Lesson
25
Analysis Of Complete Vapour Compression Refrigeration Systems

The specific objectives of this lecture are to:

1. Importance of complete vapour compression refrigeration system analysis and the methods used (*Section 25.1*)
2. Performance characteristics of reciprocating compressors (*Section 25.2*)
3. Performance characteristics of reciprocating condensers (*Section 25.3*)
4. Performance characteristics of evaporators (*Section 25.4*)
5. Performance characteristics of expansion valves (*Section 25.5*)
6. Performance characteristics of condensing unit (*Section 25.6*)
7. Performance characteristics of complete system by matching characteristics of evaporator and condensing unit (*Section 25.7*)
8. Effect of expansion valve on complete system performance (*Section 25.8*)
9. Meaning of sensitivity analysis (*Section 25.9*)

At the end of the lecture, the student should be able to:

1. Explain the concept of complete system analysis and the characteristics of graphical and analytical methods
2. Express or plot the performance characteristics of individual components such as compressors, condensers and evaporators and enumerate the influence of operating parameters such as cooling water and brine flow rates, inlet temperatures etc.
3. Obtain balance point for a condensing unit by matching the characteristics of compressors and condensers
4. Obtain the balance point and characteristics curves for a complete system assuming an ideal expansion valve
5. Explain the effect of expansion device on system performance
6. Explain the meaning of sensitivity analysis and its importance in system design and optimization

25.1. Introduction

A basic vapour compression refrigeration system consists of four essential components, namely compressor, condenser, expansion valve and evaporator. The individual performance characteristics of these components have been discussed in earlier lectures. However, in an actual system these components work in unison. The performance of a complete system is a result of the balance between these four components. For example, when the heat sink temperature varies, it affects the performance of the condenser, which in turn, affects the performance of the expansion device, evaporator and the compressor.

It is seen in Chapter 24 that expansion valve and compressor work in such a manner that the mass flow rate through the two components is the same at steady state. The balance point at steady state was obtained by equating the mass flow rates through these components. This is an example of balancing two components. Similar procedure can be extended to include the other two components also, so that a balance point for the entire system can be obtained by taking into account the individual characteristics. In principle, the balance point for the system can be obtained either by a graphical method or by an analytical method.

In graphical method, the performance of two interdependent components is plotted for the same two variables of common interest. For example, mass flow rate and evaporator temperature (or pressure) are plotted along y and x axes respectively for combination of compressor – expansion device at constant condenser temperature. The point of intersection of the two resulting curves will indicate the conditions at which the mass flow rate and evaporator temperature will be same for the two components. This point is called the balance point and in steady-state the combination will achieve these conditions.

In analytical method, the mass flow rate through expansion valve can be represented by an algebraic equation in terms of evaporator and condenser temperatures. Similarly, the mass flow rate through a given compressor can also be represented by an algebraic equation in terms of evaporator and condenser temperatures by regression analysis of experimental or analytical data. The balance point of the two components can be obtained by simultaneous solution of the two algebraic equations.

Since the graphical method uses two-dimensional plots, it considers only two components at a time while the system analysis by mathematical means can consider more than two components simultaneously. Further, considering time variation of parameters in form of differential equations can simulate the dynamic performance also. Steady–state system analysis will involve simultaneous solution of algebraic equations.

In this chapter, balance points of condensing unit, compressor-evaporator combination have been considered for illustration. As a first step the performance data of industrial components is presented in the form of plots or equations. The raw data for this purpose can be obtained from the catalogues of manufacturers. These are plotted either directly or after processing in terms of required variables.

