Lesson
34

Cooling And Heating Load Calculations - Heat Transfer Through Buildings - Fabric Heat Gain/Loss

The specific objectives of this chapter are to:

1. Discuss the general aspects of heat transfer through buildings (*Section 34.1*)
2. Discuss one-dimensional, steady state heat transfer through homogeneous, non-homogeneous walls, through air spaces and through composite walls of the buildings (*Section 34.2*)
3. Discuss unsteady heat transfer through opaque walls and roofs (*Section 34.3*)
4. Discuss one-dimensional, unsteady heat transfer through opaque walls and roofs with suitable initial and boundary conditions (*Section 34.4*)
5. Describe the analytical method used to solve the 1-D, transient heat transfer problem through building walls and roofs (*Section 34.4.1*)
6. Briefly discuss the numerical methods used to solve the transient heat transfer problem (*Section 34.4.2*)
7. Discuss the semi-empirical methods based on Effective Temperature Difference or Cooling Load Temperature Difference, discuss the physical significance of decrement and time lag factors and present typical tables of CLTD for walls and roof (*Section 34.4.3*)

At the end of the chapter, the student should be able to:

1. Calculate the steady heat transfer rates through homogeneous and non-homogeneous walls, through composite walls consisting of a combination of homogeneous and non-homogeneous walls and air spaces
2. Explain the need for considering transient heat transfer through buildings
3. Derive one-dimensional, transient heat conduction equation for building walls and roof and indicate suitable initial and boundary conditions
4. Discuss the general aspects of the analytical, numerical and semi-empirical methods used to solve the transient building heat transfer problem
5. Use the ETD/CLTD methods to estimate heat transfer rate through opaque walls and roof of the buildings

34.1. Introduction:

Whenever there is a temperature difference between the conditioned indoor space of a building and outdoor ambient, heat transfer takes place through the building structure (walls, roof, floor etc.). This is known as fabric heat gain or loss, depending upon whether heat transfer is to the building or from the building, respectively. The fabric heat transfer includes sensible heat transfer through all the structural elements of a building, but does not include radiation heat transfer through fenestration. Exact analysis of heat transfer through building structures is very complex, as it has to consider:

a) Geometrically complex structure of the walls, roofs etc. consisting of a wide variety of materials with different thermo-physical properties.
b) Continuously varying outdoor conditions due to variation in solar radiation, outdoor temperature, wind velocity and direction etc.

c) Variable indoor conditions due to variations in indoor temperatures, load patterns etc.

For cooling and heating load calculations, the indoor conditions are generally assumed to be constant to simplify the analysis. However, the variation in outdoor conditions due to solar radiation and ambient temperature has to be considered in the analysis to arrive at realistic cooling loads during summer. In winter, the heating load calculations are based on peak or near-peak conditions, which normally occur early in the morning before sunrise, in addition, in cold countries, the ambient temperature variation during the winter months is not significant. Hence, in conventional heating load calculations, the effects of solar radiation and ambient temperature variation are not considered and the heat transfer is assumed to be steady. However by this steady state method, the calculated heating capacity will be more than required. Thus for higher accuracy, it is essential to consider the transient heat transfer effects during winter also. In the present lecture, first steady state heat transfer through buildings will be discussed followed by the unsteady state heat transfer.

34.2. One-dimensional, steady state heat transfer through buildings:

Heat transfer through the building is assumed to be steady, if the indoor and outdoor conditions do not vary with time. The heat transfer is assumed to be one-dimensional if the thickness of the building wall is small compared to the other two dimensions. In general, all building walls are multi-layered and non-homogeneous and could be non-isotropic. To start with we consider a single layered, homogeneous wall and then extend the discussion to multi-layered, non-homogenous walls.

34.2.1. Homogeneous wall:

Figure 34.1(a) shows a homogeneous wall separating the conditioned indoor space from the outdoors. As shown in the figure, the wall is subjected to radiation

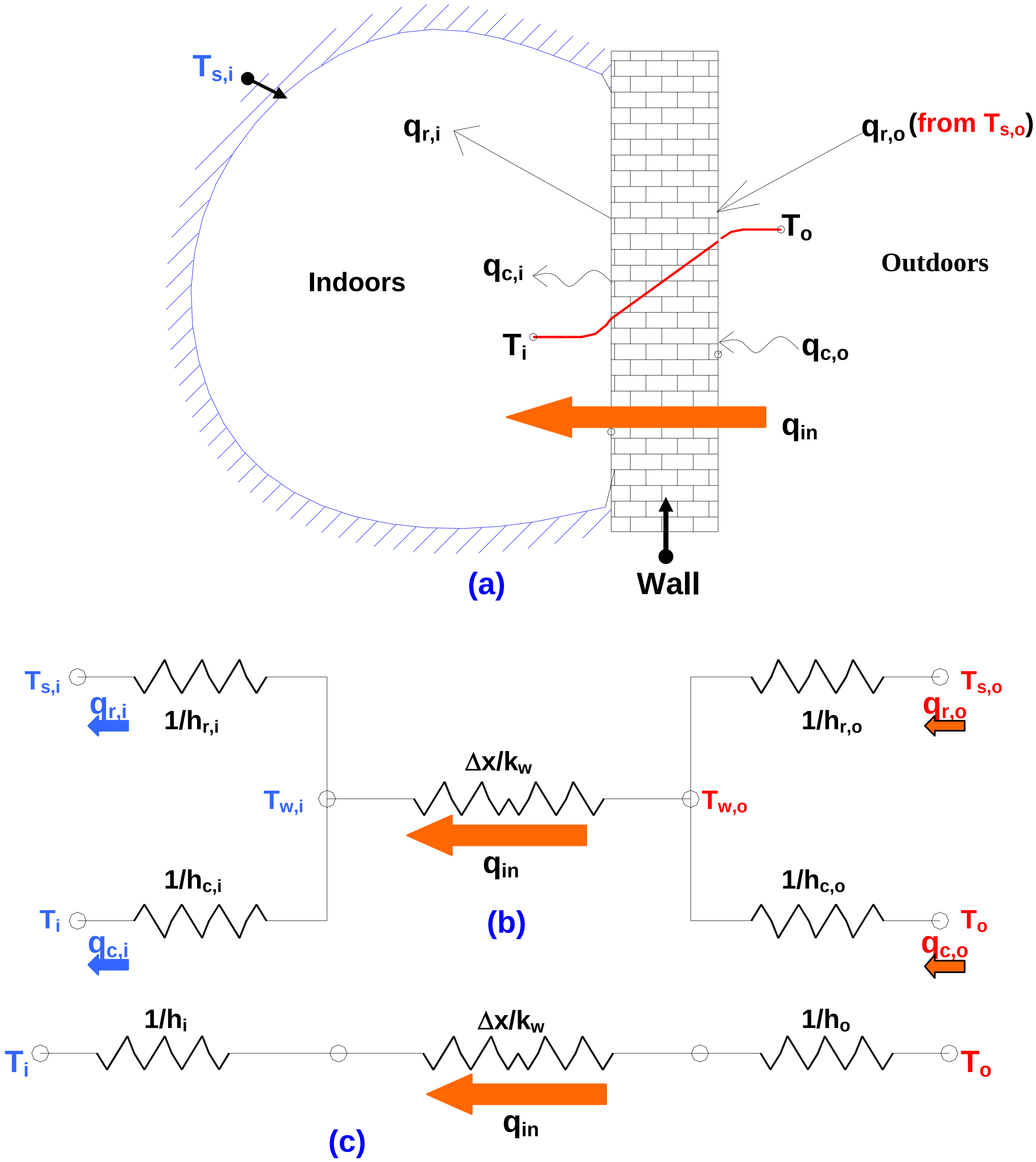

***Fig.34.1**: Steady state heat transfer through a building wall and the resistance network*

and convection heat transfer on both sides, while heat transfer through the wall is by conduction.

If outside and inside conditions do not vary with time, then the heat transfer through the wall is steady, and we can construct a heat transfer network considering various heat transfer resistances as shown in Fig.34.1(b). The heat transfer rate per unit area of the wall $\mathbf{q_{in}}$ under steady state is given by:

$$q_{in} = \{q_{c,o} + q_{r,o}\} = \{q_{c,i} + q_{r,i}\} \quad W/m^2 \tag{34.1}$$

where $q_{c,o}$ and $q_{r,o}$ are the convective and radiative heat transfers to the outer surface of the wall from outside and $q_{c,i}$ and $q_{r,i}$ are the convective and radiative heat transfers from the inner surface of the wall to the indoors, respectively. Writing the radiative heat transfer in terms of a linearized radiative heat transfer coefficient, we can write the heat transfer rate per unit area as:

$$q_{in} = h_o(T_o - T_{w,o}) = h_i(T_{w,i} - T_i) \quad W/m^2 \tag{34.2}$$

where T_i and T_o are the indoor and outdoor air temperatures, $T_{w,i}$ and $T_{w,o}$ are the inner and outer surface temperatures of the wall respectively. In the above equation, h_i and h_o are the inner and outer surface heat transfer coefficients or surface conductances, which take into account both convection and radiation heat transfers. From the resistance network, it can easily be shown that the surface conductances h_i and h_o are given by:

$$h_i = h_{c,i} + h_{r,i}\left(\frac{T_{w,i} - T_{s,i}}{T_{w,i} - T_i}\right) \tag{34.3}$$

$$h_o = h_{c,o} + h_{r,o}\left(\frac{T_{s,o} - T_{w,o}}{T_o - T_{w,o}}\right) \tag{34.4}$$

The convective heat transfer coefficient depends on whether heat transfer is by natural convection or forced convection. Normally the air inside the conditioned space is assumed to be still as the required air velocities in the conditioned space are very small. Hence, the inside convective heat transfer coefficient $h_{c,i}$ can be calculated using heat transfer correlations for natural convection. For example, for still air $h_{c,i}$ can be estimated using the following simple correlation:

$$h_{c,i} = 1.42\left(\frac{\Delta T}{L}\right)^{\frac{1}{4}} \quad W/m^2.K \tag{34.5}$$

where ΔT is the temperature difference between the inner surface of the wall and the still air, and L is the length of the wall. Of course, the actual heat transfer coefficient will be slightly higher due to the finite air motion inside the conditioned space.

