Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : 40786

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

Third Semester

Civil Engineering

MA 8353 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to : Aeronautical Engineering/Aerospace Engineering/ Agriculture Engineering/Automobile Engineering/Electrical and Electronics Engineering/Electronics and Instrumentation Engineering/ Industrial Engineering/Industrial Engineering and Management/ Instrumentation and Control Engineering/Manufacturing Engineering/ Marine Engineering/Material Science and Engineering/Mechanical Engineering/Mechanical Engineering (Sandwich)/Mechanical and Automation Engineering/Mechatronics Engineering/Production Engineering/Robotics and Automation/Bio Technology/Biotechnology and Biochemical Engineering/Chemical and Electrochemical Engineering/Food Technology/Pharmaceutical Technology)

(Regulations 2017)

Time : Three hours　　　　　　　　　　　　Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Form the partial differential equation by eliminating the arbitrary constants 'a' and 'b' from the relation $z = a(x+y)+b$.

2. Find the complete solution of $pq = xy$.

3. Fourier series of a function $f(x) = \begin{cases} \pi + x, & -\pi < x < 0 \\ \pi - x, & 0 < x < \pi \end{cases}$ is given by $\frac{\pi}{2} + \frac{4}{\pi}\left(\sum_{n=1,\,3,\,5,\ldots} \frac{1}{n^2}\cos nx\right)$. What is the function represented by the same Fourier series the interval $(\pi, 3\pi)$?

4. Find the Fourier series of a function $f(x)$ up to the first harmonic from the following data $n = 12$, $\sum f(x) = 50.090$, $\sum f(x)\cos x = 14.699$ and $\sum f(x)\sin x = 18.962$.

5. Classify the partial differential equation

$$x^2 \frac{\partial^2 u}{\partial x^2} - 2xy \frac{\partial^2 u}{\partial x \, \partial y} + \left(1 + y^2\right) \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} + 3 \frac{\partial u}{\partial y} - 5u = x .$$

6. Find the steady state temperature of a rod of length 10cm whose ends are kept at 30°C and 40°C respectively.

7. State the exponential form of Fourier integral theorem.

8. Find the Fourier sine transform of a function $f(x) = e^{-2x} + 4e^{-3x}$.

9. Find the Z-transform of $(-1)^n$.

10. State initial value theorem in Z-transforms.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the singular solution of $z = px + qy + p^2 + pq + q^2$. (8)

(ii) Solve $\left(D^3 - 6D^2D' + 12DD'^2 - 8D'^3\right)z = e^{2x+y}$. (8)

Or

(b) (i) Find the general solution of $\left(z^2 - 2yz - y^2\right)\frac{\partial z}{\partial x} + x(y+z)\frac{\partial z}{\partial y} = x(y - z)$. (8)

(ii) Solve $\left(D^2 - 5DD' + 6D'^2\right) z = y \sin x$. (8)

12. (a) (i) Find the Fourier series for the function $f(x) = |x|, -l < x < l$. Hence find the value of $1^{-2} + 3^{-2} + 5^{-2} + \ldots..$ (8)

(ii) Find the half-range Fourier sine series for $f(x) = x(\pi - x)$ in $(0, \pi)$, and hence show that $1 - \frac{1}{3^3} + \frac{1}{5^3} - \ldots. = \frac{\pi^3}{32}$. (8)

Or

(b) (i) Find the complex form of the Fourier series of a function $f(x) = e^{-ax}$ in the interval $(-l, l)$. (8)

(ii) Find the half-range cosine series for $f(x) = \begin{cases} x, & 0 < x < \pi/2 \\ \pi - x & \pi/2 < x < \pi \end{cases}$.

Hence deduce the value $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$. (8)

13. (a) A string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form of the curve $y = x(l - x)$ and also by imparting a constant velocity 'k' to every point of the string in this position at time $t = 0$. Find the displacement function $y(x,t)$. (16)

Or

(b) Find the steady state temperature distribution in a rectangular plate of sides 'a' and 'b' which is insulated on the lateral surface and three of whose edges $x = 0$, $x = a$, $y = b$ are kept at zero temperature, if the temperature in the edge $y = 0$ is given by $3\sin\frac{2\pi x}{a} + 2\sin\frac{3\pi x}{a}$. (16)

14. (a) (i) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & -1 < x < 1 \\ 0 & otherwise \end{cases}$. Hence evaluate $\int_0^\infty \left(\frac{\sin x - x\cos x}{x^3}\right)\cos\frac{x}{2}\,dx$. (10)

(ii) Use Fourier cosine transforms method to evaluate $\int_0^\infty \frac{1}{(x^2+1)(x^2+4)}\,dx$. (6)

Or

(b) (i) Find the Fourier transform of $f(x) = \frac{\sin ax}{x}$ and hence find the value of $\int_{-\infty}^{\infty} \frac{\sin^2 ax}{x^2}\,dx$, using Parseval's identity. (8)

(ii) Find the Fourier cosine transform of e^{-ax}. Use it to find the Fourier transform of $e^{-a|x|}\cos bx$. (8)

15. (a) (i) Find the Z-transform of $\frac{2n+3}{(n+1)(n+2)}$. (8)

(ii) Solve, by using Z-transform, the equation $y_{n+2} + 4y_{n+1} + 4y_n = n$, given that $y_0 = 0$ and $y_1 = 1$. (8)

Or

(b) (i) Find the Z – transform of $f(n) * g(n)$, where $f(n) = \begin{cases} (1/3)^n, & n \geq 0 \\ (1/2)^{-n}, & n < 0 \end{cases}$ and $g(n) = (1/2)^n U(n)$. (8)

(ii) Find the inverse Z - transform of $\dfrac{4z^3}{(2z-1)^2 (z-1)}$, by using the method of partial fractions. (8)
