Reg. No. : | | | | | | | | | | | |

Question Paper Code : 90338

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2019

Third Semester

Civil Engineering

MA 8353 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Aeronautical Engineering/Aerospace Engineering/Agriculture Engineering/Automobile Engineering/Electrical and Electronics Engineering/Electronics and Instrumentation Engineering/Industrial Engineering/Industrial Engineering and Management/Instrumentation and Control Engineering/Manufacturing Engineering/Marine Engineering/Material Science and Engineering/Mechanical Engineering/Mechanical Engineering (Sandwich)/Mechanical and Automation Engineering/Mechatronics Engineering/Production Engineering/Robotics and Automation Engineering/Bio Technology/Chemical and Electrochemical Engineering/Food Technology/Pharmaceutical Technology)

(Regulations 2017)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions.

PART – A (10×2=20 Marks)

1. Find the complete solution of $p = 2qx$.

2. Solve $(D^2 - 6DD' + 9D'^2)\,z = 0$.

3. State the Dirichlet's conditions.

4. Sketch the even extension of the function $f(x) = \sin x$, $0 < x < \pi$.

5. Classify the two-dimensional steady state heat conduction equation.

6. Give the mathematical formulation of the problem of one-dimensional heat conduction in a rod of length l with insulated ends and with initial temperature $f(x)$.

7. State the convolution theorem for Fourier Transforms.

8. Show that $\mathfrak{I}_c\left[f(x)\cos ax\right] = \frac{1}{2}\{F_c(s+a)+F_c(s-a)\}$ where $\mathfrak{I}_c\left[f(x)\right] = F_c(s)$ is the Fourier cosine transform of f (x).

9. Show that $Z\left[a^n f(n)\right] = F\left(\frac{z}{a}\right)$ where $Z\left[f(n)\right] = F(z)$ is the Z-transform of f (x).

10. State the initial and final value theorems of Z-transforms.

PART – B **(5×16=80 Marks)**

11. a) i) Solve $(D^3 - 2D^2 D')z = \sin(x + 2y) + 3x^2 y$. **(10)**

ii) Form the partial differential equation by eliminating the arbitrary functions from $u = f(x + ct) + g(x - ct)$. **(6)**

(OR)

b) i) Solve $(x^2 - yz)p + (y^2 - zx)q = (z^2 - xy)$. **(10)**

ii) Solve $p - x^2 = q + y^2$. **(6)**

12. a) i) Find the Fourier series of $f(x) = x^2$ in $(0, 2l)$. Hence deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots\infty = \frac{\pi^2}{6}$. **(10)**

ii) Find the complex form of the Fourier series of $f(x) = \cos ax$ in $(-\pi, \pi)$, where 'a' is neither zero nor an integer. **(6)**

(OR)

b) i) Obtain the constant term and the first three harmonics in the Fourier Cosine series of $y = f(x)$ in (0, 6) from the following table. **(10)**

x	0	1	2	3	4	5
y	4	8	15	7	6	2

ii) Find the Fourier series expansion of $f(x) = \sin ax$ in $(-l, l)$. **(6)**

13. a) i) Solve $u_t = a^2 u_{xx}$ by the method of separation of variables and obtain all possible solutions. **(8)**

ii) A rectangular plate with insulated surfaces is 8 cm wide and so long compared to its width that it may be considered as an infinite plate. If the temperature along the short edge y = 0 is $u(x, 0) = 100\sin\left(\frac{\pi x}{8}\right)$, $0 < x < 8$ while two long edges x = 0 & x = 8 as well as the other short edge are kept at 0°C, then find the steady state temperature at any point of the plate. **(8)**

(OR)

b) i) Solve the problem of a tightly stretched string with fixed end points $x = 0$ & $x = l$ which is initially in the position $y = f(x)$ and which is initially set vibrating by giving to each of its points a velocity $\frac{dy}{dt} = g(x)$ at $t = 0$. **(10)**

ii) Classify the partial differential equation $(1 - x^2) f_{xx} - 2xyf_{xy} + (1 - y^2) f_{yy} = 0$. **(6)**

14. a) i) Find the Fourier transform of $f(x)$ where $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a > 0 \end{cases}$ and hence evaluate $\int_0^{\infty} \frac{\sin x}{x} dx$. **(10)**

ii) Show that $\frac{1}{\sqrt{x}}$ is self-reciprocal under the Fourier cosine transform. **(6)**

(OR)

b) i) Find the Fourier cosine and sine transforms of e^{-ax}, $a > 0$ and hence deduce their inversion formulae. **(10)**

ii) Using Parseval's identity, evaluate $\int_0^{\infty} \frac{dx}{(x^2 + a^2)^2}$, $a > 0$. **(6)**

15. a) i) Find $Z\{\sin bt\}$ and hence find $Z\{e^{-at} \sin bt\}$. **(8)**

ii) Find $Z^{-1}\left\{\frac{8z^2}{(2z-1)(4z+1)}\right\}$ using convolution theorem. **(8)**

(OR)

b) i) Using Z-transforms, solve the difference equation $y_{n+2} - 7y_{n+1} + 12y_n = 2^n$ given $y_0 = y_1 = 0$. Use partial fraction method to find the inverse Z-transform. **(8)**

ii) Using residue method, find $Z^{-1}\left\{\frac{z}{z^2 + 2z + 2}\right\}$. **(8)**