Normally due to wind speed, the heat transfer from the outside air to the outer surface of the wall is by forced convection. Hence to estimate the outer convective heat transfer coefficient $h_{c,o}$, suitable forced convective heat transfer correlations should be used.

The linearized radiative heat transfer coefficient is calculated from the equation:

$$h_r = \left(\frac{\varepsilon\sigma}{T_1 - T_2}\right)\left(T_1^4 - T_2^4\right) \quad W/m^2.K \tag{34.6}$$

where ε is the emissivity of the surface, σ is the Stefan-Boltzmann's constant (5.673 x 10^{-8} $W/m^2.K^4$), T_1 and T_2 are the hot and cold surface temperatures (in K) respectively.

Table 34.1 shows typical surface conductance values, which can be used for estimating inner and outer heat transfer coefficients (h_i and h_o). When the air is still (i.e., for the inside heat transfer coefficient), the order-of-magnitude of convective heat transfer is almost same as that of the radiative heat transfer coefficient, as a result, the emissivity of the surface plays an important role and the surface conductance increases with emissivity as shown in the table. On the other hand, when the air is blowing at considerable speed (i.e., for external heat transfer coefficient), the convection heat transfer coefficient is many times larger than the radiative heat transfer coefficient, as a result, the effect of emissivity of the surface is not important.

Orientation of Surface	Air Velocity	Direction of heat flow	Surface emissivity		
			0.9	0.7	0.5
Horizontal	Still Air	Up	9.4	5.2	4.4
Horizontal	Still Air	Down	6.3	2.2	1.3
Vertical	Still Air	Horizontal	8.5	4.3	3.5
Any position	3.7 m/s	Any	23.3	-	-
Any position	6.4 m/s	Any	35	-	-

Table 34.1: *Surface conductance values in W/m^2.K for different orientations, air velocities and surface emissivity (C.P. Arora)*

Eliminating the surface temperatures of the wall ($T_{w,i}$ and $T_{w,o}$), the steady state heat transfer rate per unit area of the wall can be written in terms of the indoor and outdoor air temperatures and the overall heat transfer coefficient, i.e.,

$$q_{in} = U(T_o - T_i) = \frac{(T_o - T_i)}{R_{tot}} \quad W/m^2 \qquad (34.7)$$

where U is the overall heat transfer coefficient and R_{tot} is the total resistance to heat transfer. From the heat transfer network, the expression for overall heat transfer coefficient is given by:

$$\left(\frac{1}{U}\right) = \left(\frac{1}{h_i} + \frac{\Delta x}{k_w} + \frac{1}{h_o}\right) = R_{tot} \qquad (W/m^2.K) \qquad (34.8)$$

where Δx and k_w are the thickness and thermal conductivity of the wall, respectively.

If the wall consists of windows, doors etc., then the overall heat transfer Uo is obtained using the individual U-values and their respective areas as:

$$U_o = (U_{wall}.A_{wall} + U_{door}.A_{door} + U_{window}.A_{window} ...)/A_{total} \qquad (34.9)$$

where U_{wall}, U_{door}, U_{window} etc. are the overall heat transfer coefficients for the wall, door, window etc., which are obtained using Eqn.(34.8), and A_{wall}, A_{door}, A_{window} are the corresponding areas. A_{total} is the total area of the wall that includes doors, windows etc. The above equation for overall heat transfer coefficient (Eqn.(34.9)) is valid when the temperature difference across the wall components are same and the heat transfer paths through these elements are parallel.

34.2.2. Non-homogeneous walls:

In general the building walls may consist of non-homogeneous materials such as hollow bricks. Heat transfer through non-homogeneous materials such as hollow bricks is quite complicated as it involves simultaneous heat transfer by convection, conduction and radiation as shown in Fig.34.2. The heat transfer network consists of series as well as parallel paths due to the simultaneous modes of heat transfer. In practice, all these effects are lumped into a single parameter called thermal conductance, C, and the heat flux through the hollow brick is given by:

$$q = C(T_{w,o} - T_{w,i}) \quad W/m^2 \qquad (34.10)$$

The conductance values of common building materials have been measured and are available in tabular form in ASHRAE and other handbooks. Table 34.2 shows thermo-physical properties of some commonly used building materials.

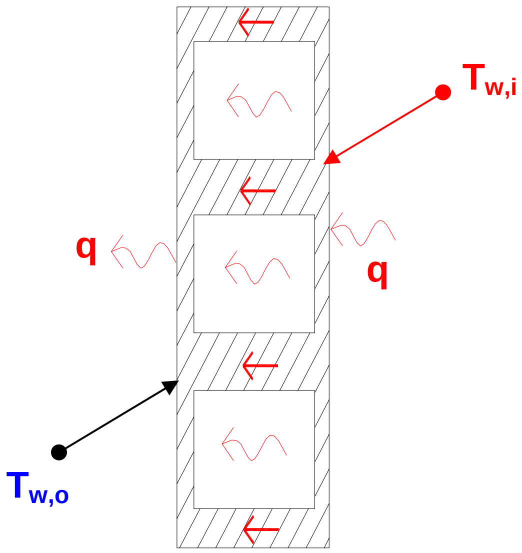

***Fig.34.2**: Heat transfer through a non-homogeneous wall*

Material	Description	Specific heat kJ/kg.K	Density kg/m^3	Thermal conductivity k_w, W/m.K	Conductance C, W/m^2.K
Bricks	Common Face brick Firebrick	0.84 0.84 0.96	1600 2000 2000	0.77 1.32 1.04 – 1.09	
Woods	Ply Hard Soft	- 2.39 2.72	544 720 512	0.1 0.158 0.1	
Masonry Materials	Concrete Plaster, Cement	0.88 0.796	1920 1885	1.73 8.65	
	Hollow Clay tiles a) 10 cm b) 20 cm c) 30 cm	 - - -	 - - -	 - - -	 5.23 3.14 2.33
	Hollow Concrete blocks d) 10 cm e) 20 cm f) 30 cm	- - -	- - -	- - -	8.14 5.23 4.54
	Foam concrete (Pre-cast slabs for roof)		210-704	0.043-0.128	
Glass	Window Borosilicate	0.84	2700 2200	0.78 1.09	
Insulating Materials	Mineral or glass wool Fiberglass board Cork board Cork granulated Thermocole (EPS)	0.67 0.7 1.884 1.88 -	24-64 64-144 104-128 45-120 30	0.038 0.038 0.038 0.045 0.037	
	Diatomaceous Earth Felt Magnesia Asbestos	- - - 0.816	320 330 270 470-570	0.061 0.052 0.067 0.154	

***Table 34.2**: Thermo-physical properties of some common building and insulating materials (C.P. Arora)*

34.2.3. Air spaces:

Buildings may consist of air spaces between walls. Since air is a bad conductor of heat, the air space provides effective insulation against heat transfer. Heat transfer through the air space takes place by a combined mechanism of conduction, convection and radiation as shown in Fig. 34.3.

