Reg. No. :

**Question Paper Code : 41310**

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

Third Semester

Civil Engineering

MA 6351 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Mechanical Engineering (Sandwich)/Aeronautical Engineering/ Agriculture Engineering/Automobile Engineering/Biomedical Engineering/ Computer Science and Engineering/Computer and Communication Engineering/ Electrical and Electronics Engineering/Electronics and Communication Engineering/Electronics and Instrumentation Engineering/Geoinformatics Engineering/Industrial Engineering/Industrial Engineering and Management/ Instrumentation and Control Engineering/Manufacturing Engineering/Marine Engineering/Materials Science and Engineering/Mechanical Engineering/ Mechanical and Automation Engineering/Mechatronics Engineering/Medical Electronics/Petrochemical Engineering/Production Engineering/Robotics and Automation Engineering/Bio Technology/Chemical Engineering/Chemical and Electrochemical Engineering/Food Technology/Information Technology/ Petrochemical Technology/Petroleum Engineering/Plastic Technology/ Polymer Technology)

(Regulations 2013)

Time : Three Hours | Maximum : 100 Marks

Answer ALL questions.

PART – A (10×2=20 Marks)

1. Find the complete integral of the PDE : $z = px + qy + \sqrt{pq}$
2. Solve : $(D^3 - 3DD'^2 + 2D'^3)\, z = 0$
3. Find $b_n$ in the expansion of $f(x) = x^2$ as a Fourier series in $(-\pi, \pi)$.
4. Define Root mean square value of a function.
5. Classify the partial differential equation $u_{xy} = u_x u_y + xy$.
6. State possible solutions of the one dimensional heat equation.
7. Find the Fourier transform of $f(x) = \begin{cases} 1 & \text{in } |x| < a \\ 0 & \text{in } |x| > a \end{cases}$

8. State the convolution theorem for Fourier transforms.

9. Find the Z-transform of {n}.

10. Prove that $Z\{nf(n)\} = -z \frac{d}{dz} F(z)$, where $Z\{f(n)\} = F(z)$.

PART – B (5×16=80 Marks)

11. a) i) Find the singular solution of the equation $z = px + qy + p^2 + pq + q^2$. (8)

ii) Solve : $x(y - z)p + y(z - x)q = z(x - y)$. (8)

(OR)

b) i) Solve : $(D^2 + 4DD' - 5D'^2)z = \sin(x - 2y) + e^{2x-y}$. (8)

ii) Solve : $(D^2 + DD' - 6D'^2)z = y\cos x$. (8)

12. a) i) Find the Fourier series for $f(x) = x^2$ in $-\pi < x < \pi$. (8)

ii) Find the half range cosine series for $f(x) = x(\pi - x)$ in $(0, \pi)$. (8)

(OR)

b) Find the Fourier series expansion upto the first three harmonics for the function defined in the following table (16)

| x | 0 | $\frac{\pi}{3}$ | $\frac{2\pi}{3}$ | $\pi$ | $\frac{4\pi}{3}$ | $\frac{5\pi}{3}$ | $2\pi$ |
|---|---|---|---|---|---|---|---|
| y | 1 | 1.4 | 1.9 | 1.7 | 1.5 | 1.2 | 1.0 |

13. a) A string is stretched and fastened to two points $x = 0$ and $x = l$ apart. Motion is started by displacing the string into the form $y = k(lx - x^2)$ from which it is released at time $t = 0$. Find the displacement of any point on the string at a distance of x from one end at time t. (16)

(OR)

b) A bar, 10 cm long with insulated sides, has its ends A and B kept at 20°C and 40°C respectively until steady state conditions prevail. The temperature at A is then suddenly raised to 50°C and at the same instant that at B is lowered to 10°C. Find the subsequent temperature at any point of the bar at any time. (16)

14. a) Find the Fourier transform of the function f(x) defined by $f(x)=\begin{cases}1-x^2 & \text{in } |x|\le 1\\ 0 & \text{in } |x|>1\end{cases}$.

Hence prove that $\int_0^\infty \frac{\sin s - s\cos s}{s^3}\cos\left(\frac{s}{2}\right)ds=\frac{3\pi}{16}$. Also show that

$\int_0^\infty \frac{(x\cos x-\sin x)^2}{x^6}dx=\frac{\pi}{15}$. **(16)**

(OR)

b) i) Show that the Fourier transform of $f(x) = e^{\frac{-x^2}{2}}$ is $e^{\frac{-s^2}{2}}$. **(8)**

ii) Find the Fourier cosine transform of $e^{-a^2x^2}$ and hence find the Fourier sine transform of $xe^{-a^2x^2}$. **(8)**

15. a) i) Find the inverse Z-transform of $\frac{8z^2}{(2z-1)(4z+1)}$ using convolution theorem for Z-transforms. **(8)**

ii) Find the inverse Z-transform of $\frac{z^2-3z}{(z-5)(z+2)}$ using residue theorem. **(8)**

(OR)

b) i) Solve : $y_{n+2}-4y_{n+1}+4y_n=0$, $y_0=1$, $y_1=0$, using Z-transform. **(10)**

ii) Find the Z-transform of {n} and $\left\{\frac{1}{n+1}\right\}$. **(6)**