick and one of red brick. The inner and outer surface tempratures are 900°c and 30°c respectively. The respective co-efficient of thermal conductivity of the layers are 1.2, 0.14 and 0.9 W/mK and thickness of 20 cm, 8 cm, and 11 cm. Assuming close bonding of the layers at the interfaces. Find the heat loss per square meter and the interface tempratures.

Ans:

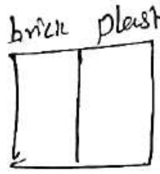
$$Q/A = 1011.25\ W/m^2$$

$$T_2 = 1004.5\ K$$

$$T_3 = 426.59\ K$$

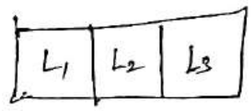
(12)

Ex: A Composite wall consists of 10 cm thick layer of building brick $k = 0.7$ W/mK and 3 cm thick plaster $k = 0.5$ W/mK. ~~An insulating material of k = 0.08 W/mK~~. An insulating material of $k = 0.08$ W/mK is to be added to reduce the heat transfer through the wall by 40%. Find its thickness.



Given data:

$L_1 = 0.1\,m$ | $k_1 = 0.7\,W/mK$
$L_2 = 0.03\,m$ | $k_2 = 0.5\,W/mK$
$L_3 = ?$ | $k_3 = 0.08\,W/mK$.



To find:

L_3 (when heat loss reduced to 40%)

Solution:

$$Q = \frac{\Delta T}{R}$$

(neglect h_a, h_b)

First consider two slab,

$$Q = \frac{\Delta T}{\frac{L_1}{k_1} + \frac{L_2}{k_2}} \qquad (\because A = 1\,m^2)$$

$$100 = \frac{\Delta T}{\frac{0.1}{0.7} + \frac{0.03}{0.5}} \qquad (\text{Assume } Q = 100\,W)$$

$$\Rightarrow \Delta T = 20.28\,K$$

Heat loss reduced by 40%. due to insulation.
So heat transfer flow is low.

$$Q = \frac{\Delta T}{R}$$

$$Q = \frac{\Delta T}{\frac{1}{h}\left[\frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3}\right]}$$

$$60 = \frac{20.28}{\left[\frac{0.1}{0.7} + \frac{0.03}{0.5} + \frac{L_3}{0.08}\right]} \quad (\because A = 1\,m^2)$$

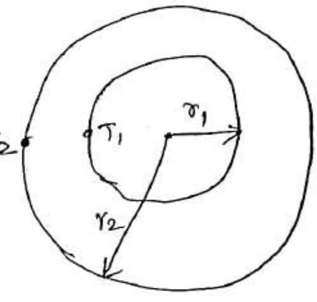
$$\Rightarrow \boxed{L_3 = 0.0108\,m}$$

(13)

PROBLEMS ON CYLINDERS:

$$Q = \frac{\Delta T}{R}$$

$$R = \frac{1}{2\pi L K} \ln\left[\frac{r_2}{r_1}\right]$$



For two layers:

$$Q = \frac{\Delta T}{R}$$

$$R = \frac{1}{2\pi L}\left[\frac{1}{h_a r_1} + \frac{\ln\left[\frac{r_2}{r_1}\right]}{k_1} + \frac{\ln\left[\frac{r_3}{r_2}\right]}{K_2} + \frac{1}{h_b r_3}\right]$$

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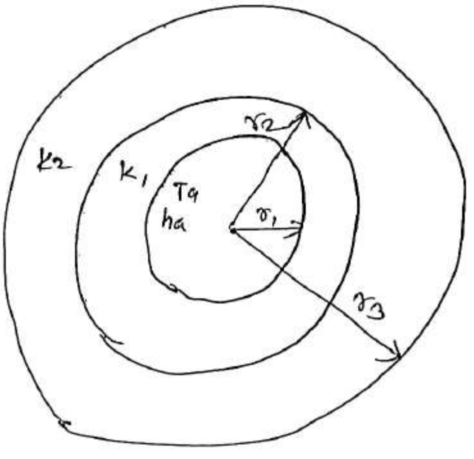


$$\frac{Q}{L} = \frac{[T_a - T_b]}{\frac{1}{2\pi}\left[\frac{1}{h_a r_1} + \frac{\ln\left[\frac{r_2}{r_1}\right]}{k_1} + \frac{\ln\left[\frac{r_3}{r_2}\right]}{k_2} + \frac{1}{h_b r_3}\right]}$$

here, (Q/L unit = W/m)

Ex: A hallow cylinder 5 cm inner radius and 10 cm outer radius has inner surface temprature 200°c and outer surface temprature 100°C. If the thermal conductivity is 70 W/mK, find heat transfer per unit length.

Given data:

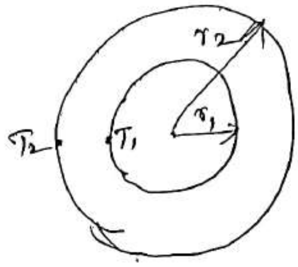
$r_1 = 0.05\,m$

$r_2 = 0.1\,m$

$T_1 = 473\,K$

$T_2 = 373\,K$

$K = 70\,W/mK$



Solution:

$$Q = \frac{\Delta T_{overall}}{R} \qquad (HMT\ 44)$$

$$Q = \frac{T_1 - T_2}{\frac{1}{2\pi L K} \ln\left[\frac{r_2}{r_1}\right]}$$

$$Q/L = \frac{T_1 - T_2}{\frac{1}{2\pi k} \ln\left(\frac{r_2}{r_1}\right)}$$

$$Q/L = \frac{473 - 373}{\frac{1}{2\pi \times 70} \cdot \ln\left[\frac{0.1}{0.05}\right]}$$

$$\Rightarrow \boxed{Q/L = 63420.87\ W/m}$$

(14).

Ex: Hot air at 40°c flowing through a steel pipe of 10 cm diameter. The pipe is covered with two layer of insulating material of thickness 4 cm and 3 cm and their corresponding thermal conductivities are 0.1 and 0.32 W/mK. The inside and outside convective heat transfer co-efficient are 50 W/m²K and 15 W/m²K. The outer temp is 10°C. Find the heat loss per meter length of steam pipe.