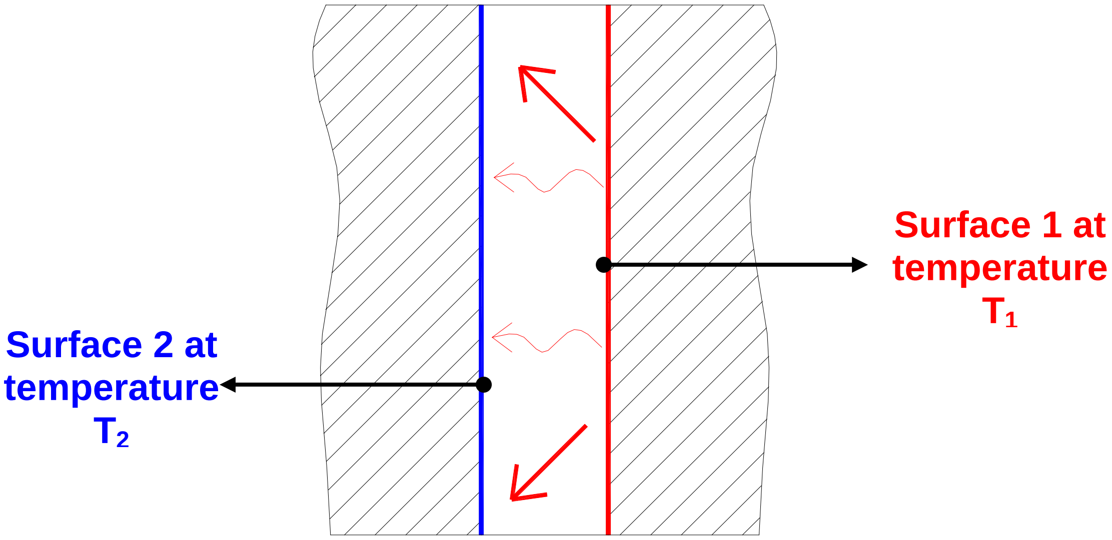

Fig.34.3: *Heat transfer through an air space in the wall*

Thus the heat transfer rate through the air spaces depends upon its width, orientation and surface emissivities of the wall surfaces and the temperature difference between the two surfaces. Heat transfer by conduction is considerable only when the thickness of the air space is very small. Studies show that beyond an air gap of about 2 cms, the effect of conduction heat transfer is negligible, and heat transfer is predominantly by convection and radiation. Since the thickness of the air spaces varies normally from 5 cms to 55 cms (e.g. for false ceilings), the effect of conduction may be neglected. In such a case, the heat flux through the air space is given by:

$$q = C(T_1 - T_2) \quad W/m^2 \tag{34.11}$$

where C is the conductance of the air space that includes the radiation as well as convection effects. Assuming the heat transfer coefficient h_c to be same for both the surfaces (i.e., when air is well-mixed in the air space), the air temperature to be uniform and the surfaces 1 and 2 to be infinite parallel planes, it can be shown that the conductance C is given by:

$$C = \left(\frac{h_c}{2} + h_r\right) \quad W/m^2.K \tag{34.12}$$

The linearized radiative heat transfer coefficient h_r is given by:

$$h_r = \left(\frac{F_{12}\sigma}{T_1 - T_2}\right)\left(T_1^4 - T_2^4\right) \quad W/m^2.K \tag{34.13}$$

where the view factor F_{12} is given by:

$$F_{12} = \frac{1}{\left[\left(1/\varepsilon_1\right) + \left(1/\varepsilon_2\right) - 1\right]} \tag{34.14}$$

where ε_1 and ε_2 are the emissivities of surfaces 1 and 2, respectively. Table 34.3 shows the typical conductance values for the air spaces commonly encountered in buildings.

Position & Mean Temp. difference	Direction of heat flow	Width of air space, cm	Conductance, W/m^2.K
Horizontal, 10°C	Up	2.1 11.6	6.7 6.2
	Down	2.1 4.2 11.6	5.7 5.1 4.8
Vertical, 10°C	Horizontal	2.1 11.6	5.8 5.8
Horizontal, 32°C	Up	2.1 11.6	7.7 7.2
	Down	2.1 4.2 11.6	7.0 6.2 5.8
Vertical, 32°C	Horizontal	2.1 11.6	7.0 6.9

Table 34.3: *Typical conductance values of air spaces (C.P. Arora)*

34.2.4. Multi-layered, composite walls:

In general, a building wall may consist of several layers comprising of layers of homogeneous and non-homogeneous wall materials made up of structural and insulating materials and air spaces. For such a multi-layered wall, one can write the heat transfer rate per unit area as:

$$q_{in} = U(T_o - T_i) = \frac{(T_o - T_i)}{R_{tot}} \quad W/m^2 \tag{34.15}$$

where the overall heat transfer coefficient U is given as:

$$\left(\frac{1}{U}\right) = R_{tot} = \left(\frac{1}{h_i}\right) + \sum_{i=1}^{N}\left(\frac{\Delta x_i}{k_{w,i}}\right) + \sum_{j=1}^{M}\left(\frac{1}{C_j}\right) + \left(\frac{1}{h_o}\right) \tag{34.16}$$

Thus from the structure of the wall, various material properties and conductance values of non-homogeneous materials and air spaces and inner and outer surface temperatures and conductance, one can calculate the heat transfer rate under steady state conditions. It should be kept in mind that the equations given above are limited to plane walls. For non-planar walls (e.g. circular walls), the contour of the walls must be taken into account while calculating heat transfer rates.

34.3. Unsteady heat transfer through opaque walls and roofs:

In general, heat transfer through building walls and roof is unsteady, this is particularly so in summer due to solar radiation and varying ambient temperature. In the calculation of unsteady heat transfer rates through buildings, it is essential to consider the thermal capacity of the walls and roof. Due to the finite and often large thermal capacity of the buildings, the heat transfer rate from outside to the outer surface is not equal to the heat transfer rate from the inner surface to the indoor space[1]. In addition, the thermal capacity of the buildings introduces a time lag. These aspects have to be considered in realistic estimation of building cooling loads. This makes the problem mathematically complex. Though the conduction through the building walls and roof could be multi-dimensional, for the sake of simplicity a one-dimensional heat transfer is normally considered. It is to be noted that when the heat transfer is not steady, the concept of simple resistance network as discussed before cannot be used for obtaining heat transfer rate through the wall.

34.4. One-dimensional, unsteady heat transfer through building walls and roof:

For the sake of simplicity, it is assumed that the wall is made of a homogeneous material. It is also assumed that the temperature of the conditioned space is kept constant by using a suitable air conditioning system. Figure 34.4 shows a wall of thickness L subjected to unsteady heat transfer. As shown in the figure the outer surface of the wall (x=L) is subjected to direct and diffuse radiations from the sun (**$\alpha_D I_D$ and $\alpha_d I_d$**), reflected radiation from the outer wall to the surrounding surfaces (**R**) and convective heat transfer from outdoor air to the outer surface of the wall (**$h_o(T_o-T_{w,o})$**). Heat transfer from the inner surface to the conditioned space takes place due to combined effects of convection and radiation (**$h_i(T_{w,i}-T_o)$**).

Applying energy balance equation to the outer surface of the wall (x = L) at any instance of time θ, we can write:

$$q_{x=L,\theta} = -k_w \left(\frac{\partial T}{\partial x}\right)_{x=L,\theta} = h_o(T_o - T_{x=L}) + \alpha_D I_D + \alpha_d I_d - R \qquad (34.17)$$

Applying energy balance equation to the inner surface of the wall (x = 0), we can write:

$$q_{x=0,\theta} = -k_w \left(\frac{\partial T}{\partial x}\right)_{x=0,\theta} = h_i(T_{x=0} - T_i) \qquad (34.18)$$

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[1] If the thermal capacity of the wall is small (e.g. for a thin door), the heat transfer will still be transient due to changing outdoor conditions. However, at any point of time the heat transfer rate at the outer surface is equal to the heat transfer rate at the inner surface, i.e., **$q_{o,\theta} = q_{i,\theta}$** due to negligible thermal storage effect

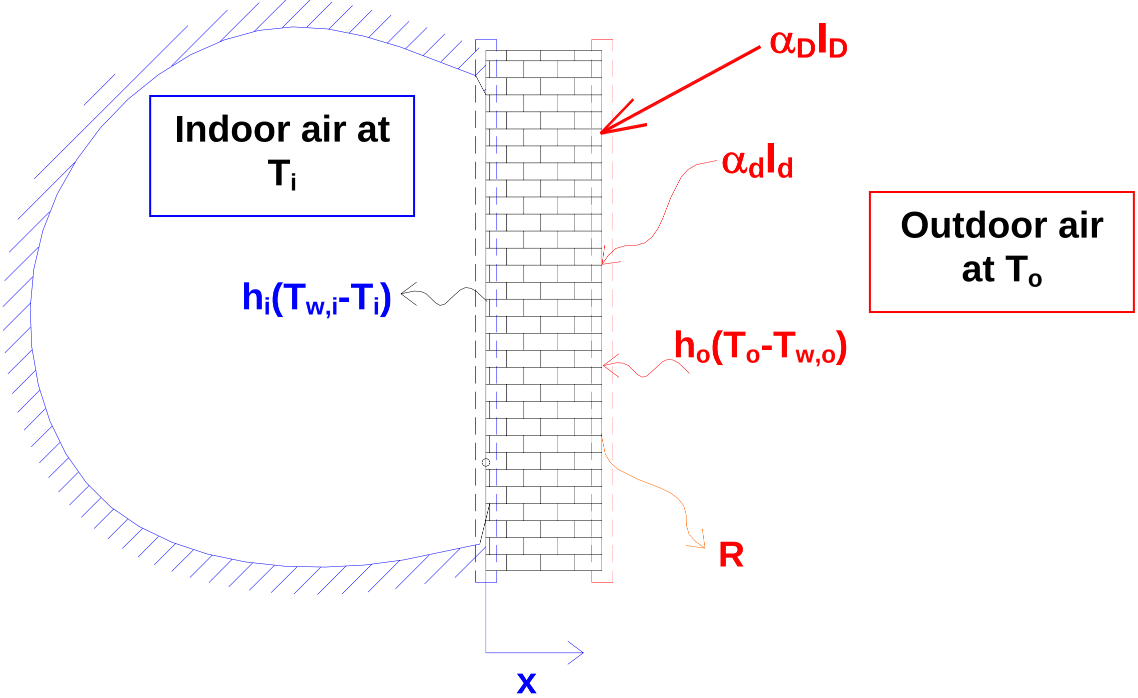

Fig.34.4. *Unsteady heat transfer through a building wall*

In general due to the finite thermal capacity of the walls; at any point of time θ, the heat transfer rate at the outer surface is not equal to the heat transfer rate at the inner surface, i.e.,

$$\mathbf{q}_{x=L,\theta} \neq \mathbf{q}_{x=0,\theta} \tag{34.19}$$

For cooling load calculations, we need to know the heat transfer rate from the inner surface of the wall to the conditioned space at a given time θ, i.e., $\mathbf{q}_{x=0,\theta}$. From Eq.(34.18), to calculate $\mathbf{q}_{x=0,\theta}$, we need to know the temperature distribution $(\partial T/\partial x)$ inside the wall so that we can calculate $(\partial T/\partial x)_{x=0,\theta}$ and $\mathbf{q}_{in}$ from Eq.(34.18). To find the temperature distribution inside the wall, one has to solve the transient heat conduction equation as the mode of heat transfer through the solid wall is assumed to be by conduction only. Assuming the variation in thermal properties of the solid wall to be negligible, the one-dimensional, transient heat conduction equation through the plane wall is given by:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\left(\frac{\partial T}{\partial \theta}\right) \tag{34.20}$$

