Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 80765

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

Third Semester

Civil Engineering

MA 2211/MA 1201 A/080100008/080210001/10177 MA 301/CK 201/MA 31 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to All Branches)

(Regulations 2008/2010)

(Also Common to PTMA 2211 for B.E. (Part-Time) Second Semester — All Branches — Regulations 2009)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Find the constant term in the expansion of $\cos^2 x$ as a Fourier series in the interval $(-\pi, \pi)$.

2. Define Root Mean square value of a function $f(x)$ over the interval (a, b).

3. Find the Fourier Sine Transform of e^{-3x}.

4. If $\mathcal{F}\{f(x)\} = F(s)$, prove that $\mathcal{F}\{f(ax)\} = \frac{1}{a} . F\left(\frac{s}{a}\right)$.

5. Form the PDE by eliminating the arbitrary constants 'a', 'b' from the relation $4\left(1+a^2\right)z = (x + ay + b)^2$.

6. Solve $\left(D^3 - 4D^2D' + 4DD'^2\right)z = 0$.

7. An insulated rot of length 60 cm has its ends at A and B maintained at 20°C and 80°C respectively. Find the steady state solution of the rod.

8. A plate is bounded by the lines $x = 0, y = 0$, $x = l$ and $y = l$. Its faces are insulated. The edge coinciding with x-axis is kept at 100°C. The edge coinciding with y-axis is kept at 50°C. The other two edges are kept at 0°C. Write the boundary conditions that are needed for solving two dimensional heat flow equation.

9. Find the Z-transform of $\frac{1}{n}$.

10. Find the inverse Z-transform of $\frac{z}{(z+1)^{2}}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the Fourier series of $f(x) = (\pi - x)^2$ in $(0, 2\pi)$ of periodicity 2π. (8)

(ii) Obtain the Fourier series to represent the function $f(x) = |x|$, $-\pi < x < \pi$ and deduce $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$. (8)

Or

(b) (i) Find the half-range Fourier cosine series of $f(x) = (\pi - x)^2$ in the interval $(0, \pi)$. Hence find the sum of the series $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots + \infty$. (8)

(ii) Find the Fourier series upto second harmonic for the following data for y with period 6. (8)

x:	0	1	2	3	4	5
y:	9	18	24	28	26	20

12. (a) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$. Hence show that

(i) $\int_0^{\infty} \frac{\sin s - s\cos s}{s^3} \cos\left(\frac{s}{2}\right) ds = \frac{3\pi}{16}$ and

(ii) $\int_0^{\infty} \frac{\left(x\cos x - \sin x^2\right)}{x^6} dx = \frac{\pi}{15}$. (16)

Or

(b) (i) Using Fourier cosine transform, evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)^2}$. (8)

(ii) Find the function whose Fourier series transform is $\frac{e^{-as}}{s}(a>0)$. (8)

13. (a) (i) Find the singular integral of $z = px + qy + p^2 + pq + q^2$. (8)

(ii) Solve the partial differential equation $(x-2z)p + (2z-y)q = q - x$. (8)

Or

(b) (i) Solve : $(D^2 + 3DD' - 4D'^2)z = \cos(2x+y) + xy$. (8)

(ii) Solve : $(D^2 - DD' + 2D)z = e^{2x+y} + 4$. (8)

14. (a) A uniform elastic string of length 60 cms is subjected to a constant tension of 2 Kg. If the ends fixed and the initial displacement $y(x,0) = 60x - x^2$, $0 < x < 60$, while the initial velocity is zero, find the displacement function $y(x,t)$. (16)

Or

(b) Solve the problem of heat conduction in a rod given that the temperature function $u(x,t)$ is subject to the condition, $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, $0 \le x \le l$, $t > 0$.

(i) u is finite as $t \to \infty$

(ii) $\frac{\partial u}{\partial x} = 0$ for $x = 0$ and $x = l$, $t > 0$

(iii) $u = lx - x^2$ for $t = 0, 0 \le x \le l$. (16)

15. (a) (i) Find $Z(\cos n\theta)$ and hence deduce $Z\left(\cos\frac{n\pi}{2}\right)$. (8)

(ii) Using Z-transform solve : $y_{n+2} - 3y_{n+1} - 10y_n = 0$, $y_0 = 1$, and $y_1 = 0$. (8)

Or

(b) (i) State and prove the second shifting property of Z-transform. (6)

(ii) Using convolution theorem, find $Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$. (10)