25.2. Reciprocating compressor performance characteristics:

The power requirement and mass flow rate as function of evaporator temperature with condenser temperature as a parameter were presented in the chapter on compressors. For the purpose of balancing, the refrigeration capacity is required as a function of evaporator and condenser temperatures. This can be easily determined by considering the refrigeration cycle or from the catalogue data of the manufacturer. Figure 25.1 shows a theoretical single stage saturated cycle on T-s chart.
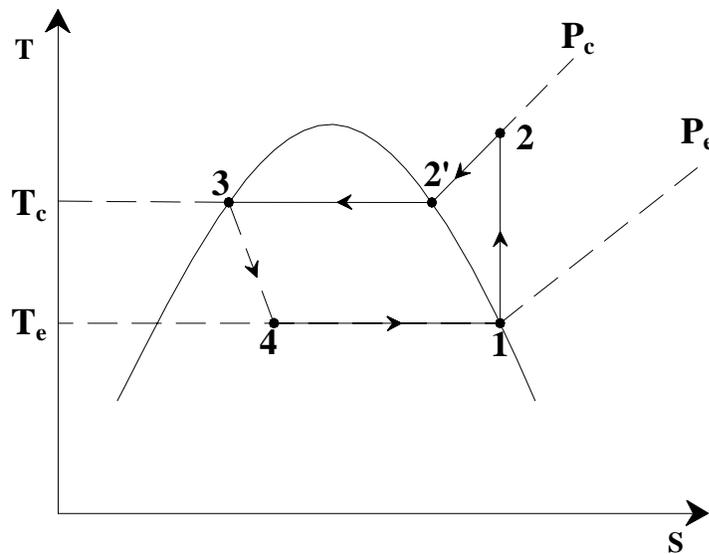
Fig.25.1*: A single stage, saturated vapour compression refrigeration cycle*

For the above cycle, the refrigeration capacity and power input to compressor are given by:

$$\dot{Q}_e = \dot{m}_r (h_1 - h_4) = \dot{V}_1 \left(\frac{h_1 - h_4}{v_1} \right) \qquad (25.1)$$

where $\dot{Q}_e$ is the refrigeration capacity, $\dot{m}_r$ **and** $\dot{V}_1$ are the refrigerant mass flow rate and volumetric flow rate of refrigerant at compressor inlet, respectively,

v_1 is the specific volume of refrigerant at compressor inlet, and h_1 and h_4 are the enthalpies of refrigerant at the exit and inlet of evaporator.

The volumetric flow rate of a reciprocating compressor is given by:

$$\dot{V}_1 = n.\eta_V \left(\frac{\pi D^2 L}{4} \right) \left(\frac{N}{60} \right)$$

(25.2)

where n is the number of cylinders, η_V is the volumetric efficiency, D, L and N are the bore, stroke and speed (in RPM) of the compressor, respectively.

It is seen in Chapter 19 that at a given condenser temperature the cooling capacity associated with mass flow rate given by a compressor increases as the evaporator temperature increases. On the other hand, for a given evaporator temperature, the cooling capacity decreases with increase in condenser temperature. These characteristics are shown graphically in Fig.25.2.
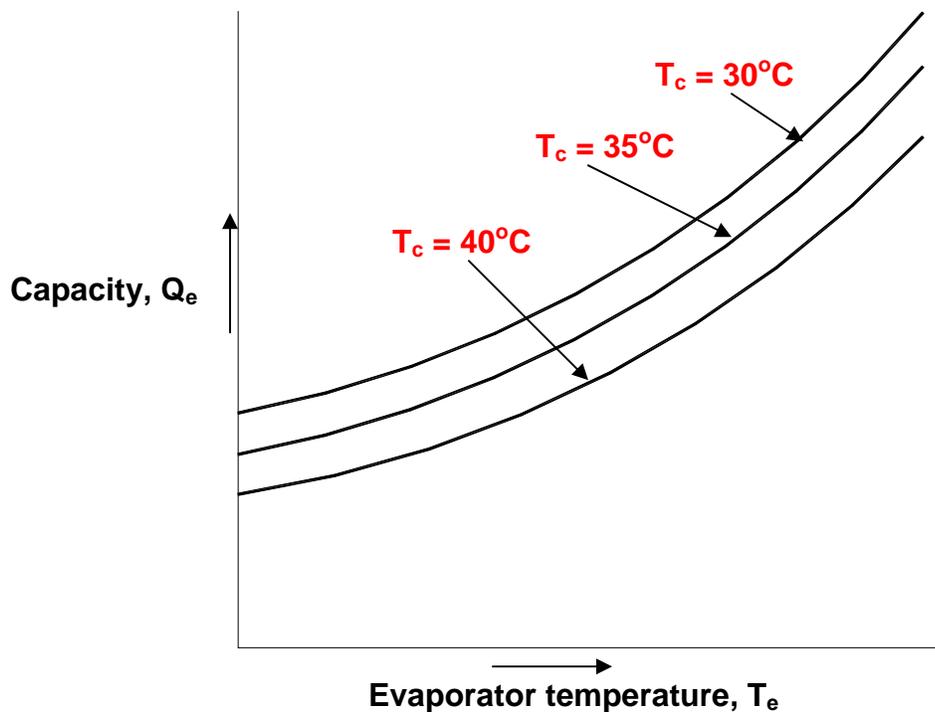
Fig.25.2. Variation of refrigeration capacity of a reciprocating compressor with evaporator and condenser temperatures at a fixed RPM