In the above equation, α is the thermal diffusivity ($\alpha = k_w/\rho_w c_{pw}$), x is the length coordinate and θ is the time coordinate. To solve the above partial differential equation, an initial condition and two boundary conditions are required to be specified. The initial condition could be a known temperature gradient at a particular time, θ = 0, i.e.,

$$T_{x,\theta=0} = T_i(x) \tag{34.21}$$

The two boundary conditions at x = L and x = 0 are given by Eqs.(34.17) and (34.18). The boundary condition at x = L, i.e., Eq.(34.17) can be written as:

$$q_{x=L,\theta} = -k_w \left(\frac{\partial T}{\partial x}\right)_{x=L,\theta} = h_o(T_o - T_{x=L}) + \alpha_D I_D + \alpha_d I_d - R = h_o(T_{sol-air} - T_{x=L}) \quad (34.22)$$

where $T_{sol-air}$ is known as the **sol-air temperature** and is an equivalent or an effective outdoor temperature that combines the effects of convection and radiation. From the above equation the sol-air temperature is given by:

$$T_{sol-air} = T_o + \left(\frac{\alpha_D I_D + \alpha_d I_d - R}{h_o}\right) \quad (34.23)$$

It can be easily seen that in the absence of any radiation, the sol-air temperature is simply equal to the outdoor air temperature. The difference between the sol-air temperature and ambient air temperature increases as the amount of radiation incident on the outer surface increases and/or the external heat transfer coefficient decreases. Since on any given day, the outdoor air temperature and solar radiation vary with time, the sol-air temperature also varies with time in a periodic manner.

In terms of the sol-air temperature the boundary condition at x=L is written as:

$$q_{x=L,\theta} = -k_w \left(\frac{\partial T}{\partial x}\right)_{x=L,\theta} = h_o(T_{sol-air} - T_{x=L}) \quad (34.24)$$

Thus the one-dimensional unsteady heat transfer equation through the plane wall given by Eq.(34.20) should be solved using the initial condition given by Eq.(34.21) and the boundary conditions given by Eqs.(34.18) and (34.24). This problem can be solved by an analytical method involving an infinite harmonic series or by using numerical techniques such as finite difference or finite volume methods or by using semi-empirical methods.

34.4.1. Analytical solution:

Analytical solutions to transient transfer through building walls and roof are available for simple geometries. To simplify the problem further it is generally assumed that the outdoor air temperature and solar radiation intensity vary in a periodic manner. In addition, normally the indoor temperature and thermal properties of the wall materials are assumed to be constant. Though the variation of ambient temperature and solar radiation is highly erratic and hence non-periodic due mainly to the presence of clouds and other climatic factors, the assumption of periodic variation is justified if one assumes a clear sky. For example Fig.34.5, shows the direct, diffuse and total radiation intensity on a horizontal roof under clear sky conditions. It can be seen that the variation is periodic with the peak occurring at the solar noon. Applying periodic boundary condition at the outer surface, the analytical solution is obtained in terms of an infinite Fourier series consisting of various harmonics.

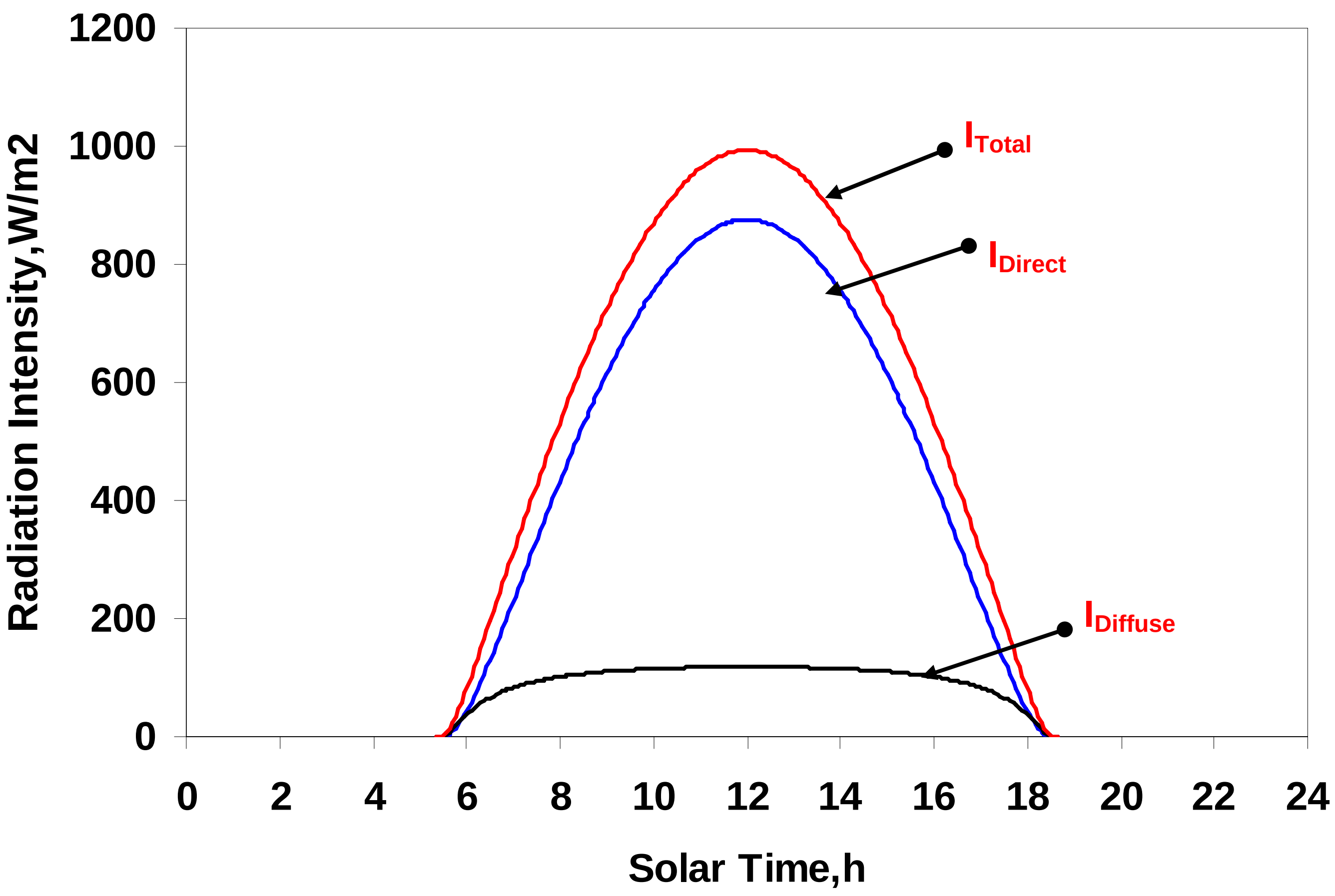

***Fig.34.5**: Variation of direct, diffuse and total solar radiation on a horizontal surface with time under clear sky conditions*

The sol-air temperature at any instant θ is given by (**Threlkeld**):

$$T_{sol-air,\theta} = T_{sol-air,m} + M_1 \cos\varpi_1\theta + N_1 \sin\varpi_1\theta + M_2 \cos\varpi_2\theta + N_2 \sin\varpi_2\theta + ... \quad (34.25)$$

Where the mean sol-air temperature $T_{sol-air,m}$ is obtained by averaging the instantaneous sol-air temperature over a 24-hour period, i.e. by integrating $T_{sol-air}$ using Eqn.(34.23) over a 24 hour period. Hence it is given by:

$$T_{sol-air,m} = \frac{1}{24}\int_0^{24} T_{sol-air} d\theta \quad (34.26)$$

The coefficients M_n and N_n are given by:

$$M_n = \frac{1}{12}\int_0^{24} T_{sol-air} \cos \varpi_n\theta \, d\theta \quad (34.27)$$

$$N_n = \frac{1}{12}\int_0^{24} T_{sol-air} \sin \varpi_n\theta \, d\theta \quad (34.28)$$