Given data:

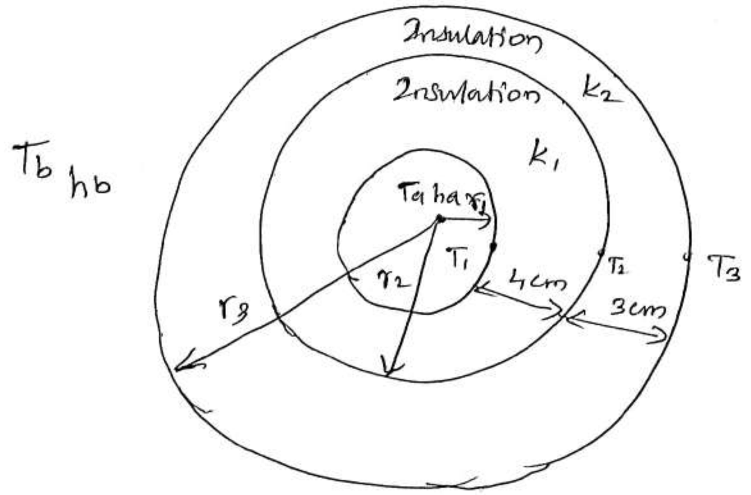
$T_a = 40+273 = 313K$ | $T_b = 10+273 = 283 K$

$r_1 = 0.05 m$ | $r_2 = r_1 + 4 = 9cm = 0.09 m$

$r_3 = r_2 + 3 cm = 12 cm = 0.12 m$

$k_1 = 0.1 W/mK$, $k_2 = 0.32 W/mK$, $h_a = 50 W/m^2K$,

$h_b = 15 W/m^2K$.



Solution:

$$Q = \frac{\Delta T}{R}$$

$$Q = \frac{T_a - T_b}{\frac{1}{2\pi L}\left[\frac{1}{h_a r_1} + \frac{\ln\left[\frac{r_2}{r_1}\right]}{k_1} + \frac{\ln\left[\frac{r_3}{r_2}\right]}{k_2} + \frac{1}{h_b r_3}\right]}$$

$$\Rightarrow \frac{Q}{L} = \frac{313 - 283}{\frac{1}{2\pi}\left[\frac{1}{50 \times 0.05} + \frac{\ln\left[\frac{0.09}{0.05}\right]}{0.1} + \frac{\ln\left[\frac{0.12}{0.09}\right]}{0.32} + \frac{1}{15 \times 0.12}\right]}$$

$$\Rightarrow \boxed{\frac{Q}{L} = 24.37 \text{ W/m}}$$

Ex: An Insulated steel pipe carrying a hot liquid. Inner diameter of the pipe is 25cm, wall thick 2cm, thickness of insulation 5cm, temprature of hot liquid is 100°C, temprature of sorrounding is 20°C, Inside heat transfer co-efficient is 730 W/m²K and outside heat transfer co-efficient is 12 W/m²K, Calculate the heat loss per meter length of pipe. Take K_{steel} = 55 W/mK, $K_{insulating\ material}$ = 0.22 W/mK.

Ans: Q/L = 281.178 W/m.

(15)

Ex: A steel pipe of 120 mm inner diameter, 140 mm outer diameter with thermal conductivity 55 W/mK is covered with two layers of insulation each having a thickness of 55 mm. The thermal conductivity of the first insulation material is 0.05 W/mK and that of the second is 0.11 W/mK. The temperature of the inside tube surface is 240°C and that of outside surface of the insulation is 60°C. Calculate loss of heat per meter length of pipe and interface temperature between the two layers of insulation.

Given data:

$r_1 = 0.06 m$

$r_2 = 0.07 m$

$r_3 = r_2 + 0.055 = 0.125 m$

$r_4 = r_3 + 0.055 = 0.18 m$

$K_1 = 55 W/mK$

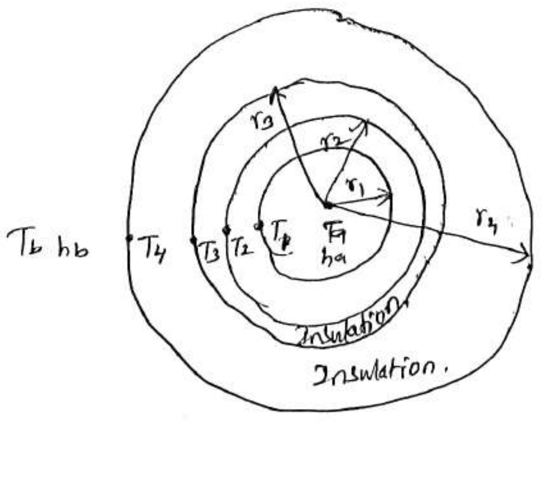
$K_2 = 0.05 W/mK$

$K_3 = 0.11 W/mK$

$T_1 = 513 K$

$T_4 = 333 K$.

Insulated pipe.



To find:

Q/L, T_3

Solution:

$$Q = \frac{\Delta T_{overall}}{R}$$

(5)

$$Q/L = \frac{T_1 - T_4}{\frac{1}{2\pi}\left[\frac{\ln\left[\frac{r_2}{r_1}\right]}{55} + \frac{\ln\left[\frac{r_3}{r_2}\right]}{k_2} + \frac{\ln\left[\frac{r_4}{r_3}\right]}{k_3}\right]}$$

$$\boxed{Q/L = 75.83 \ W/m}$$

WKT,

Interface temprature relation.

$$Q = \frac{T_1 - T_4}{R} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_4}{R_3} \rightarrow (1)$$

$$(1) \Rightarrow \quad Q = \frac{T_1 - T_2}{R_1}$$

$$Q/L = \frac{T_1 - T_2}{\frac{1}{2\pi}\left[\frac{\ln\left[\frac{r_2}{r_1}\right]}{k_1}\right]}$$

$$\Rightarrow \boxed{T_2 = 512.7 \ K}$$

$$(1) \Rightarrow \quad Q = \frac{T_2 - T_3}{R_2}$$

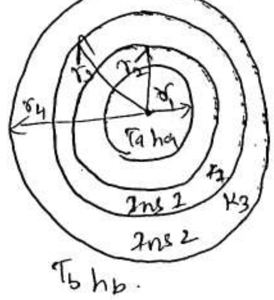
$$Q/L = \frac{T_2 - T_3}{\frac{1}{2\pi}\left[\frac{\ln\left[\frac{r_3}{r_2}\right]}{k_2}\right]}$$

$$\Rightarrow \boxed{T_3 = 372.7 K}$$

(16)

(IV). A steel pipe of 170 mm inner diameter and 190 mm outer diameter with thermal conductivity 55 W/mK is covered with two layers of insulation. The thickness of the first layer is 25 mm ($K = 0.1$ W/mK) and the second layer thickness is 40 mm ($K = 0.18$ W/mK). The temprature of the steam and inner surface of the steam pipe is 320°C and outer surface of the insulation is 80°C. Ambient air temprature is 25°C. The surface co-efficients for inside and outside surfaces are 230 W/m²K and 6 W/m²K respectively. Determine the heat loss per metre length of the steam pipe and layer of contact tempratures and also calculate the overall heat transfer co-efficient.