The following equation may represent the above trends:

$$Q_e = a_1 + a_2 T_e + a_3 T_e^2 + a_4 T_c + a_5 T_c^2 + a_6 T_e T_c + a_7 T_e^2 T_c + a_8 T_e T_c^2 + a_9 T_e^2 T_c^2 \quad (25.3)$$

where T_e and T_c are evaporator and condenser temperatures, respectively. The a_1 to a_9 are constants which can be determined by curve fitting the experimental or manufacturers' data using least square method, or by solving nine simultaneous equations of the type (25.3) for the nine constants a_i using nine values of Q_e from given catalogue data for various values of T_e and T_c. Similar expression can be obtained for power input to the compressor.

25.3. Condenser performance characteristics:

Actual representation of condenser performance can be quite complex as it consists of a de-superheating zone followed by condensing and subcooling zones. The heat transfer coefficient varies continuously along the length of the condenser due to the continuously changing state of the refrigerant. Hence a detailed analysis should include these aspects. However, as discussed in an earlier chapter on condensers, most of the time a simplified procedure is adopted by assuming the temperature to remain constant at a saturated temperature corresponding to the condensing pressure and a constant average condensing heat transfer coefficient is assumed.

For air-cooled condensers, it is possible to represent the total heat rejection rate from the condenser as a function of temperature difference and the overall heat transfer coefficient as follows:

$$Q_c = U_c A_c (T_c - T_\infty) \quad\quad\quad (25.4)$$

where, T_∞ is the ambient temperature and T_c is the condensing temperature of refrigerant.

For water-cooled condenser, one has to consider the water flow rate and inlet water temperature as additional parameters. In this case also a single region with constant condenser temperature T_c is considered. The heat transfer rate for a water-cooled condenser is expressed as follows:

$$Q_c = U_c A_c \text{LMTD} = \dot{m}_w Cp_w (T_{w,o} - T_{w,i}) \quad\quad (25.5)$$

where $\dot{m}_w$ is the water flow rate, U_c is overall heat transfer coefficient, $T_{w,i}$ and $T_{w,o}$ are the inlet and outlet water temperatures respectively. The log mean temperature difference of condenser LMTD_c is expressed as follows:

$$LMTDc = \frac{T_{w,o} - T_{w,i}}{\ln\left\{\dfrac{T_c - T_{w,i}}{T_c - T_{w,o}}\right\}} \tag{25.6}$$

From Eqns.(25.5) and (25.6) it can easily be shown that:

$$T_{w,o} = T_c - (T_c - T_{w,i})e^{-\left(\frac{U_c A_c}{m_w C_{pw}}\right)} = T_c - (T_c - T_{w,i})e^{-NTU} \tag{25.6}$$

NTU is the Number of Transfer Units equal to $\left(U_c A_c / \dot{m}_w Cp_w \right)$