In the above expressions, the value of n can be restricted to 2 or 3 as higher order terms do not contribute significantly. In the above expressions, ω_n is the angular velocity, and **$\omega_1 = \pi/12$ radians per hour or 15^o per hour and $\omega_n = n\omega_1$.** The coefficients M_1, M_2.. and N_1, N_2.. are obtained from Eqns.(34.27) and (34.28). All the calculations are based on solar time, and θ is taken as 0 hours at 12'O clock midnight. Thus using the above equations, the sol-air temperature at an instance can be calculated for clear days at any location. Now using the above series expression for sol-air temperature, the solution of the unsteady heat conduction equation yields expression for wall temperature as a function of x and θ as:

$$T_{x,\theta} = A + Bx + \sum_{n=1}^{\infty}\left(C_n \cos P_n.mx + D_n \sin P_n.mx\right).e^{\left(-m^2\varpi_n\theta\right)} \qquad (34.29)$$

where A,B, C and D are constants, and $m = \sqrt[4]{-1}$. The coefficients A, B, Cn and Dn can be either real or complex. However, in the solution only the real parts are considered. Then it is shown that the inner wall temperature (i.e, at x = 0) the temperature is given by:

$$T_{x=0,\theta} = T_{x=0,0} + \frac{1}{h_i}\begin{bmatrix} U(T_{e,m} - T_{x=0,0}) + V_1 T_{e,1}\cos(\varpi_1\theta - \psi_1 - \phi_1) + \\ V_2 T_{e,2}\cos(\varpi_2\theta - \psi_2 - \phi_2)... \end{bmatrix} \qquad (34.30)$$

where T_e stands for the sol-air temperature ($T_{sol\text{-}air}$) and ;

$$U = \frac{1}{\frac{1}{h_i} + \frac{L}{k_w} + \frac{1}{h_o}} \qquad (34.31)$$

$$V_n = \frac{h_i h_o}{\sigma_n k_w \sqrt{Y_n^2 + Z_n^2}} \qquad (34.32)$$

$$\sigma_n = \sqrt{\frac{\varpi_n}{2\alpha_w}}; \text{ and } \alpha_w = \frac{k_w}{\rho_w c_{p,w}} = \text{Thermal diffusivity of the wall} \qquad (34.33)$$

The constants Y_n and Z_n in Eqn.(34.34) are expressed in terms of h_i, h_o, σ_n, L, k_w.

The term ϕ_n in Eqn.(34.30) is called as **Time Lag** factor and is given by:

$$\phi_n = \tan^{-1}\left(\frac{Z_n}{Y_n}\right) \qquad (34.34)$$

The rate of heat transfer from the inner surface is also shown to be in the form of an infinite series as shown below:

$$q_{x=0,\theta} = U\{[T_{e,m} + \lambda_1 T_{e,1}\cos(\varpi_1\theta - \psi_1 - \phi_1) + \lambda_2 T_{e,2}\cos(\varpi_2\theta - \psi_2 - \phi_2)] - T_{x=0,0}\}$$

(34.35)

In the above expression the quantity λ_n is called as **decrement factor** and as mentioned before, ϕ_n is the as time lag factor. The factor ψ_n takes into account the inner and outer heat transfer coefficients, thickness and thermal properties of the wall etc. The expressions for decrement factor λ_n and factor ψ_n are given by:

$$\lambda_n = \frac{V_n}{U}; \quad \text{and} \quad \psi_n = \tan^{-1}\left(\frac{N_n}{M_n}\right) \qquad (34.36)$$

34.4.2. Numerical methods:

The analytical method discussed above, though gives an almost exact solution, becomes very complex for other geometries or boundary conditions. The numerical techniques are very powerful and are very useful for solving the unsteady conduction equations with a wide variety of boundary conditions, variable properties and irregular shapes. However, the use of numerical methods requires a powerful computer, and the solution obtained is not exact and is prone to errors if not applied properly. Nevertheless, at present with the advent of computers, the numerical method is the preferred method due to its versatility and flexibility. The commonly used numerical methods are: finite difference method, finite element method, finite volume method etc. In general, the principle of all numerical methods is to write the continuous functions such as temperature in discrete forms by dividing the domain of interest into a large number of grids or elements. Due to discretization, the governing partial differential equations get converted into a set of algebraic equations, which then are solved to get the parameters of interest in the domain. The reader should refer to any book on Numerical Methods for further details on these techniques.

34.4.3. Semi-empirical methods:

The semi-empirical methods use the form suggested by the analytical method along with experimental observations on standard walls. These semi-empirical methods based on Equivalent Temperature Difference (**ETD**) or Cooling Load Temperature Difference (**CLTD**), are widely used by air conditioning industry due to their simplicity. However, the empirical data covers only standard walls and is suitable for specific location, orientation and day. In the present lecture, this method is used to estimate unsteady heat transfer through building walls and roofs. Before presenting this method, one has to consider the physical significance of decrement factor and time lag factor mentioned under analytical methods.

Decrement factor and Time Lag:

Based on the form suggested by analytical methods, the heat transfer rate to the conditioned space at any time θ can be written as:

$$Q_{x=0,\theta} = UA(T_{sol-air,m} - T_i) + UA\lambda(T_{sol-air,\theta-\phi} - T_{sol-air,m}) \qquad (34.37)$$

In the above expression, $T_{sol\text{-}air,m}$ is the time averaged sol-air temperature, $T_{sol\text{-}air,\theta-\phi}$ is the sol-air temperature ϕ hours before θ, U and A are the overall heat transfer coefficient and area of the wall, λ is the decrement factor and ϕ is the time lag.

The **decrement factor, λ** accounts for the fact that due to finite thermal capacity, the heat transferred to the outer surface of the wall is partly stored and partly transferred to the conditioned space. Due to the thermal energy storage, the temperature of the wall increases, and if it exceeds the outdoor air temperature then a part of the energy stored is transferred to outside and not to the conditioned space. Thus finally the heat transferred to the conditioned space from the inner surface (cooling load) is smaller than the heat transferred to the outer surface. This implies that the finite thermal capacity of the wall introduces a decrement in heat transfer.

The decrement factor, that varies between 0 to 1, increases as the thermal capacity of the wall increases. Thus thicker walls have lower decrement factor and thinner walls have higher decrement factor.

The finite thermal capacity of the building walls and roof also introduces a **time lag, ϕ**. The time lag is the difference between the time at which the outer surface receives heat and the time at which the inner surface senses it. Due to the effect of time lag, if the outdoor temperature is maximum at noon, the indoor temperature of a non-air conditioned room reaches a maximum somewhere in the afternoon.

As mentioned both decrement factor and time lag depend on the thermal capacity (**mass x specific heat**) of the wall. Most of the commonly used building structural materials have a specific heat of about 840 J/kg.K, then, the thermal capacity of these walls depend mainly on the thickness and density of the wall material. For these standard wall materials, the decrement factor decreases and the time lag increases as the wall thickness and density increase as shown in Fig. 34.6. Thus from the comfort point of view it is always advantageous to construct buildings with thick walls as this will yield low decrement factor and large time lag. In the limiting case, when the thermal capacity of the wall is very large, then the decrement factor becomes zero, then the heat transferred to the conditioned will remain constant throughout the day at the mean value as given by the Eqn.(34.37), i.e.,

$$\mathbf{Q_{x=0,\theta} = UA(T_{sol-air,m} - T_i) \quad when \ \lambda = 0.0} \qquad (34.38)$$

On the other extreme, if the wall has negligible thermal capacity, then the decrement factor will be 1.0 and the time lag will be 0, and the heat transfer rate to the conditioned space at any point is equal to the heat transferred to the outer surface of the wall at that instant, i.e.,

$$\mathbf{Q_{x=0,\theta} = UA(T_{sol-air,\theta} - T_i) \quad when \ \lambda = 1.0 \ and \ \phi = 0} \qquad (34.39)$$

