

Reg. No. :

Question Paper Code : X 60767

B.E./B.Tech. DEGREE EXAMINATIONS, NOV./DEC. 2020 Second/Third Semester Civil Engineering MA 2211/10177 MA 301/MA 1201 A/080100008/080210001/ MA 31 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS (Common to all Branches) (Regulations 2008/2010) (Also Common to PTMA2211 for B.E.(Part-Time) Second Semester-All Branches - Regulations 2009) hree Hours Maximum : 100 Marks

Time : Three Hours

Answer ALL questions.

PART - A

(10×2=20 Marks)

- 1. Find the value of a_0 in the Fourier series expansion of $f(x) = e^x$ in $(0, 2\pi)$.
- 2. Find the half range sine series expansion of f(x) = 1 in (0, 2).
- 3. State Fourier integral theorem.
- 4. Find the Fourier sine transform of $f(x) = e^{-x/2}$.
- 5. Form the PDE from $(x a)^2 + (y b)^2 + z^2 = r^2$.
- 6. Find the complete integral of p + q = pq.
- 7. What is the basic difference between the solutions of one dimensional wave equation and one dimensional heat equation with respect to the time ?
- 8. Write down the partial differential equation that represents steady state heat flow in two dimensions and name the variables involved.
- 9. Find the Z-transform of a^n .
- 10. Solve $y_{n+1} 2 y_n = 0$, given that y(0) = 2.

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PART – B (5×16=80 Marks)

11. a) i) Find the Fourier series of x^2 in $(-\pi, \pi)$ and hence deduce that $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots \frac{\pi^4}{90}.$ (8)

ii) Obtain the Fourier cosine series of $f(x) = \begin{cases} kx, & 0 < x < \frac{1}{2} \\ k(l-x), & \frac{1}{2} < x < l \end{cases}$ (8) (OR)

- b) i) Find the complex form of Fourier series of $\cos ax in (-\pi, \pi)$, where "a" is not an integer. (8)
 - ii) Obtain the Fourier cosine series of $(x 1)^2$, 0 < x < 1 and hence show that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$. (8)

12. a) i) Find the Fourier transform of $f(\mathbf{x}) = \begin{cases} 1 - |\mathbf{x}| & \text{if } |\mathbf{x}| < 1 \\ 0, & \text{if } |\mathbf{x}| > 1 \end{cases}$ and hence evaluate $\int_{0}^{\infty} \frac{\sin^{4} t}{t^{4}} dt$. (8)

ii) Find the Fourier transform of $f(\mathbf{x}) = \begin{cases} a^2 - \mathbf{x}^2, & |\mathbf{x}| \le a \\ 0, & |\mathbf{x}| > a > 0 \end{cases}$. Hence deduce that $\int_{0}^{\infty} \frac{\sin t - t \cos t}{t^3} dt = \frac{\pi}{4}$. (8)

- b) i) Find the Fourier cosine and sine transforms of $f(x) = e^{-ax}$, a > 0 and hence deduce the inversion formula. (8)
 - ii) Find the Fourier cosine transform of $e^{-a^2x^2}$, a > 0. Hence show that the function $e^{-x^2/2}$ is self-reciprocal. (8)
- 13. a) i) From the PDE by eliminating the arbitrary functions f_1 , f_2 from the relation $Z = xf_1(x + t) + f_2(x + t)$. (8)

ii) Solve
$$\left(\frac{p}{2} + x\right)^2 + \left(\frac{q}{2} + y\right)^2 = 1$$
. (8)
(OR)

- b) i) Solve $x^2p + y^2q = z(x + y)$. (8)
 - ii) Solve $(r + s 6t) = y \cos x$. (8)

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- 14. a) A tightly stretched string with fixed end points x = 0 and x = l is initially in
 - a position given by $y(x, 0) = K(lx x^2)$. It is released from rest from this positions. Find the expression for the displacement at any time 't'. (16)

(OR)

b) Find the solution to the equation $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ that satisfies the conditions,

$$u(0, t) = 0, u(l, t) = 0 \text{ for } t > 0 \text{ and } u(x, 0) = \begin{cases} x, & 0 \le x \le l/2 \\ l-x, & l/2 < x < l \end{cases}$$
(16)

15. a) i) Find
$$Z[n(n-1)(n-2)]$$
.

ii) Using Convolution theorem, find the inverse Z-transform of

$$\frac{8z^2}{(2z-1)(4z-1)}.$$
(OR)
(8)

- b) i) Solve the difference equation y(k + 2) + y(k) = 1, y(0) = y(1) = 0, using Z-transform. (8)
 - ii) Solve $y_{n+2} + y_n = 2^n.n$, using Z-transform. (8)

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(8)