Reg. No. : [ ]

Question Paper Code : 31522

ECE

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2013.

Third Semester

Civil Engineering

MA 2211/MA 31/MA 1201 A/CK 201/10177 MA 301/080100008/080210001 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to all branches)

(Regulation 2008/2010)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the value of a_0 in the Fourier series expansion of $f(x) = e^x$ in $(0, 2\pi)$.
2. Find the half range sine series expansion of $f(x) = 1$ in $(0, 2)$.
3. Define self reciprocal with respect to Fourier Transform.
4. Prove that $F\left[f(x-a)\right] = e^{ias}F\left[f(x)\right]$.
5. Form a PDE by eliminating the arbitrary constants 'a' and 'b' from $z = ax^2 + by^2$.
6. Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x\, \partial y} + \frac{\partial z}{\partial x} = 0$.
7. Define steady state condition on heat flow.
8. An insulated rod of length l cm has its ends A and B maintained at 0°C and 80°C respectively. Find the steady state solution of the rod.
9. Find the Z-transform of $\frac{1}{n}$.
10. Find the inverse Z-transform of $\frac{z}{(z+1)^2}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the Fourier Series Expansion of $f(x) = \begin{cases} 1 \text{ for } 0 < x < \pi \\ 2 \text{ for } \pi < x < 2\pi \end{cases}$. (8)

(ii) Find the Fourier series expansion of $f(x) = \begin{cases} -x+1, & -\pi < x < 0 \\ x+1, & 0 < x < \pi. \end{cases}$. (8)

Or

(b) (i) Find the half range sine series of $f(x) = lx - x^2$ in $(0, l)$. (8)

(ii) Find the first two harmonies of the Fourier series expansion for the following data. (8)

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

12. (a) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$

Hence show that

(i) $\int_0^\infty \frac{\sin s - s\cos s}{s^3} \cos\left(\frac{s}{2}\right) ds = \frac{3\pi}{16}$ and

(ii) $\int_0^\infty \frac{(x\cos x - \sin x)^2}{x^6} dx = \frac{\pi}{15}$. (16)

Or

(b) (i) Using Fourier Cosine Transform, evaluate $\int_0^\infty \frac{dx}{(x^2 + a^2)^2}$. (8)

(ii) Find the function whose Fourier Sine Transform is $\frac{e^{-as}}{s}(a > 0)$. (8)

13. (a) (i) Form the PDE by eliminating the arbitrary function 'f' and 'g' from $z = x^2 f(y) + y^2 g(x)$. (8)

(ii) Solve $\left[D^2 - DD' - 2D'^2\right] z = 2x + 3y + e^{2x+4y}$. (8)

Or

(b) (i) Solve $\left(y^2 + z^2\right) p - xyq + xz = 0$. (8)

(ii) Find the singular integral of $z = px + qy + \sqrt{1 + p^2 + q^2}$. (8)

14. (a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y(x, 0) = K\left(lx - x^2\right)$. It is released from rest from this position. Find the expression for the displacement at any time 't'. (16)

Or

(b) Find the solution to the equation $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ that satisfies the conditions $u(0, t) = 0$, $u(l, t) = 0$, for $t > 0$ and

$$u(x, 0) = \begin{cases} x, & 0 \le x \le l/2 \\ l - x, & l/2 < x < l \end{cases}$$ (16)

15. (a) (i) Find the Z-transform of $\frac{1}{(n+1)(n+2)}$. (8)

(ii) Using Z-transform solve the difference equation $Y_{n+2} + 2Y_{n+1} + Y_n = n$ given $Y_0 = 0 = Y_1$. (8)

Or

(b) (i) Form the difference equation from $Y(n) = (A + Bn)\, 2^n$. (8)

(ii) Using convolution theorem find $Z^{-1}\left[\frac{z^2}{(z-1)(z-3)}\right]$. (8)

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