The matching or the determination of balance point requires that its characteristics be represented in the same form as done for compressor, that is, cooling capacity vs. evaporator temperature. The condenser by itself does not give cooling capacity. One finds out the condensation rate of liquid refrigerant from the heat rejection capacity of condenser. The condensate rate multiplied by refrigeration capacity gives the cooling capacity. Hence from the given heat rejection capacity Q_c, one finds the condensate rate $\dot{m}_{ref}$ for the SSS cycle as follows:

$$\dot{m}_r = Q_c/(h_2 - h_3) \tag{25.7}$$

The corresponding refrigeration of the condenser is given by,

$$Q_e = \dot{m}_r (h_1 - h_4) \tag{25.8}$$

The condenser characteristics are shown in Fig.25.3 for a fixed value of $\dot{m}_w$ and $T_{w,i}$. It is observed that for a fixed evaporator temperature the capacity is higher at higher condenser temperature. A higher condenser temperature leads to a larger value of $LMTD_c$, which in turn gives a larger heat transfer rate and a larger condensate rate.

Further it is observed that at fixed condenser temperature, the cooling capacity increases with increase in evaporator temperature. The heat rejection ratio decreases with increase in evaporator temperature hence less heat rejection Q_c is required per unit cooling capacity, therefore the condensate rate of condenser can give larger cooling capacity. Figure 25.4 shows the effect of entering water temperature $T_{w,i}$ on cooling capacity for various condenser temperatures. The cooling capacity is zero when the entering water temperature

is equal to condenser temperature. As the inlet water temperature increases for a fixed condenser temperature, the $LMTD_c$ decreases, which decreases the cooling capacity. The following algebraic equation representing the curves of Fig. 25.3 at constant inlet temperature and flow rate of water can represent these characteristics.

$$Q_e = b_1 + b_2 T_e + b_3 T_e^2 + b_4 T_c + b_5 T_c^2 + b_6 T_e T_c + b_7 T_e^2 T_c + b_8 T_e T_c^2 + b_9 T_e^2 T_c^2 \quad (25.9)$$
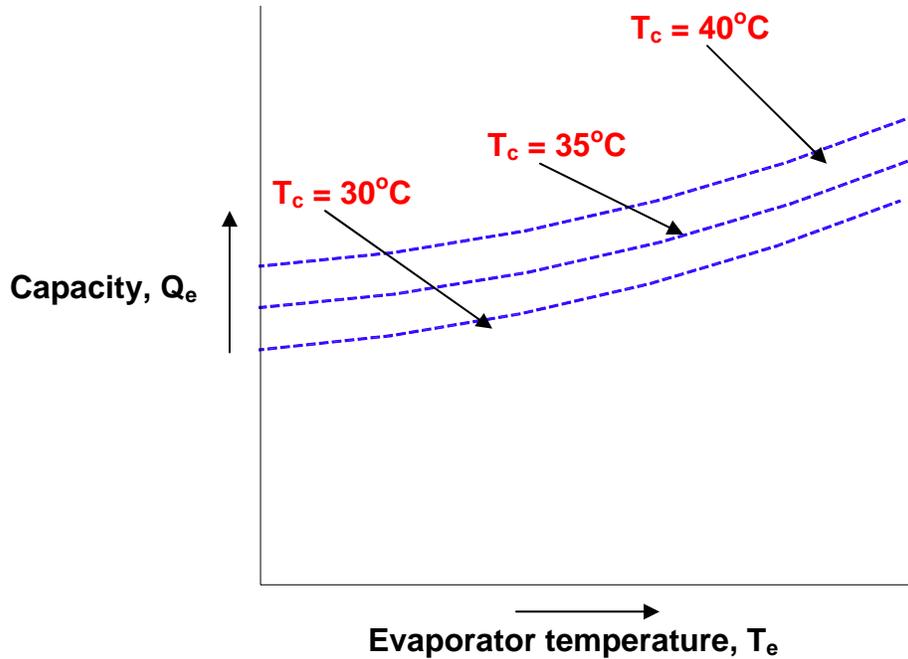
Fig.25.3. *Condenser performance at fixed water inlet temperature and flow rate*
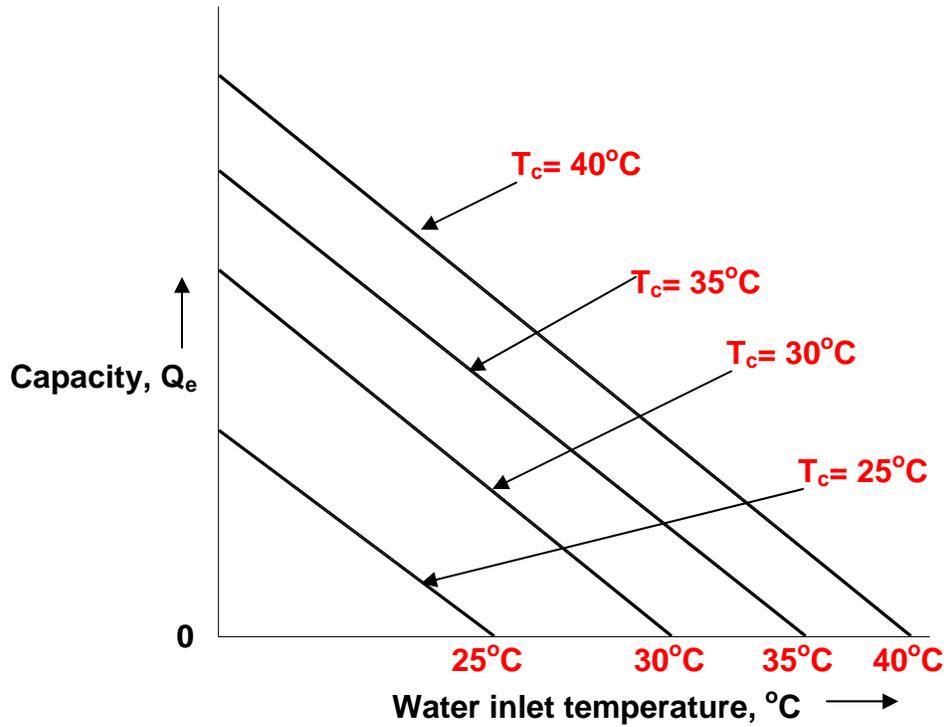
Fig.25.4. *Condenser performance with water inlet temperature at fixed flow rate*