Given data:



$r_1 = 0.085$ m

$r_2 = 0.095$ m

$r_3 = r_2 + 0.025 \Rightarrow r_3 = 0.12$ m

$r_4 = r_3 + 0.04 \Rightarrow r_4 = 0.16$ m

$K_3 = 0.18$ W/mK, $k_2 = 0.1$ W/mK, $k_1 = 55$ W/mK

$T_a = T_1 = 593$ K

$T_4 = 353$ K

$T_b = 298$ K

$h_a = 230$ W/m²K

$h_b = 6$ W/m²K

To find:

Q/L, T_2, T_3, U

Solution:

$$Q = \frac{\Delta T}{R}$$

$$\frac{Q}{L} = \frac{T_a - T_b}{\frac{1}{2\pi}\left[\frac{1}{h_a r_1} + \frac{\ln\left[\frac{r_2}{r_1}\right]}{k_1} + \frac{\ln\left[\frac{r_3}{r_2}\right]}{k_2} + \frac{\ln\left[\frac{r_4}{r_3}\right]}{k_3} + \frac{1}{h_b r_4}\right]}$$

$$\boxed{Q/L = 368.5\ W/m}$$

WKT,

$$Q = UA\,\Delta T \qquad \because \Delta T = T_a - T_b$$

$$Q/L = U \times 2\pi r_4 (T_a - T_b) \qquad A = 2\pi r_4 L$$

$$368.5 = U \times 2 \times \pi \times 0.16\,(593 - 298)$$

$$\boxed{U = 1.24\ W/m^2 K}$$

WKT,

Interface temperature relation,

$$Q = \frac{T_a - T_b}{R} = \frac{T_a - T_1}{R_a} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_4}{R_3}$$

$$= \frac{T_4 - T_b}{R_b} \longrightarrow (1)$$

(17).

$$Q = \frac{T_1 - T_2}{R_1}$$

$$Q/L = \frac{T_1 - T_2}{\frac{1}{2\pi}\left[\frac{\ln\left[\frac{r_2}{r_1}\right]}{k_1}\right]}$$

$$Q/L = \frac{593 - T_2}{\frac{1}{2\pi}\left[\frac{\ln\left[\frac{0.095}{0.085}\right]}{55}\right]}$$

$$\Rightarrow \boxed{T_2 = 592.9K}$$

III^ly

$$Q = \frac{T_2 - T_3}{R_2}$$

$$Q/L = \frac{T_2 - T_3}{\frac{1}{2\pi}\left[\frac{\ln\left[\frac{r_3}{r_2}\right]}{k_2}\right]}$$

$$\Rightarrow \boxed{T_3 = 455.88\ K}$$

(AU). A hot steam pipe having an inside surface temprature of 250°C has an inside diameter of 80mm and a wall thickness of 5.5mm. It is covered with 90mm layer of insulation having thermal conductivity of 0.5 W/mK followed by a 40mm layer of insulation having thermal conductivity of 0.25 W/mK. The outside surface temprature of insulation is 20°C. Calculate the heat loss per meter length. Assume thermal conductivity of the pipe as 47 W/mK.

Ans: $Q/L = 4248.8 \text{ W/m}$.

(AU). A thick walled tube of stainless steel ($k = 77.85$ kJ/hr m°C) 25 mm ID and 50 mm OD is covered with a 25 mm layer of asbestos [$k = 0.88$ kJ/hr m°C) If the inside wall temperature of the pipe is maintained at 550°C and the outside of the insulator at 45°C. Calculate the heat loss per meter length of pipe.

Given data:

$r_1 = 0.0125 \text{ m}$

$r_2 = 0.025 \text{ m}$

$r_3 = 0.050 \text{ m}$

$k_1 = 77.85 \text{ kJ/hr m°C}$

$= \frac{77.85}{3600} \text{ kJ/s m°C}$

$= 0.0216 \text{ kJ/s m°C}$

$= 0.0216 \times 10^3 \text{ W/m°C}$, $T_a = 550°C$, $T_b = 45°C$

Solution:

$$\boxed{Q/L = 1103.9 \text{ W/m}}$$

18

HEAT CONDUCTION WITH HEAT GENERATION ;

Ex:

1. Electric coils
2. Resistance heater.
3. Nuclear Reactor.
4. Combustion of fuel in the bed of boiler.

FORMULAE USED:

For plane wall ; $\dot{q} = Q/A$

1. Surface temprature $T_w = T_\infty + \frac{\dot{q} L}{2h}$

2. maximum temprature $T_{max} = T_w + \frac{\dot{q} L^2}{8K}$

Where, T_∞ - Fluid temprature K

q - Heat generation W/m^3

L - Thickness, m

h - Heat transfer Co-efficient $[W/m^2K]$

K - thermal Conductivity W/mK.

For cylinder ;

1. Heat generation $\dot{q} = Q/V$

2. maximum temprature $T_{max} = T_w + \frac{\dot{q} r^2}{4K}$

3. Surface temprature $T_w = T_\infty + \frac{r \dot{q}}{2h}$

where, V - Volume $\pi r^2 L$

r - radius - m

For sphere,

1. Temprature at the centre,

$$T_c = T_w + \frac{\dot{q} r^2}{6K}$$

- problems on plane wall : (Internal heat generation).