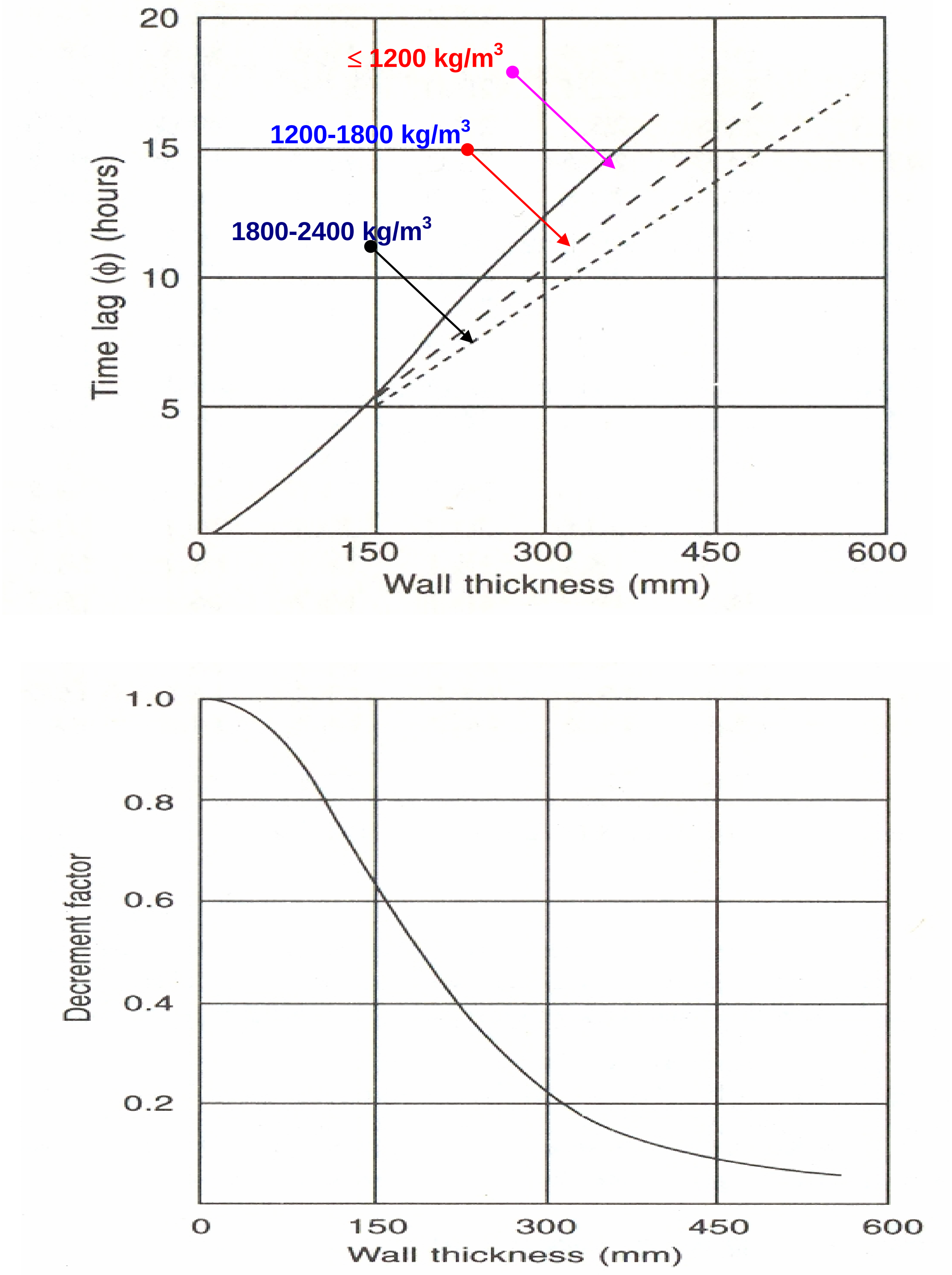

***Fig.34.6**: Variation of time lag and decrement factor with wall thickness and density*

In general the decrement factor of building walls and roof lies between 0 to 1 and the time lag will be greater than 0 hours. However, for windows and thin doors etc, which are exposed to outdoors, the decrement factor may be taken as 1.0 and the time lag factor as 0.0, as the thermal storage capacity of these elements is very small. Figure 34.7 shows the variation of heat transfer rate to the conditioned space with solar time for walls of different thickness. It can be seen that for thin walls with small time lag, the peak heat transfer occurs sometime around 4 P.M, whereas for thick walls with large time lags, the peak occurs well after midnight. Since the outside temperatures will be much smaller during the night the building can reject heat to the outside during night, thus for thicker walls due to the thermal storage effect a major portion of the heat absorbed by the outer surface during the daytime can be rejected to the outside, while a relatively small amount is transferred to the conditioned space (small decrement factor). The net effect is a greatly reduced cooling load on the building for thick walls. It can also be observed that due to large decrement factor the peak heat transfer for thin walled structures is much higher compared to the thick walled buildings. This implies the requirement of cooling system of much larger capacity (hence high initial cost) for buildings with thin walls compared to thick walls.

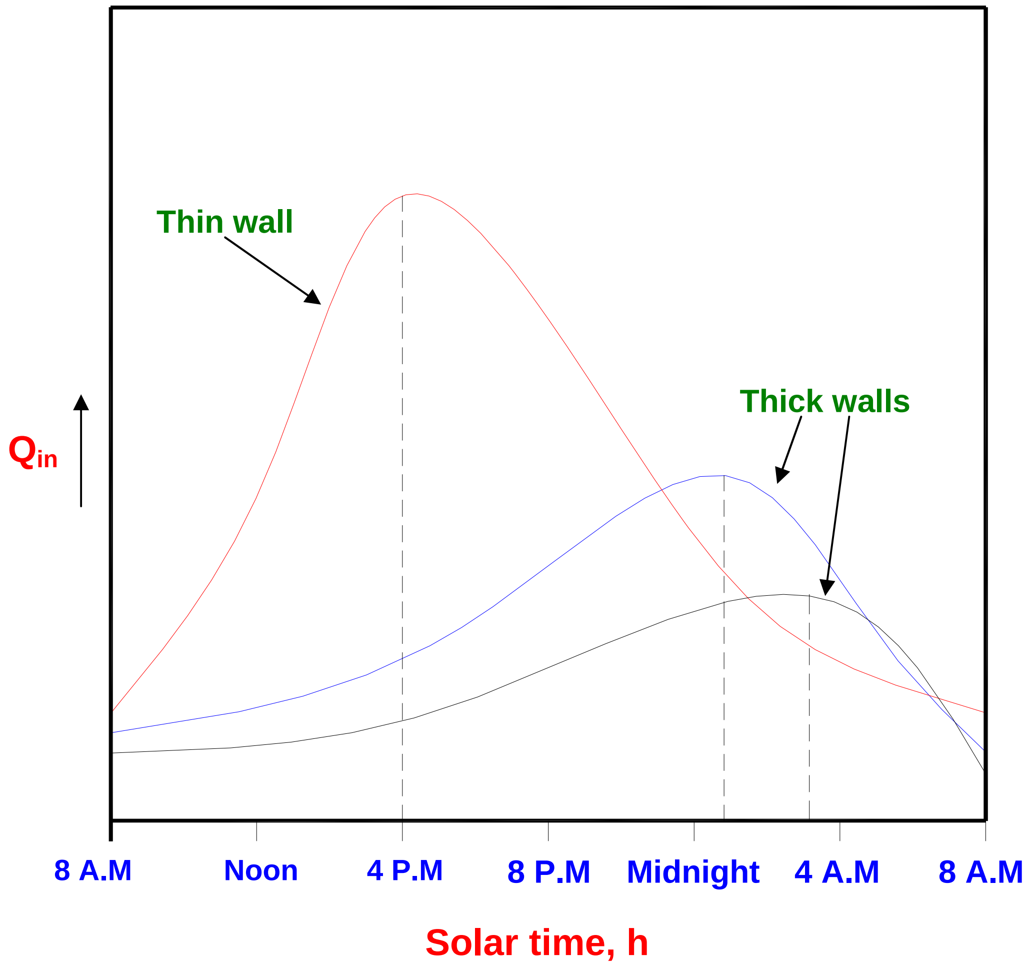

Fig.34.7*: Variation of heat transfer rate with time for thick and thin walled buildings*

When the thermal capacity of the building is sufficiently large, then it is also possible to maintain reasonably comfortable temperatures inside the building even without an air conditioning system during both winter and summer. This is the principle behind old temples and buildings, which are comfortable throughout the year without any artificial air conditioning systems. However, the effect of the thermal capacity becomes significant mainly in locations, which have large variation in diurnal temperatures (i.e., T_{max}-T_{min} on a particular day is large). This is generally the case in dry areas, where thick walled buildings are highly beneficial. In costal areas with large humidity the diurnal temperature variation is not very large, as a result the decrement factor will be high even with thick walled buildings as the building cannot loose significant amount of heat to the outside even during the night due to the relatively high night temperatures. Thus thick walled buildings are not as effective in coastal areas as in dry areas.

Empirical methods for cooling load estimation:

Equation (34.37) can be written as:

$$Q_{x=0,\theta} = UA(T_{sol-air,m} - T_i) + UA\lambda(T_{sol-air,\theta-\phi} - T_{sol-air,m}) = UA.\Delta T_{eff} \quad (34.40)$$

where ΔT_{eff}, called as Equivalent Temperature Difference (**ETD**) or Cooling Load Temperature Difference (**CLTD**) is given by:

$$\Delta T_{eff} = (T_{sol-air,m} - T_i) + \lambda(T_{sol-air,\theta-\phi} - T_{sol-air,m}) \quad (34.41)$$

It can be seen from the above expression that ETD or CLTD depends on:

i. Decrement (λ) and Time Lag (ϕ) factors
ii. Solar radiation and outside ambient temperature (through sol-air temperature), and
iii. Inside temperature, T_i

Tables of ETD and CLTD have been prepared for fixed values of inside and outside temperatures, for different latitudes, orientations and different types of walls and roofs. For example, a typical CLTD table for a roof without suspended ceiling prepared and presented by ASHRAE is shown in Table 34.4:

Roof type	Mass per unit area, kg/m²	Heat capacity, kJ/m².K	Solar Time, h													
			07	**08**	**09**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
3	**90**	**90**	-2	1	5	11	18	25	31	36	39	40	40	37	32	25
4	**150**	**120**	1	0	2	4	8	13	18	24	29	33	35	36	35	32
5	**250**	**230**	4	4	6	8	11	15	18	22	25	28	29	30	29	27
6	**365**	**330**	9	8	7	8	8	10	12	15	18	20	22	24	25	26

