Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 51571

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2014.

Third Semester

Civil Engineering

MA 2211/MA 31/MA 1201 A/CK 201/080100008/080210001/10177 MA 301 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS/ MATHEMATICS — III

(Common to all branches)

(Regulation 2008/2010)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. State the conditions for a function $f(x)$ to be expanded as a Fourier series in a given interval.

2. Expand $f(x) = 1$ as a half range sine series in the interval $(0, \pi)$.

3. Find the Fourier sine transform of $f(x) = \frac{1}{x}$.

4. State the Fourier integral theorem.

5. Form the PDE by eliminating the arbitrary constants a, b from the relation $z = ax^3 + by^3$.

6. Solve : $(D^4 - D'^4)z = 0$.

7. Write all the solutions of the one-dimensional wave equation $y_{tt} = a^2 y_{xx}$.

8. State the assumptions in deriving the one-dimensions heat flow equation (unsteady state).

9. Find the Z-transform of n^2.

10. State the convolution theorem on Z-transforms.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Expand $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi < x < 0 \\ 1 - \frac{2x}{\pi}, & 0 < x < \pi \end{cases}$ as a full range Fourier series in the interval $(-\pi, \pi)$. Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\infty = \frac{\pi^2}{8}$. (8)

(ii) Find the half-range sine series of $f(x) = 4x - x^2$ in the interval $(0, 4)$. Hence deduce the value of the series $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \ldots\infty$. (8)

Or

(b) (i) Expand $f(x) = \sin x$ as a complex form Fourier series in $(-\pi, \pi)$. (8)

(ii) Compute the first three harmonics of the Fourier series for $f(x)$ from the following data : (8)

x :	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
$f(x)$:	1.0	1.4	1.9	1.7	1.5	1.2	1.0

12. (a) (i) Find the Fourier transform of $e^{-a|x|}$, $a > 0$ and hence deduce that

(1) $\int_0^\infty \frac{\cos xt}{a^2 + t^2}\, dt = \frac{\pi}{2a} e^{-a|x|}$

(2) $F\{xe^{-a|x|}\} = i\sqrt{\frac{2}{\pi}} \frac{2as}{(s^2 + a^2)^2}$, here F stands for Fourier transform. (8)

(ii) Solve for $f(x)$ from the integral equation (8)

$$\int_0^\infty f(x) \sin sx\, dx = \begin{cases} 1, & 0 \le s < 1 \\ 2, & 1 \le s < 2 \\ 0, & s \ge 2. \end{cases}$$

Or

(b) (i) Find the Fourier transform of $f(x) = \frac{1}{\sqrt{|x|}}$. (8)

(ii) Using Parseval's identity evaluate the following integrals

(1) $\int_0^\infty \frac{dx}{(a^2 + x^2)^2}$

(2) $\int_0^\infty \frac{x^2}{(a^2 + x^2)^2} dx$ where $a > 0$. (8)

13. (a) (i) Form the PDE by eliminating the arbitrary function from the relation $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$. (8)

(ii) Solve the Lagrange's equation $(x + 2z)p + (2xz - y)q = x^2 + y$. (8)

Or

(b) (i) Solve : $x^2p^2 + y^2q^2 = z^2$. (8)

(ii) Solve : $(D^2 + DD' - 6D'^2)z = y\cos x$. (8)

14. (a) A string is stretched and fastened to points at a distance 'l' apart. Motion is started by displacing the string in the form $y = a\sin\left(\frac{\pi x}{l}\right)$, $0 < x < l$, from which it is released at time $t = 0$. Find the displacement at any time t. (16)

Or

(b) An infinitely long rectangular plate with insulated surfaces is 10 cm wide. The two long edges and one short edge are kept at 0°C, while the other short edge $x = 0$ is kept at temperature

$u = 20y, \quad 0 \le y \le 5$

$u = 20(10 - y), \quad 5 < y \le 10$.

Find the steady state temperature distribution in the plate. (16)

15. (a) (i) Find the Z-transforms of $r^n \cos n\theta$ and $e^{-at} \cos bt$. (8)

(ii) Solve $u_{n+2} - 3u_{n+1} + 2u_n = 4^n$, given that $u_0 = 0$, $u_1 = 1$. (8)

Or

(b) (i) Using convolution theorem find inverse Z-transform of $\frac{z^2}{(z-a)(z-b)}$. (8)

(ii) Solve $y_{n+2} - 3y_{n+1} - 10y_n = 0$, given $y_0 = 1$, $y_1 = 0$. (8)