Ex: An Electric coil current is passed through a plane wall of thickness 150 mm which generates heat at the rate of 50,000 W/m^3. The convective heat transfer co-efficient b/w wall and ambient air is 65 W/m^2K, ambient air temprature is 28°C and the thermal conductivity of wall material is 22 W/mK. Calculate, (1) Surface temprature (2) maximum temprature in the wall.

Given data:

$L = 0.150$ m

$\dot{q} = 50,000\ W/m^3$

$h = 65\ W/m^2 K$

$T_\infty = 28°C + 273 \Rightarrow T_\infty = 301\ K$

$K = 22\ W/mK$.

Solution:

Find Surface temprature,

$$T_w = T_\infty + \frac{\dot{q} L}{2h}$$

$$T_w = 301 + \frac{50000 \times 0.150}{2 \times 65} \rightarrow \boxed{T_w = 358.6\ K}$$

(19).

maximum temprature,

$$T_{max} = T_w + \frac{\dot{q}L^2}{8K}$$

$$= 358.6 + \frac{\dot{q}L^2}{8K}$$

$$\boxed{T_{max} = 364.9K}$$

Ex: An Electric current is passed through a plane of wall thickness 25mm and 120mm wide, which is used to heat a fluid at 30°C. The heat generation is $65 \times 10^5\ W/m^3$. Thermal conductivity of the plate is 25 W/mK. Calculate the heat transfer co-efficient to maintain the temprature of the plane below 150°C.

Given data:

$L = 0.025\ m$

$W = 0.12\ m$

$T_\infty = 30°C + 273 = 303\ K$

$\dot{q} = 65 \times 10^5\ W/m^3$

$k = 25\ W/mK$

$T_{max} = 150°C + 273$

$T_{max} = 423\ K$

Solution:

$$T_{max} = T_w + \frac{\dot{q}L^2}{8K}$$

$$\Rightarrow \boxed{T_w = 402.6K}$$

$$T_w = T_\infty + \frac{\dot{q}L}{2h} \Rightarrow \boxed{h = 815.7\ W/m^2K}$$

overall heat transfer co-efficient $h = 815.7\ W/m^2K$

Ex: An Electric current is passed through a composite wall made up of two layers. First layer is steel of 10 cm thickness and second layer is brass of 8 cm thickness. The outer surface temprature of the steel and brass are maintained at 120°C and 65°C respectively. Assuming that the contact between two slab is perfect and the heat generation is 1,65,000 W/m². Determine (1) heat flux through the outer surface of brass slab. (2) Inter surface temprature, Take K for steel is 45 W/mK for brass is 80 W/mK.

Given data:

$L_1 = 0.10\ m$

$L_2 = 0.08\ m$

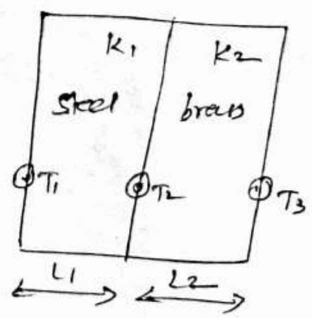
$T_1 = 120°C + 273 \Rightarrow T_1 = 393K$

$T_3 = 65°C + 273 \Rightarrow T_3 = 338K$

$q_g = 1,65,000\ W/m^2$

$K_1 = 45\ W/mK$

$K_2 = 80\ W/mK$



To find:

Heat flux q_2

Innerface temprature T_2.

Solution:

Take, q_1 - Heat flux on steel slab.

q_2 - Heat flux on brass slab.

(2d).

Heat generation $\dot{q}_g = \dot{q}_1 + \dot{q}_2 \rightarrow (1)$

Heat transfer through steel,

$$Q_1 = \frac{\Delta T}{R_1}$$

$$Q_1 = \frac{T_1 - T_2}{L_1/k_1 A}$$

$$Q_1 = \frac{T_2 - T_1}{L_1/k_1 A} \quad [\because T_2 \text{ greater than } T_1] \longrightarrow (2)$$

Heat transfer through brass,

$$Q_2 = \frac{\Delta T}{R_2}$$

$$Q_2 = \frac{T_2 - T_3}{L_2/k_2 A} \longrightarrow (3)$$

Total heat transfer,

$$Q = Q_1 + Q_2$$

$$\frac{Q}{A} = \frac{T_2 - T_1}{L_1/k_1} + \frac{T_2 - T_3}{L_2/k_2}$$

Heat flux (or) Heat generation,

$$\dot{q}_g = \frac{Q}{A} = \frac{T_2 - T_1}{L_1/k_1} + \frac{T_2 - T_3}{L_2/k_2}$$

$$\dot{q}_g = \frac{T_2 - 393}{0.10/45} + \frac{T_2 - 338}{0.08/80}$$

$$1{,}65{,}000 = \frac{T_2 - 393}{2.2\times10^{-3}} + \frac{T_2 - 338}{1\times10^{-3}}$$

$$= \frac{T_2}{2.2\times10^{-3}} - \frac{393}{2.2\times10^{-3}} + \frac{T_2}{1\times10^{-3}} - \frac{338}{1\times10^{-3}}$$

$$= T_2\left[454.54 + 1000\right] - 1{,}78{,}636.3 - 338000$$

$$1{,}65{,}000 = T_2\left[1454.54\right] - \left[5{,}16{,}636\right]$$

$$\Rightarrow \boxed{T_2 = 468.6\,K}$$

Heat transfer in steel,

$$\frac{Q_1}{A} = \frac{T_2 - T_1}{L_1/K_1}$$

$$q_1 = \frac{468.6 - 393}{0.10/45}$$

$$\boxed{q_1 = 34020\ W/m^2}$$

(1) $\Rightarrow$

$$\dot{q}_g = \dot{q}_1 + \dot{q}_2$$

$$1{,}65{,}000 = 34{,}020 + \dot{q}_2$$

$$\Rightarrow \boxed{\dot{q}_2 = 1{,}30{,}980\ W/m^2}$$

Heat flux through brass slab $\dot{q}_2 = 1{,}30{,}980\ W/m^2$

(21).

Problems on cylinder : (Internal Heat generation).

Ex: A Copper wire of 40mm diameter carries 250 A and has resistance of 0.25×10^{-4} Ω cm/length Surface temprature of copper is 250°c and the ambient air temprature is 10°c. If the thermal conductivity of the copper wire is 175 W/mK. Calculate, (1) Heat transfer co-efficient between wire surface and air. (2) Maximum temprature in the drive.

