Reg. No. : [| | | | | | | | | | | |]

Question Paper Code : 21522

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2013.

Third Semester

Civil Engineering

MA 2211/MA 31/MA 1201 A/CK 201/10177 MA 301/080100008/080210001 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to all branches)

(Regulation 2008/2010)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. State the Dirichlet's conditions for Fourier series.

2. What is meant by Harmonic Analysis?

3. Find the Fourier Sine Transform of e^{-3x}.

4. If $\mathfrak{F}\{f(x)\} = F(s)$, prove that $\mathfrak{F}\{f(ax)\} = \frac{1}{a} \cdot F\left(\frac{s}{a}\right)$.

5. Form the PDE from $(x-a)^2 + (y-b)^2 + z^2 = r^2$.

6. Find the complete integral of $p + q = pq$.

7. In the one dimensional heat equation $u_t = c^2 \cdot u_{xx}$, what is c^2?

8. Write down the two dimensional heat equation both in transient and steady states.

9. Find $Z(n)$.

10. Obtain $Z^{-1}\left[\frac{z}{(z+1)(z+2)}\right]$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the Fourier series of x^2 in $(-\pi, \pi)$ and hence deduce that $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots \frac{\pi^4}{90}$. (8)

(ii) Obtain the Fourier cosine series of $f(x) = \begin{cases} kx, & 0 < x < l/2 \\ k(l-x), & l/2 < x < l \end{cases}$. (8)

Or

(b) (i) Find the complex form of Fourier series of $\cos ax$ in $(-\pi, \pi)$, where "a" is not an integer. (8)

(ii) Obtain the Fourier cosine series of $(x-1)^2$, $0<x<1$ and hence show that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots=\frac{\pi^2}{6}$. (8)

12. (a) (i) Find the Fourier transform of $f(x)=\begin{cases}1, & |x|<a\\ 0, & |x|>a\end{cases}$ and hence find $\int_0^\infty \frac{\sin x}{x}\,dx$. (8)

(ii) Verify the convolution theorem under Fourier Transform, for $f(x)=g(x)=e^{-x^2}$. (8)

Or

(b) (i) Obtain the Fourier Transform of $e^{-x^2/2}$. (8)

(ii) Evaluate $\int_0^\infty \frac{dx}{\left(x^2+a^2\right)^2}$ using Parseval's identity. (8)

13. (a) (i) Solve : $x\left(y^2-z^2\right)p+y\left(z^2-x^2\right)q=z\left(x^2-y^2\right)$. (8)

(ii) Solve : $\left(D^2+DD'-6D'^2\right)z=y\cdot\cos x$. (8)

Or

(b) (i) Solve : $z=px+qy+\sqrt{p^2+q^2+1}$. (8)

(ii) Solve : $\left(D^3-7DD'^2-6D'^3\right)z=\sin(2x+y)$. (8)

14. (a) A tightly stretched string between the fixed end points $x=0$ and $x=l$ is initially at rest in its equilibrium position. If each of its points is given a velocity $kx(l-x)$, find the displacement $y(x,t)$ of the string.

Or

(b) An infinitely long rectangular plate is of width 10 cm. The temperature along the short edge $y=0$ is given by

$u=\begin{cases}20x, & 0<x<5\\ 20(10-x), & 5<x<10\end{cases}$. If all the other edges are kept at zero temperature, find the steady state temperature at any point on it.

15. (a) (i) Find $Z(\cos n\theta)$ and hence deduce $Z\left(\cos\frac{n\pi}{2}\right)$. (8)

(ii) Using Z-transform solve : $y_{n+2}-3y_{n+1}-10y_n=0$, $y_0=1$ and $y_1=0$. (8)

Or

(b) (i) State and prove the second shifting property of Z-transform. (6)

(ii) Using convolution theorem, find $Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$. (10)

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