The characteristics in Fig. 25.4 are straight lines with almost same slope for all the condenser temperatures. These may be represented by the following equation with a constant G.

$$Q_e = G(T_c - T_{w,i})$$
(25.10)

25.4. Evaporator Performance

Evaporator is also a heat exchanger just like condenser. For the sake of illustration, consider an evaporator that is used for chilling a brine. The cooling capacity of brine chiller is shown in Fig. 25.5 as a function of brine flow rate for different values of LMTD of evaporator. The brine side heat transfer coefficient h_b increases as the brine flow rate increases as a result, the overall heat transfer coefficient of the evaporator increases. Figure 25.5 shows that the cooling capacity increases with flow rate for fixed $LMTD_e$ for this reason.
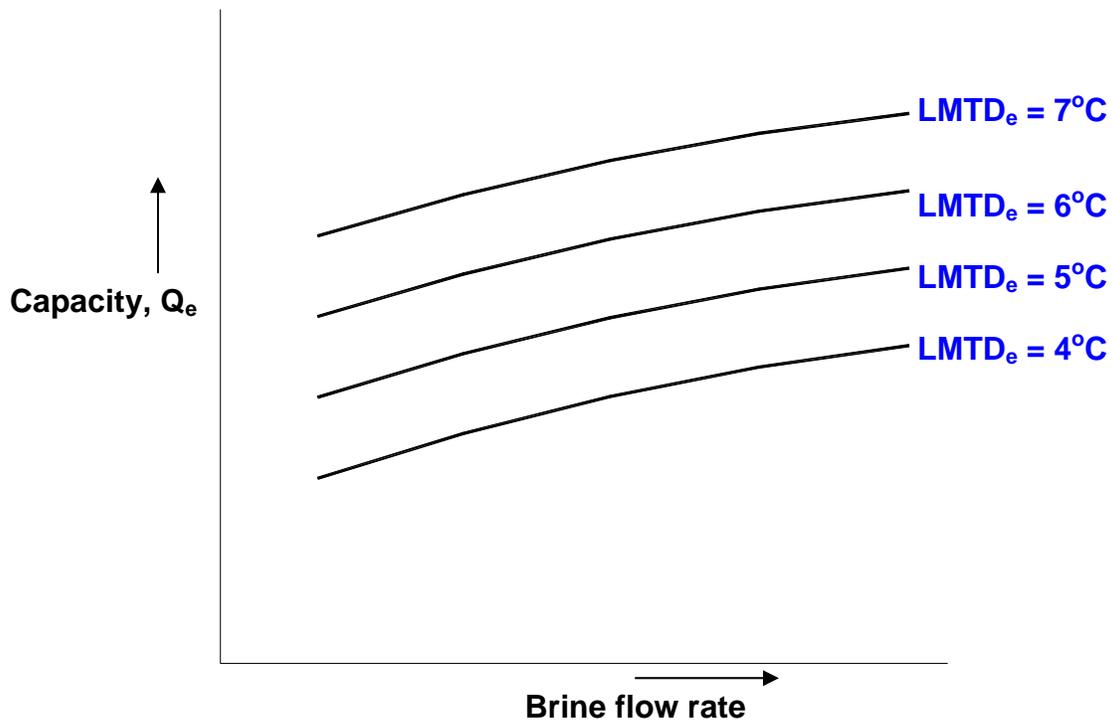
Fig.25.5: *Evaporator performance with brine flow rate and $LMTD_e$*