Given data:

$d = 0.040m \Rightarrow r = 0.020$ m

current $I = 250$ A

Resistance $R = 0.25 \times 10^{-4}$ Ω cm/length.

$T_w = 250°c + 273 = 523$ K

Ambient air temprature $T_\infty = 10°c + 273 = 283$ K

$K = 175$ W/mK.

To find:

(1) h, (2) T_{max}.

Solution:

Heat transfer $Q = I^2 R$

$= 250^2 \times 0.25 \times 10^{-4}$

$= 1.562$ W/cm

$$\boxed{Q = 1.562 \times 10^2 \text{ W/m}}$$

$$\dot{q} = \frac{Q}{V} = \frac{156}{\pi \times r^2 \times L} \qquad (\because V = \pi r^2 L)$$

$$= \frac{156}{\pi \times 0.02^2 \times 1}$$

$$\boxed{\dot{q} = 124203 \ W/m^3}$$

WKT,

$$T_{max} = T_w + \frac{\dot{q} r^2}{4k}$$

$$\Rightarrow \boxed{T_{max} = 523.07 \ K}$$

WKT,

$$T_w = T_\infty + \frac{r\dot{q}}{2h}$$

$$\Rightarrow \boxed{h = 5.17 \ W/m^2 K}$$

Ex: A copper wire of 1m long is used as a heating element in a 13 kW heater. The copper surface temprature is 1300°C, ambient air temprature is 20°C, out side surface co-efficient is 1.1 kW/m²K. Thermal conductivity and resistance of the copper are 15 W/mK and 0.21 ohm respectively. Calculate the following, (i) Diameter of copper wire.
(ii) Rate of current flow..

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Given data:

$L = 1m$

$Q = 13 \times 10^3 W$

$T_W = 1573 K$

$T_\infty = 295 K$

$h = 1.1 \times 10^3 W/m^2K$

$R = 0.21 \Omega$

To find:

1) d, I

Solution:

$$\dot{q} = \frac{Q}{V} = \frac{13 \times 10^3}{\pi r^2 L}$$

$$\Rightarrow \dot{q} = \frac{4140}{r^2}$$

Surface temprature,

$$T_W = T_\infty + \frac{r\dot{q}}{2h}$$

$$1573 = 295 + \frac{r \times \frac{4140}{r^2}}{2 \times 1.1 \times 10^3}$$

$$1278 = \frac{4140}{r \times 2200}$$

$$\Rightarrow r = 1.47 \times 10^{-3} m \Rightarrow \boxed{d = 2.94 \times 10^{-3} m}$$

WKT,

$$Q = I^2 R$$

$$13 \times 10^3 = I^2 \times 0.21$$

$$\boxed{I = 248 A}$$

FINS:

It is possible to increase the heat transfer rate by increasing the surface of heat transfer. The surfaces used for increasing heat transfer are called extended surfaces (or) fins.

TYPES OF FINS:

(i) Uniform straight fin.
(ii) Tapered straight fin.
(iii) Splines.
(iv) Annular fin.
(v) pin fins.

FIN EFFICIENCY:

Fin efficiency $\eta_{fin} = \frac{Q_{fin}}{Q_{max}}$

Insulated fin efficiency $\eta_{fin} = \frac{\tanh(mL)}{mL}$

FIN EFFECTIVENESS:

$\varepsilon = \frac{Q_{with\ fin}}{Q_{without\ fin}}$

for insulated fin

$\varepsilon = \frac{\tanh(mL)}{\sqrt{\frac{hA}{KP}}}$

Common three types:

1. infinitely long fin.
2. Short fin (end insulated)
3. Short fin (end not insulated).

Applications:

1. Cooling of electronic components.
2. Cooling of motor cycle Engines.
3. Cooling of small capacity compressors.
4. Cooling of transfers.
5. Cooling of radiators.

(23).

Formulae :

1. Infinitely long fin (or) long fin.

(a) Temprature distribution:

$$\frac{T - T_\infty}{T_b - T_\infty} = e^{-mx}$$

where,

T_b - Base temprature, K

T_∞ - Sorrounding temprature K

T - Intermediate temprature K

x - Distance, m

$$m = \sqrt{\frac{hP}{KA}}$$

h - heat transfer co-efficient W/m^2K

P - perimeter (m)

K - Thermal conductivity W/mK

A - Area. m^2

(b) Heat transfered,

$$Q = (T_b - T_\infty)\sqrt{hPKA}$$

2. Short fin [end Insulated ($\frac{L}{d} < 30$)]

(a) Temprature distribution,

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh m[L-x]}{\cosh[mL]}$$

(b) Heat transfered,

$$Q = (hPKA)^{1/2}(T_b - T_\infty)\tanh(mL)$$

Ex: Find the heat loss from a rod of 3mm in dia. and infinitely long when its base is maintained at 140°C. The conductivity of the material is 150 W/mK and the heat transfer coefficient on the surface of the rod is 300 W/m²K. The temprature of the air surrounding the rod is 15°C.

Given:

$d = 3 \times 10^{-3}$ m

$T_b = 140 + 273 = 413$ K

$T_\infty = 15 + 273 = 288$ K

$K = 150$ W/mK

$h = 300$ W/m²K

To find:

Q

Solution:

$Q = (T_b - T_\infty)\sqrt{hPKA}$ (HMT book 50)

Area $= \frac{\pi}{4} d^2$

$= \frac{\pi}{4}(3 \times 10^{-3})^2 \Rightarrow \boxed{A = 7.06 \times 10^{-6}\ m^2}$

perimeter $= P = \pi d$

$\boxed{P = 9.42 \times 10^{-3}\ m}$

Apply all values,

$\boxed{Q = 6.83\ W}$

(24)

Ex: A long rod ~~5mm~~ 5 cm diameter its base is connected to a furnace wall at 150°C, while the end is projecting into the room at 20°C. The temprature of the rod at distance of 20 cm apart from its base is 60°C. The conductivity of the material is 200 W/mK. Determine convective heat transfer co-efficient.

