Reg. No. :

## Question Paper Code : 50779

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2017
Third Semester
Civil Engineering
MA 6351 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Mechanical Engineering (Sandwich)/ Aeronautical Engineering/ Agriculture Engineering/ Automobile Engineering/ Biomedical Engineering/ Computer Science and Engineering/ Electrical and Electronics Engineering/ Electronics and Communication Engineering/ Electronics and Instrumentation Engineering/ Geoinformatics Engineering/ Industrial Engineering/ Industrial Engineering and Management/ Instrumentation and Control Engineering/ Manufacturing Engineering/ Marine Engineering/ Materials Science and Engineering/Mechanical Engineering/Mechanical and Automation Engineering/ Mechatronics Engineering/ Medical Electronics/ Petrochemical Engineering/ Production Engineering/ Robotics and Automation Engineering/ Biotechnology, Chemical Engineering/ Chemical and Electrochemical Engineering/ Food Technology/ Information Technology/ Petrochemical Technology/ Petroleum Engineering/ Plastic Technology/Polymer Technology)
(Regulations 2013)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions.

PART – A (10×2=20 Marks)

1. Find the partial differential equation by eliminating the arbitrary function 'f' from the relation $z = f(x^2 - y^2)$.

2. Find the complete integral of $\sqrt{p} + \sqrt{q} = 1$.

3. State Dirichlet's conditions for a given function f(x) to be expanded in Fourier series.

4. Write the complex form of Fourier series for a function f(x) defined in $-l < x < l$.

5. What is the basic difference between the solutions of one dimensional wave equation and one dimensional heat equation ?

6. State any two solutions of the Laplace equation $u_{xx} + u_{yy} = 0$ involving exponential terms in x or y.

7. If F [f(x)] = F(s), then find F[f(ax)].

8. State the convolution theorem for Fourier transforms.

9. Find the Z-transform of the function f(n) = 1/n.

10. Form the difference equation by eliminating arbitrary constant 'a' from $y_n = a.\,2^n$.

PART – B (5×16=80 Marks)

11. a) i) Find the singular integral of $z = px + qy + p^2 - q^2$. (8)

ii) Find the general integral of $(x - 2z)\,p + (2z - y)\,q = y - x$. (8)

(OR)

b) Solve the following equations.

i) $(D^2 + 2DD' + D'^2)\,z = e^{x-y} + xy$ (8)

ii) $(D^2 - 5DD' + 6D'^2)\,z = y \sin x$. (8)

12. a) i) Find the Fourier series for a function $f(x) = x + x^2$ in $(-\pi, \pi)$ and hence deduce the value of $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots.$ (8)

ii) Find the Fourier series of y = f(x) up to first harmonic which is defined by the following data in $(0, 2\pi)$ : (8)

| x | 0 | $\pi/3$ | $2\pi/3$ | $\pi$ | $4\pi/3$ | $5\pi/3$ | $2\pi$ |
|---|---|---|---|---|---|---|---|
| f(x) | 1 | 1.4 | 1.9 | 1.7 | 1.5 | 1.2 | 1 |

(OR)

b) i) Find the half-range cosine series for f(x) = x in $(0, \pi)$. Hence deduce the value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots.$ (8)

ii) Find the Fourier series for a function $f(x) = \begin{cases} l - x, & 0 < x \le l \\ 0, & l < x \le 2l \end{cases}$ in $(0, 2l)$. (8)

13. a) A tightly stretched string of length $l$ has its end fastened at $x = 0$, $x = l$. At $t = 0$, the string is in the form $f(x) = kx(l - x)$ and then released. Find the displacement at any point of the string at a distance $x$ from one end and at any time $t > 0$. **(16)**

(OR)

b) A rod of length $l$ cm has its ends A and B kept at 0°C and 100°C respectively, until steady state conditions prevail. If the temperature at B is suddenly reduced to 0°C and maintained at 0°C, find the temperature distribution $u(x, t)$ at a distance x from A at any time t. **(16)**

14. a) i) If $F_S(s)$ and $F_C(s)$ denote Fourier sine and cosine transform of a function $f(x)$ respectively, then show that

$$F_S\{f(x) \sin ax\} = \frac{1}{2}\{F_C(s-a) - F_C(s+a)\}$$ **(4)**

ii) Find the Fourier transform of a function $f(x) = \begin{cases} 1-|x| & \text{if } -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$ and hence

find the value of $\int_0^\infty \frac{\sin^4 t}{t^4} dt$ by Parseval's identity. **(12)**

(OR)

b) Find the Fourier sine and cosine transforms of a function $f(x) = e^{-x}$. Using Parseval's identity, evaluate :

(1) $\int_0^\infty \frac{dx}{(x^2+1)^2}$ and (2) $\int_0^\infty \frac{x^2 dx}{(x^2+1)^2}$ **(16)**

15. a) i) Find the Z-transform of $\frac{2n+3}{(n+1)(n+2)}$. **(8)**

ii) Find $Z^{-1}\left[\frac{z^2}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}\right]$ by using convolution theorem. **(8)**

(OR)

b) i) Find the inverse Z-transform of $\frac{z^3}{(z-1)^2(z-2)}$ by method of partial fraction. **(6)**

ii) Solve the difference equation $y(n+2) - 7y(n+1) + 12y(n) = 2^n$, given that $y(0) = 0$ and $y(1) = 0$, by using Z-transform. **(10)**