One can obtain the data for cooling capacity at various brine inlet temperatures from the characteristics of evaporator as shown in Fig.25.5. For example, if a plot for brine inlet temperature $T_{b,i}$ of 10°C is required, then we may choose an $LMTD_e$ of 5°C and read the capacity Q_e for the chosen brine flow rate $\dot{m}_b$. Then the brine outlet temperature $T_{b,o}$ is obtained from the equation:

$$Q_e = \dot{m}_b \, C_{pb} \, (T_{b,i} - T_{b,o}) \qquad (25.11)$$

Then the evaporator temperature T_e is obtained from the expression for $LMTD_e$:

$$LMTD_e = \frac{T_{b,i} - T_{b,o}}{\ln\left\{\dfrac{T_{b,i} - T_e}{T_{b,o} - T_e}\right\}} \qquad (25.12)$$

The capacity Q_e and evaporator temperature T_e are determined for different values of $LMTD_e$ for a fixed brine flow rate and brine inlet temperature of 10^oC. Figure 25.6 shows a plot obtained by this method. In this plot the brine flow rate is constant hence the brine side heat transfer coefficient is constant. If the evaporation heat transfer coefficient was also constant then overall heat transfer coefficient will also be constant and these lines will be straight lines. The evaporation heat transfer coefficient increases with increases in evaporator temperature hence these lines deviate slightly from straight lines. The capacity for these lines may be expressed as follows:

$$Q_e = c_0(T_{b,i}-T_e)+c_1(T_{b,i}-T_e)^2 \qquad (25.13)$$
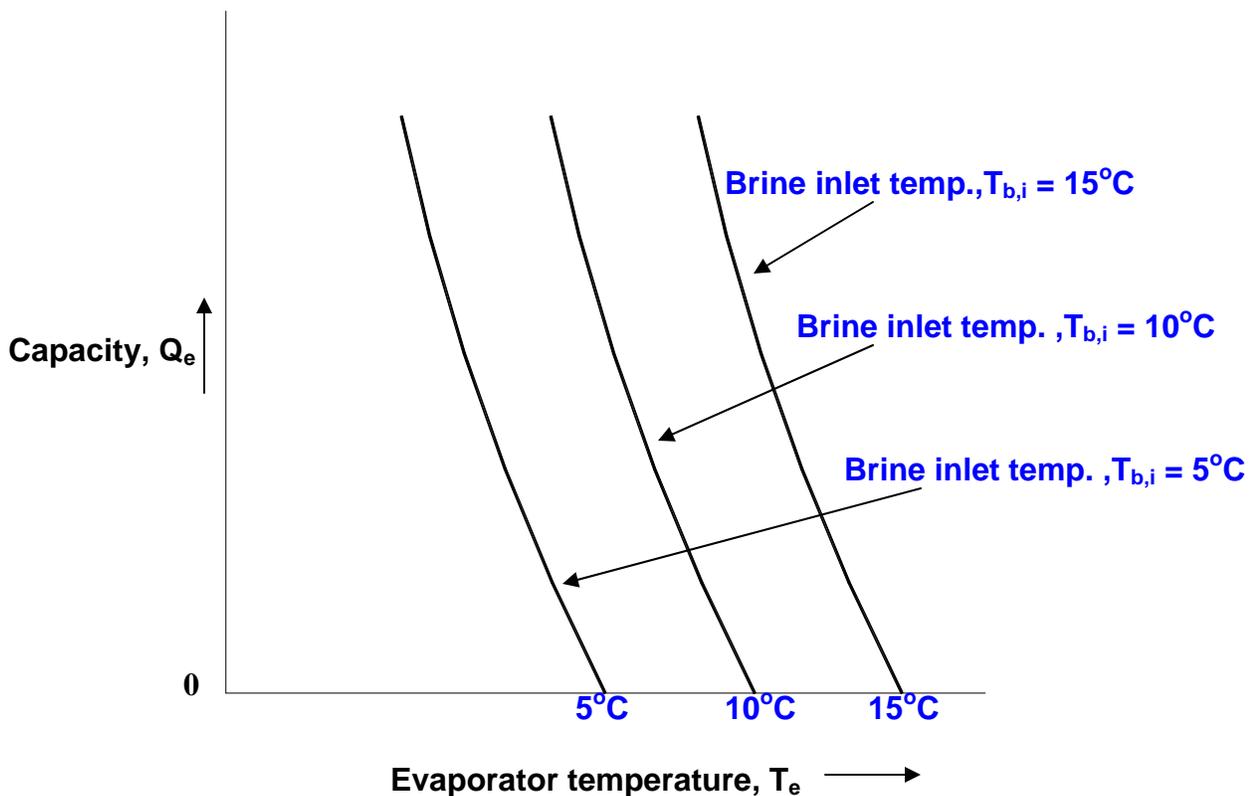
Fig.25.6: *Performance characteristics of evaporator at fixed brine flow rate*

25.5. Expansion valve Characteristics:

The characteristics of expansion valve play an important role in deciding the conditions achieved by the refrigeration system. It was shown in Chapter 24 that compressor and expansion valve seek an evaporator temperature such that under steady state conditions, the mass flow rate is same through the compressor and expansion valve. This was the result under the constraint that the condenser and evaporator have sufficiently large heat transfer areas and do not influence the performance of expansion device and compressor. In this chapter it is assumed that the expansion valve is capable of providing sufficient mass flow rate at all condenser and evaporator temperatures. This is assumed to simplify the matching problem. A float type of expansion valve or thermostatic expansion valve will meet this requirement. If the analysis is being done by computational method then the valve performance may also be included with some additional computational effort.

25.6. Condensing unit:

As mentioned before, if graphical procedure is used to find performance evaluation of various components, then only two components can be considered at a time. In view of this the first sub-system considered is the condensing unit. Condensing unit is a combination of compressor and condenser. This unit draws refrigerant from the evaporator, compresses it in the compressor, condenses it in the condenser and then feeds the condensed liquid refrigerant to the expansion valve. It is available off-the-shelf as a packaged unit from the manufacturer with matched set of compressor, compressor motor and condenser along with reservoir and controls. This may be air-cooled or water-cooled unit which may be installed as an outdoor unit.