Given:

$d = 5 \times 10^{-2}\ m$

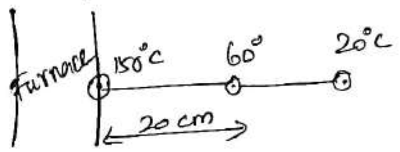
$T_b = 150 + 273 = 423\ K$

$T_\infty = 20 + 273 = 293\ K$

$x = 0.2\ m$

$T = 60 + 273 = 333\ K$

$K = 200\ W/mK.$



To find:

h

Solution:

$$\frac{T - T_\infty}{T_b - T_\infty} = e^{-mx} \qquad (\text{HMT DB } 50)$$

$$\frac{333 - 293}{423 - 293} = e^{-m \times 0.2}$$

$$0.307 = e^{-m \times 0.2}$$

$\ln(0.307) = -m \times 0.2$

$\Rightarrow \boxed{m = 5.9 \ m^{-1}}$

WKT,

$m = \sqrt{\frac{hP}{KA}}$ [From HMT DB 50]

$A = \frac{\pi}{4} d^2$

$\boxed{A = 1.96 \times 10^{-3} \ m^2}$

$P = \pi d$

$\boxed{P = 0.157}$

$m = \sqrt{\frac{hP}{KA}}$

$5.9 = \sqrt{\frac{h \times 0.157}{200 \times 1.96 \times 10^{-3}}}$

$5.9 = \sqrt{0.40 h}$ (Square on both sides)

$\Rightarrow \boxed{h = 87.02 \ W/m^2 K}$

Result:

$h = 87.02 \ W/m^2 K$

(25)

Ex.

One end of the long solid rod of 50 mm diameter is inserted into a into a furnace with the other end is projecting the atomosphere at 25°C. once the steady state is reached, the temprature of the rod is measured at two points 20 cm apart are found to be 150°C and 100°C. The Connective heat transfer Co-efficient between the rod and the sorrounding air is 30 W/m²K. Calculate the thermal conductivity of the rod material.

Given :

$d = 0.05 m$

$T_\infty = 25 + 273 = 298 K$

$x = 0.2 m$

$T_b = 150°C + 273 = 423 K$

$T = 100°C + 273 = 373 K$

$h = 30 W/m^2 K$

To find:

k

Solution:

$$\frac{T - T_\infty}{T_b - T_\infty} = e^{-mx}$$

$$0.6 = e^{-m \times 0.20}$$

$$\ln(0.6) = -m \times 0.2$$

$$-0.51 = -m \times 0.2$$

$$\boxed{m = 2.55\ m^{-1}}$$

WKT,

$$m = \sqrt{\frac{hP}{KA}} \qquad (\text{HMT DB 50})$$

$$2.55 = \sqrt{\frac{hP}{KA}}$$

$$P = \pi d = 0.157\ m$$

$$A = \frac{\pi}{4} d^2 = 1.96 \times 10^{-3}\ m^2$$

WKT,

$$m = \sqrt{\frac{hP}{KA}}$$

$$2.55 = \sqrt{\frac{30 \times 0.157}{K \times 1.96 \times 10^{-3}}}$$

$$\Rightarrow \boxed{K = 369.7\ W/mK}$$

<u>Result;</u>

$$K = 369.7\ W/mK$$

(26).

(AU). An Aluminium rod ($K = 204$ W/mK) 2 cm in diameter and 20 cm long protrudes from a wall which is maintained at $300^\circ C$. The end of the rod is insulated and the surface of the rod is exposed to air at $30^\circ C$. The heat transfer co-efficient between the rod surface and air is $10 W/m^2K$. Calculate the heat lost by rod and the temprature of the rod at a distance of 10cm from the wall.

Given :

$K = 204$ W/mK

$d = 0.02$ m

$L = 0.2$ m

$T_b = 300^\circ C + 273 = 573$ K

$T_\infty = 30^\circ C + 273 = 303$ K

$h = 10 W/m^2 K$

To find :

Heat lost Q,

T at 10 cm from wall.

Solution:

$d = 0.02$ m

$L = 0.2$ m

$L/d = 10 < 30$

Short fin (end insulated).

$$Q = [hPKA]^{1/2} (T_b - T_\infty) \tanh(mL) \quad \text{(HMT DB 50)} \longrightarrow (1)$$

$$A = \frac{\pi}{4} d^2 = 3.14 \times 10^{-4}\, m^2$$

$$P = \pi d = 0.0628\, m$$

$$m = \sqrt{\frac{hP}{KA}}$$

$$\boxed{m = 3.13\, m^{-1}}$$

$$(1) \Rightarrow \quad Q = (10 \times 0.0628 \times 204 \times 3.14 \times 10^{-4})^{1/2} \times (573 - 303) \times \tanh(3.13 \times 0.20)$$

$$\boxed{Q = 30.07\, W}$$

Temprature distribution (short fin, end insulated)

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh[m(L - x)]}{\cosh(mL)} \quad \text{[HMT DB 50]}$$

put $x = 0.10m$,

$$\frac{T - 303}{573 - 303} = \frac{\cosh[3.13(0.20 - 0.10)]}{\cosh(3.13 \times 0.20)}$$

$$\frac{T - 303}{573 - 303} = 0.8727$$

$$\Rightarrow \boxed{T = 538.63\, K}$$

(27)

Ex. A turbine blades are made of Stainless steel. Each blade carries 85W heat. The cross sectional area of each blade is $4.5\ cm^2$, perimeter of each blades is 7cm. The gas temprature flowing over the blade is 800°C. Temprature at the root of the blade is 1250°C, thermal conductivity of the blade is 22 W/mK and heat transfer co-efficient is $110\ W/m^2K$. Determine the height of the blade neglecting the heat flow from the gas to the end of blade.