Description of Roof types:

Type 3: 100 mm thick, lightweight concrete

Type 4: 150 mm thick, lightweight concrete

Type 5: 100 mm thick, heavyweight concrete

Type 6: Roof terrace systems

***Table 34.4**: CLTD values (in K) for flat roofs without suspended ceilings (ASHRAE Handbook)*

For vertical walls in addition to the other parameters, the orientation of the wall affects the incident solar radiation and hence the CLTD values. For example, Table 34.5 shows the CLTD values for a D-Type (100-mm face brick with 200-mm concrete block and interior finish or 100-mm face brick and 100-mm concrete brick with interior finish) wall with solar time for different orientations:

Solar Time,h	Orientation							
	N	NE	E	SE	S	SW	W	NW
7	3	4	5	5	4	6	7	6
8	3	4	5	5	4	5	6	5
9	3	6	7	5	3	5	5	4
10	3	8	10	7	3	4	5	4
11	4	10	13	10	4	4	5	4
12	4	11	15	12	5	5	5	4
13	5	12	17	14	7	6	6	5
14	6	13	18	16	9	7	6	6
15	6	13	18	17	11	9	8	7
16	7	13	18	18	13	12	10	8
17	8	14	18	18	15	15	13	10
18	9	14	18	18	16	18	17	12
19	10	14	17	17	16	20	20	15
20	11	13	17	17	16	21	22	17
$CLTD_{max}$	11	14	18	18	16	21	23	18

***Table 34.5**: CLTD values (in K) for D-type walls (ASHRAE Handbook)*

The above tables are valid for the following conditions:

a) Inside temperature of **25°C**, maximum outside temperature of **35°C** with an average value of **29°C** and a daily range of **12°C**. For inside and average outside temperatures (T_i and T_{av}) other than the above, the following adjustment has to be made to CLTD:

$$\mathbf{CLTD_{adj} = CLTD_{Table} + (25-T_i) + (T_{av}-29)} \qquad (34.42)$$

Where $CLTD_{Table}$ is the value obtained from the table.

b) Solar radiation typical of July 21st at 40°N latitude, but in the absence of more accurate data, the tables can be used without significant error for 0°N to 50°N and for summer months.

Similar data are available for other types of walls and roofs and for different latitudes. Adjustments are also suggested for walls and roofs with insulation, wetted roofs etc.

Thus knowing the value of the overall heat transfer coefficient and area of the wall from the building specifications, local design outdoor temperatures and suitable ETD or CLTD values from the tables, one can calculate the heat transfer rate to the conditioned space through the opaque walls and roof of the building using Eq.(34.40). It should be remembered that the use of published ETD or CLTD cannot cover all possible walls and roofs and other conditions. Hence, some error is always involved in using these data. However, by developing individual heat transfer models for the specific building and using the numerical methods, one can estimate the heat transfer rate to the building more accurately. However, since this is extremely time consuming, practising engineers generally use the published data and provide a safety factor to account for possible differences in the actual and published values.

Questions and answers:

1. Estimation of heat transfer rate through buildings is complex due to:

a) Complex structure of the walls and roofs consisting of a wide variety of materials
b) Varying indoor and outdoor conditions
c) Large size of the buildings
d) All of the above

Ans.: a) and b)

2. Heat transfer through buildings can be considered as steady, if:

a) Variation in outdoor conditions with time are not significant
b) Variation in indoor conditions with time are not significant
c) Thermal capacity of the building is large
d) All of the above

Ans.: d)

3. Which of the following statements are TRUE?

a) A wall is said to be homogeneous if its properties do not vary with temperature
b) A wall is said to be homogeneous if its properties do not vary with location
c) The heat transfer resistance of a homogeneous wall depends on its thickness and density
d) The heat transfer resistance of a homogeneous wall depends on its thickness and thermal conductivity

Ans.: b) and d)

4. Which of the following statements are TRUE?

a) Heat transfer can take place by more than one mode in a non-homogeneous wall
b) The heat transfer resistance of a non-homogeneous wall is indicated in terms of its conductance
c) In an air space, the conduction effect becomes dominant as the air gap reduces
d) In an air space, the conduction effect becomes dominant as the air gap increases

Ans.: a), b) and c)

5. Which of the following statements are TRUE?

a) Heat transfer through a building wall may be considered as steady if its thermal capacity is very small
b) When the thermal capacity of the wall is large, at any point of time the heat transferred to the outer surface of the wall is larger than the heat transfer from the inner surface
c) When the thermal capacity of the wall is large, the heat transfer rate at the outer surface of the wall can be smaller than the heat transfer rate from the inner surface

d) Due to finite thermal capacity of the wall, the outer surface temperature is always higher than the inner surface temperature

Ans.: c)

6. Which of the following statements are TRUE?

a) The sol-air temperature depends on indoor and outdoor temperatures
b) The sol-air temperature depends on outdoor temperature and incident solar radiation
c) The sol-air temperature depends on outdoor temperature, incident solar radiation and surface properties of the wall
d) The sol-air temperature depends on outdoor temperature, incident solar radiation, surface properties of the wall and the external heat transfer coefficient

Ans.: d)

7. Which of the following statements are TRUE?

a) In the analytical method, the outer boundary conditions are generally assumed to be independent of time
b) In the analytical method, the outer boundary conditions are generally assumed to vary in a periodic manner with time
c) In the analytical method, the indoor temperature is generally assumed to be independent of time
d) Analytical methods are amenable to simple geometries only

Ans.: b), c) and d)

8. Which of the following statements are TRUE?

a) For walls with negligible thermal capacity, the decrement factor is 0.0 and time lag is 1.0
b) For walls with negligible thermal capacity, the decrement factor is 1.0 and time lag is 0.0
c) The required cooling capacity of the air conditioning plant increases as decrement factor increases and time lag decreases
d) The required cooling capacity of the air conditioning plant increases as decrement factor decreases and time lag increases

Ans.: b) and c)
9. Which of the following statements are TRUE?

a) From thermal comfort point of view, thick walled structures are beneficial in hot and humid climates
b) From thermal comfort point of view, thick walled structures are beneficial in hot and dry climates
c) On a given day, the CLTD value of east facing wall reaches a peak before a west facing wall
d) On a given day, the CLTD value of west facing wall reaches a peak before a east facing wall

Ans.: b) and c)

10. Which of the following statements are TRUE?

a) Adjustments to CLTD tables have to be made if the latitude is different
b) Adjustments to CLTD tables have to be made if the indoor temperature is different
c) Adjustments to CLTD tables have to be made if the outdoor temperature is different
d) Adjustments to CLTD tables have to be made if the daily range is different

Ans.: b), c) and d)

11. A building has to be maintained at 21°C(dry bulb) and 50% relative humidity when the outside conditions are -30°C(dry bulb) and 100% relative humidity. The inner and outer surface heat transfer coefficients are 8.3 W/m^2.K and 34.4 W/m^2.K, respectively. A designer chooses an insulated wall that has a thermal resistance (R-value) of 0.3 m^2.K/W. Find whether the wall insulation is sufficient to prevent condensation of moisture on the surface. If the chosen R-value of the wall can lead to condensation, what is the minimum thickness of additional insulation (thermal conductivity 0.036 W/m.K) required to prevent condensation. Take the barometric pressure as 101 kPa.

Ans.: From the psychrometric chart; for inside conditions of 21°C and 50% RH:

$$\textbf{Dew Point Temperature, } T_{DPT,i} = 10^oC$$

The overall heat transfer coefficient for the wall U is given by:

$$U = [R_{wall} + (1/h_i) + (1/h_o)]^{-1} = [0.3 + (1/8.3) + (1/34.4)]^{-1} = 2.224\ W/m^2.K$$

Assuming steady state, the heat transfer rate through the wall is given by:

$$q_w = U(T_i - T_o) = 2.224 \times (21 - (-30)) = 113.424\ W/m^2$$

The temperature of the inner surface of the wall, $T_{s,i}$ is obtained using the equation:

$$q_w = h_i(T_i - T_{s,i}) = 113.424 \Rightarrow T_{s,I} = 7.33^oC$$

$$\text{Since } T_{s,i} < T_{DPT,i}$$

⇒ Condensation will take place on the inner surface of the wall (Ans.)

To prevent condensation, the minimum allowable temperature of inner surface is the DPT (10°C)

Under this condition, the maximum allowable heat transfer rate is given by:

$$q_{w,allowable} = h_i(T_i - T_{DPT,i}) = 8.3 \times (21 - 10) = 91.3\ W/m^2$$

Hence the required U_{req} value is:

$$U_{req} = 91.3/(T_i - T_o) = 91.3/(21 - (-30)) = 1.79\ W/m^2.K$$

Hence the required resistance of the wall, $R_{w,req}$ is given by:

$$R_{w,req} = (1/U_{req}) - (1/h_i) - (1/h_o) = 0.4091\ m^2.K/W$$

Hence the amount of additional insulation to be added is:

$$R_{add} = (t_{add}/k_{add}) = 0.4091 - R_{wall} = 0.4091 - 0.3 = 0.1091\ m^2.K/W$$

⇒ **Required insulation thickness, $t_{add} = 0.1091 \times 0.036 = 3.928\times10^{-3}$ m (Ans.)**

12. A 4m x 5m wall consists of 3 glass windows of 1.5m x 1.0 m dimensions. The wall has thickness of 0.125 m and a thermal conductivity of 0.5 W/m.K, while the glass windows are 6 mm thick with a thermal conductivity of 1.24 W/m.K. The values of internal and external surface conductance for the wall (including glass) are 8.3 $W/m^2.K$ and 34.4 $W/m^2.K$, respectively. The internal and external temperatures are 21^oC and -30^oC, respectively. Calculate the total heat transfer rate through the wall. What percentage of this heat transfer is through the windows?