Given data:

$Q = 85\ W$

$A = 4.5 \times 10^{-4}\ m^2$

$P = 7 \times 10^{-2}\ m$

$T_\infty = 800 + 273 = 1073\ K$

$T_b = 1250°C + 273 = 1523\ K$

$K = 22\ W/mK$

$h = 110\ W/m^2K$

To find:

L

Solution:

Short fin, end insulated,

$$Q = (hPKA)^{1/2}\ (T_b - T_\infty)\ \tanh\ [mL]$$

$$85 = (110 \times 7 \times 10^{-2} \times 22 \times 4.5 \times 10^{-4})^{1/2}\ (1523 - 1073)\ \tanh(mL) \rightarrow (1)$$

where, $m = \sqrt{\frac{hP}{KA}}$

$\Rightarrow \boxed{m = 27.8\ m^{-1}}$

(1) $\Rightarrow$

$85 = (110 \times 7 \times 10^{-2} \times 22 \times 4.5 \times 10^{-4})^{1/2}$

$(1523 - 1073)\ \tan h\ (27.8 \times L)$

$85 = 124.2 \times \tan h\ (27.8 \times L)$

$\tan h\ (27.8 \times L) = 0.684$

$27.8\ L = 0.836$

$\boxed{L = 0.030\ m}$

(AU) An Iron fin (end Insulated) having the length of 50 mm, width 100 mm and thickness 5 mm. Assume $K = 210$ KJ/hr.m.°C, and $h = 42$ KJ/m² h°C for the material of the fin and the temperature at fin base is 80°C. Find the amount of heat transfered through the fin. Also find the temperature at tip of the fin, if the atm temperature is 20°C.

Given data:

Short fin- end Insulated.

$L = 0.05$ m

$W = 0.1$ m

$t = 0.005$ m

$$k = 210 \text{ KJ/m h °C} \Rightarrow \frac{210}{3600} \text{ KJ/s m°C}$$

$$\Rightarrow 0.058 \times 10^3 \text{ W/m°C}$$

$$h = 42 \text{ KJ/m}^2\text{h°C} \Rightarrow \frac{42}{3600} \text{ KJ/s m}^2\text{°C}$$

$$\Rightarrow 0.011 \times 10^3 \text{ W/m}^2\text{°C}$$

$T_b = 80°C$

$T_\infty = 20°C.$

To find:

Q, T

Solution :

WKT,

$$Q = (hPKA)^{1/2} (T_b - T_\infty) \tanh(mL) \quad \rightarrow (13)$$

$$m = \sqrt{\frac{hP}{KA}}$$

$$m = 8.9 \ m^{-1}$$

$$P = 2w + 2t$$

$$P = (2 \times 0.12) + [2 \times 0.005]$$

$$\boxed{P = 0.24 \ m}$$

$$A = w \times t \Rightarrow \boxed{A = 5 \times 10^{-4} m^2}$$

Subs All values in (13)

we get $\boxed{Q = 6.5 \quad W}$

WKT,

Temperature distribution,

$$\frac{T - T_\infty}{T_b - T_\infty} = \cancel{(hPKA)^{1/2}} \frac{\cosh(m[L - x])}{\cosh(mL)}$$

(where $L = x$)

[WKT, $\cosh 0 = 1$]

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{1}{\cosh(mL)}$$

$$\Rightarrow \boxed{T = \cancel{345.52} \ 347.57 \ K}$$

(27(ii))

Ex: A Stainless steel cylindrical rod fin of 1.2 cm diameter and 6 cm height with thermal conductivity of 25 W/mK is exposed to sorrounding with a temperature of 60°C. The heat transfer co-efficient is 45 W/m²K and the temperature at the base of the fin is 100°C. Determine the

1) Fin efficiency.
2) Temperature at the edge of rod.
3) Heat dissipation.
4) Fin effectiveness.

Assume fin is end insulated.

Given data:

$d = 1.2 \times 10^{-2}$ m

$L = 6 \times 10^{-2}$ m

$k = 25$ W/mK

$T_\infty = 333$ K

$h = 45$ W/m²K

$T_b = 373$ K

To find:

$\eta_{fin}, T_{x=L}, Q, E$

Solution:

1. Fin efficiency, (end insulated).

$$\eta_{fin} = \frac{\tan h(mL)}{mL}$$ (From HMT DB 50)

$$m = \sqrt{\frac{hP}{KA}}$$

$$P = \pi d = 0.0376\, m$$

$$A = \pi r^2 = 1.13 \times 10^{-4}\, m^2$$

$$\Rightarrow \boxed{m = 24.4\, m^{-1}}$$

$$\boxed{\eta_{fin} = 61\,\%}$$

2. Temperature at the edge,

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh[m(L-x)]}{\cosh(mL)}$$

where, $(L = x)$

$$\Rightarrow \boxed{T = 350.5\, K}$$

3. Heat dissipation (Q)

$$Q = (hPKA)^{1/2}(T_b - T_\infty) \tanh(mL)$$

$$\boxed{Q = 2.48\, W}$$

4. Fin effectiveness

$$E = \frac{\tanh(mL)}{\sqrt{\frac{hA}{KP}}}$$

$$\Rightarrow \boxed{E = 12.2}$$

27 (iii)

(iv). A heating unit in the form of cylinder is 6cm diameter and 1.2m long. It is provided with 20 longitudinal fins 3mm thick which protrude 50mm from the surface of the cylinder. The temperature at the base of the fin is 80°C. The ambient temperature is 25°C. The film heat transfer co-efficient from the cylinder and fins to the surrounding air is 10 W/m²K. Calculate the rate of heat transfer from the finned wall to the surrounding. Take k = 90 W/mK.

Given data:

$D = 6 \times 10^{-2}$ m

$L_{cy} = 1.2$ m

$T_b = 353$ K

$T_\infty = 298$ K

$h = 10$ W/m²K

$L_f = 50 \times 10^{-3}$ m

$t = 3 \times 10^{-3}$ m

$k = 90$ W/mK.

Solution:

WKT,

Assume (Short-fin-end-insulated)

Q_1 = Fin Heat transfer.

Q_2 = Cylinder wall Heat transfer.

Total heat transfer,

$$Q = Q_1 + Q_2$$

WKT,

$$Q_1 = (hPKA)^{1/2} (T_b - T_\infty) \tanh(mL) \rightarrow (B).$$

Square fin.

$$P = 4 \times t = 0.012\ m$$
$$A = t^2 = 9 \times 10^{-6}\ m^2$$

where, $m = \sqrt{\frac{hP}{KA}}$. $\Rightarrow \boxed{m = 12.2\ m^{-1}}$

Then,

subs all values in (B),

we get $\boxed{Q_1 = 0.29\ W}$

for 20 fins

$$\boxed{Q_1 = 20 \times 0.29 \Rightarrow Q_1 = 5.8\ W}$$

WKT,

Heat transfer in cylinder wall,

$$Q_2 = hA\Delta T.$$

$$Q_2 = h\ [\pi \times D \times L_{cy} - 20 \times t^2]\ (T_b - T_\infty)$$

$$\boxed{Q_2 = 124.3\ W.}$$

$$\therefore Q = Q_1 + Q_2$$

$$\boxed{Q = 130.19\ W}$$

TRANSIENT HEAT CONDUCTION (OR) UNSTEADY STATE :

If the temprature of a body does not vary with time, it is said to be in a steady state.

If the temprature varies with time and the body is to be in an unsteady (or) transient state.

Transient heat conduction types:

(i) periodic flow,

The temprature, varies on regular basis.

Ex: Ic Engine, earth in 24 hrs.

(ii) Non periodic flow,

The temprature at any point with in the system varies non - linearly with time.

Ex: Heating of an ingot in a furbars, cooling of bar

Lumped heat Analysis :

The process in which the internal resistance is assumed as negligible in comparison with its surface resistance is known as newtonian heating (or) cooling process.

In this process the temprature is considered to be uniform at given time. such an analysis is called lumped parameter analysis.

Formulaes:

Biot number,,

$$B_i = \frac{hL_c}{K}$$

k - Thermal Conductivity W/mK

h - heat transfer coefficient (W/m²K)

$$\boxed{L_c = V/A}$$

Lc - Significant length.

$$L_c = \frac{L}{2} \quad (\text{For slab})$$

$$L_c = \frac{R}{2} \quad (\text{For cylinder})$$

$$L_c = \frac{R}{3} \quad (\text{For Sphere})$$

$$L_c = \frac{L}{6} \quad (\text{For Cube})$$

Note:

1. In lumped parameter system, Biot number values is less than 0.1.

$$B_i < 0.1 \Rightarrow \frac{hL_c}{k} < 0.1$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[-\frac{hA}{\rho C_p V} \times t\right]} \quad [\text{HMT DB 58}]$$

T_0 - Initial temprature K

T - Intermediate temprature K.

T_∞ - Surface temprature. K

Ex. (29). A Copper rod of outer diameter 20mm initially at a temprature of 380°C is suddently immersed in a water at 100°C. Determine the time required for the rod to reach 210°C. Take convective heat transfer co-efficient is 95 W/m²K.

Given data:

$D = 0.02\ m$

$R = 0.01\ m$

$T_0 = 653\ K$

$T_\infty = 373\ K$ (Final temprature)

$T = 483\ K$

$h = 95\ W/m^2 K$

To find:

Time (t) required to reach (210°C).

Solution:

Properties of copper are,

Density $\rho = 8954\ kg/m^3$

Specific heat $c_p = 383\ J/kg K$ [HMT DB 2]

Thermal Conductivity $k = 386\ W/mK$

For cylinder,

$$L_c = \frac{R}{2}$$

$$L_c = 5 \times 10^{-3}\ m$$

$$B_i = \frac{h L_c}{K} = \frac{95 \times 5 \times 10^{-3}}{386} \Rightarrow B_i = 1.23 \times 10^{-3}$$

$B_i < 0.1$

So, this is Lumped analysis problem,

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[\frac{-hA}{\rho C_p V} \times t\right]} \quad \text{(HMTT PB 58)}$$

$$\frac{483 - 373}{653 - 373} = e^{\left[\frac{-95}{\underbrace{5\times10^{-3}}_{L_c} \times 383 \times 8954} \times t\right]} \quad \left(\because L_c = \frac{V}{A}\right)$$

$$\ln(0.392) = \frac{-95 \times t}{5\times10^{-3} \times 383 \times 8954}$$

$$\boxed{t = 169.03\ s}$$

Result:

Time required to reach 210° C is 169.03 s

Ex:

A 5cm thick copper slab is at 200°c initially and it is suddenly immersed in water. So its surface temprature lowered to 90°C. In one test run, the initial temprature is decreased by 40°c and the time taken is 6minutes. Determine the heat transfer co-efficients by using lumped capacity method of analysis.

(30).

Given data:

$L = 0.05\,m$

$T_0 = 473K$

$T_\infty = 363K$

$T = 200^\circ C - 40^\circ C = 160^\circ C + 273 = 433\,K$

$t = 6\,min = 360$ seconds.

To find:

h,

Solution:

properties of copper are,

$\rho = 8954\ kg/m^3$

$C_p = 383\ J/kg\,K$ (HMT DB 2)

$K = 386\ W/mK$

For Slab,

$L_c = \frac{L}{2}$

$L_c = 0.025$

For lumped system,

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[\frac{-hA}{\rho C_p V} \times t\right]}$$

(HMT DB 58)

$$\frac{433 - 363}{473 - 363} = e^{\left[\frac{-h \times 360}{8954 \times 383 \times 0.025}\right]}$$ $\because L_c = \frac{V}{A}$

$$\ln(0.636) = \frac{-h \times 360}{8954 \times 383 \times 0.025}$$

$$\Rightarrow \boxed{h = 107.77 \ W/m^2K}$$

(A0) An Aluminium cube 6cm on a side is originally at a temprature of 500°c. It is suddenly immersed in a liquid at 10°c for which h is 120 W/m²K. Estimate the time required for the cube to reach a temprature of 250°c. For aluminium $\rho = 2700$ kg/m³, $C_p = 900$ J/kgK, $K = 204$ W/mK.

Given data:

$L = 0.06 \, m$

$T_0 = 500 + 273 = 773 \, K$

$T_\infty = 10°c + 273 = 283 \, K$

$T = 250°c + 273 = 523 \, K$

$h = 120 \, W/m^2K$

$\rho = 2700 \, kg/m^3$

$C_p = 900 \, J/kgK$

$K = 204 \, W/mK.$

Solution:

$$L_c = \frac{L}{6} = 0.01 \, m$$

$$B_i = \frac{hL_c}{K} \Rightarrow B_i = 5.88 \times 10^{-3} < 0.1$$

WKT,

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[\frac{-hA}{C_p V \rho} \times t\right]}$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[\frac{-h \, t}{C_p L_c \, \rho}\right]}$$

$$\Rightarrow \boxed{t = 144.86 \, s}$$