Ans.: The total heat transfer rate through the wall is given by:

$$Q_{total} = U_oA_{total}(T_i - T_o)$$

The value of U_oA_{total} is given by:

$$U_oA_{total} = U_{wall}A_{wall} + U_{glass}A_{glass}$$

The U values for the wall and glass are obtained from their individual resistance values as:

$$U_{wall} = [(0.125/0.5) + (1/8.3) + (1/34.4)]^{-1} = 2.503\ W/m^2.K$$

$$U_{glass} = [(0.006/1.24) + (1/8.3) + (1/34.4)]^{-1} = 6.48\ W/m^2.K$$

The area of glass, $A_{glass} = 3 \times 1.5 \times 1.0 = 4.5\ m^2$

The area of wall, $A_{wall} = 4 \times 5 - 4.5 = 15.5\ m^2$

Hence, $U_oA_{total} = U_{wall}A_{wall} + U_{glass}A_{glass} = 2.503 \times 15.5 + 6.48 \times 4.5 = 67.96$ W/K

Hence, $Q_{total} = U_oA_{total}(T_i - T_o) = 67.96(21+30) = 3465.96$ W (Ans.)

% of heat transfer rate through glass = $\{U_{glass}A_{glass}(T_i - T_o)/Q_{total}\}\times100 = 42.9\%$

(Ans.)

13. A multi-layered wall consists (from inside to outside) 6mm thick plywood, 125 mm thick common brick, 2.1 mm thick air space, 125 mm thick common brick and 6 mm thick cement plaster. The values of internal and external surface conductance for the wall are 8.3 $W/m^2.K$ and 34.4 $W/m^2.K$, respectively. Find the overall heat transfer coefficient of the wall. What is the value of U, if the air space is replaced by 20 mm thick EPS board? Assume the temperature difference across the air space to be 10 K.

Ans.: For the composite wall, the overall heat transfer coefficient U is given by:

$$\left(\frac{1}{U}\right) = R_{tot} = \left(\frac{1}{h_i}\right) + \sum_{i=1}^{N}\left(\frac{\Delta x_i}{k_{w,i}}\right) + \sum_{j=1}^{M}\left(\frac{1}{C_j}\right) + \left(\frac{1}{h_o}\right)$$

Substituting the values of individual resistances using the input values of wall thickness and thermal conductivity and thermal conductance (From Tables 34.2 and 34.3), the overall heat transfer coefficient is given by:

$$\left(\frac{1}{U}\right) = \left(\frac{1}{8.3}\right) + \left(\frac{0.006}{0.1}\right) + \left(\frac{0.125}{0.77}\right) + \left(\frac{1}{5.8}\right) + \left(\frac{0.125}{0.77}\right) + \left(\frac{0.006}{8.65}\right) + \left(\frac{1}{34.4}\right) = \mathbf{0.7073\, m^2K/W}$$

$$\Rightarrow \mathbf{U = 1.414\ W/m^2.K} \qquad \textbf{(Ans.)}$$

If the air space is replaced by 20 mm EPS (k = 0.037 W/m.K), then the new U-value is:

$$\mathbf{U_{EPS} = [(1/U) - (1/5.8) + (0.02/0.037)]^{-1} = 0.93\ W/m^2.K} \qquad \textbf{(Ans.)}$$

Thus replacing the air gap with EPS leads to **a decrease in the U-value by about 34 percent.**

14. Determine the sol-air temperature for a flat roof if the direct radiation normal to the sun's rays (I_{DN}) is 893 W/m^2 and the intensity of scattered radiation normal to the roof (I_d) is 112 W/m^2. Take the absorptivity of the roof for direct and scattered radiation as 0.9, the heat transfer coefficient of the outside surface as 34.4 W/m^2, the outside air temperature as 37^{o}C and the solar altitude angle as 80^o. If the time lag of the roof structure is zero and its decrement factor is unity, calculate the heat gain to the room beneath the roof if the U-value of the roof is 0.5 W/m^2.K and the room temperature is 25^{o}C.

Ans.: For a flat roof, the angle of incidence θ is given by:

$$\theta = (\pi/2) - \beta = (\pi/2) - 80 = 10^o$$

where β is the altitude angle

Total solar irradiation on the flat roof I_t is given by:

$$\mathbf{I_t = I_{DN}.\cos(\theta) + I_d = 893 \times \cos(10) + 112 = 991.43\ W/m^2}$$

Hence the sol-air temperature is given by:

$$T_{sol-air} = T_o + \left(\frac{\alpha_D I_D + \alpha_d I_d - R}{h_o}\right) = 37 + \frac{0.9 \times 991.43}{34.4} = 62.94^o \qquad \textbf{(Ans.)}$$

Since the time lag is 0 and decrement factor is 1.0 for the roof, the heat transfer rate through the roof is given by:

$$\mathbf{q = U(T_{sol\text{-}air} - T_i) = 18.97\ W/m^2} \qquad \textbf{(Ans.)}$$

15. A building has its north, west facing walls and the roof exposed to sun. The dimensions of the building are 12 m X 12 m X 5 m (WXLXH). The U-value of the walls are 0.5 W/m^2.K, while it is 0.4 W/m^2.K for the roof. There are no windows on north and west walls, and the other two walls are exposed to air conditioned spaces. The outside design temperature is 41^{o}C while the indoor is maintained at 25^{o}C, while the average temperature for the design day is 31^{o}C. Calculate heat transfer rate to the building at 5 P.M., 6 P.M and & P.M. Assume the walls are of D-Type and the roof is of Type 5.

Ans.: Since the average outside temperature is different from 29oC, adjustments have to be made to the values obtained from the CLTD tables.

$$\mathbf{CLTD_{adj} = CLTD_{Table} + (T_{av} - 29) = CLTD_{Table} + 2}$$

<u>a) Heat transfer rate through the roof:</u>

From the Table of CLTD values for roof (Table 34.5), the CLTD values at 5 P.M., 6 P.M. and 7 P.M. are 29^{o}C, 30^{o}C and 29^{o}C, respectively.

$$\therefore Q_{roof,5\,P.M.} = U_{roof}A_{roof}CLTD_{adj,5\,P.M.} = 0.4 \times 144 \times (29 + 2) = 1785.6\ W$$
$$Q_{roof,\,6.P.M.} = U_{roof}A_{roof}CLTD_{adj,6\,P.M.} = 0.4 \times 144 \times 32 = 1843.2\ W$$

$$Q_{roof,\,7\,P.M.} = Q_{roof,\,5\,P.M.} = 1785.6\ W \text{ (as the CLTD values are same)}$$

b) Heat transfer rate through north facing wall:

Table 34.6 is used for obtaining CLTD values for the walls

$$Q_{north,\,5\,P.M.} = U_{wall}A_{wall}CLTD_{adj,\,5\,P.M.} = 0.5 \times 60 \times 10 = 300\ W$$

$$Q_{north,\,6.\,P.M.} = U_{wall}A_{wall}CLTD_{adj\,6\,P.M.} = 0.5 \times 60 \times 11 = 330\ W$$

$$Q_{north,7\,P.M.} = U_{wall}A_{wall}CLTD_{adj,\,7\,P.M.} = 0.5 \times 60 \times 12 = 360\ W$$

c) Heat transfer rate through the west facing wall:

Similar to the north facing wall, the heat transfer rates through the west facing walls are found to be:

$$Q_{west,5\,P.M.} = 450\ W$$

$$Q_{west,6\,P.M.} = 570\ W$$

$$Q_{west,7\,P.M.} = 660\ W$$

$\therefore$ Total heat transfer through the building is:

$$Q_{total,\,5\,P.M.} = 1785.6 + 300 + 450 = 2535.6\ W \qquad \text{(Ans.)}$$

$$Q_{total,\,6\,P.M.} = 1843.2 + 330 + 570 = 2743.2\ W \qquad \text{(Ans.)}$$

$$Q_{total,\,7\,P.M.} = 1785.6 + 360 + 660 = 2805.6\ W \qquad \text{(Ans.)}$$

Comments:

1. The difference in design dry bulb temperature between outdoor and indoor is 17^{o}C, it is observed that the CLTD value ranges between 31 to 32^{o}C for the roof, 10 to 12^{o}C for the north facing the wall and 15 to 22^{o}C for the west facing wall. The difference between CLTD values and $(T_o - T_i)_{design}$ is due to varying outdoor temperatures, varying solar radiation and finally due to the thermal capacity of the walls.

2. It is seen that the maximum amount of heat transfer rate is through the roof, hence, putting additional insulation on the roof will reduce the cooling load

3. Due to the thermal lag effect of the building, the peak heat transfer takes place not during sunshine, but after sunset.