The performance of condensing unit as function of evaporator temperature is obtained by combining the cooling capacity versus evaporator temperature characteristics of compressor and condenser. First we consider cooling capacity versus evaporator temperature assuming the compressor sped, the temperature and mass flow rate and entering water to condenser to be constant. This matching is obtained by superimposing the compressor performance curve given in Fig.25.2 on the condenser performance given in Fig.25.3 as shown in Fig.25.7. The intersection of compressor and condenser characteristics is at point A for 30°C condenser temperature. The combination of compressor and condenser will achieve a cooling capacity and evaporator temperature corresponding to this point at a condensing temperature of 30°C. Similarly, points B and C are the intersections at condenser temperatures of 35 and 40°C, respectively. These points are called balance points and the line A-B-C is called the performance characteristics of the condensing unit.
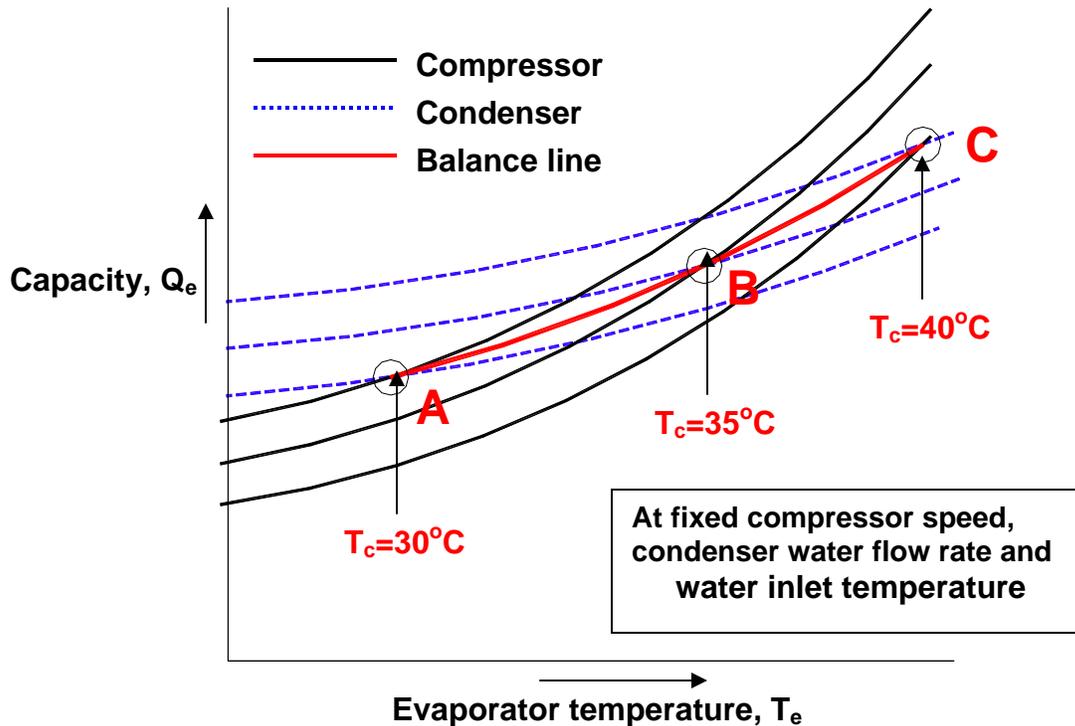
Fig.25.7: *Performance characteristics of a condensing unit as a function of evaporator and condensing temperatures*

It is observed that as the evaporator temperature decreases, the condensing temperature for the combination also decreases. This is explained as follows: at lower evaporator temperatures, the volumetric efficiency and the mass flow rate through the compressor decreases. This decreases the load on the condenser. A large condenser heat transfer area is available for small mass flow rate, hence condensation can occur at lower condenser temperature. It is also seen that as the evaporator temperature decreases, the refrigeration capacity of the condensing unit also decreases. This is due to the lower mass flow rate through the compressor due to lower volumetric efficiency and lower vapour density at compressor inlet.

Figure 25.8 shows the variation of refrigeration capacity of the condensing unit with variation in inlet water temperature to the condenser. This is obtained by superimposition of compressor characteristics of Fig.25.2 on the variation of condenser performance with inlet water temperature given in Fig.25.4. The two figures are shown side-by-side. At constant evaporator temperature of say, -5°C and condenser temperature of 30°C, the inlet water temperature corresponding to point D is required to match the two components. Points E and F are the balance points at condenser temperatures of 35 and 40°C respectively. Line DEF is the characteristics of the condensing unit at an evaporator temperature of -5°C. It is observed that the cooling capacity decreases as the inlet water temperature to condenser increases.
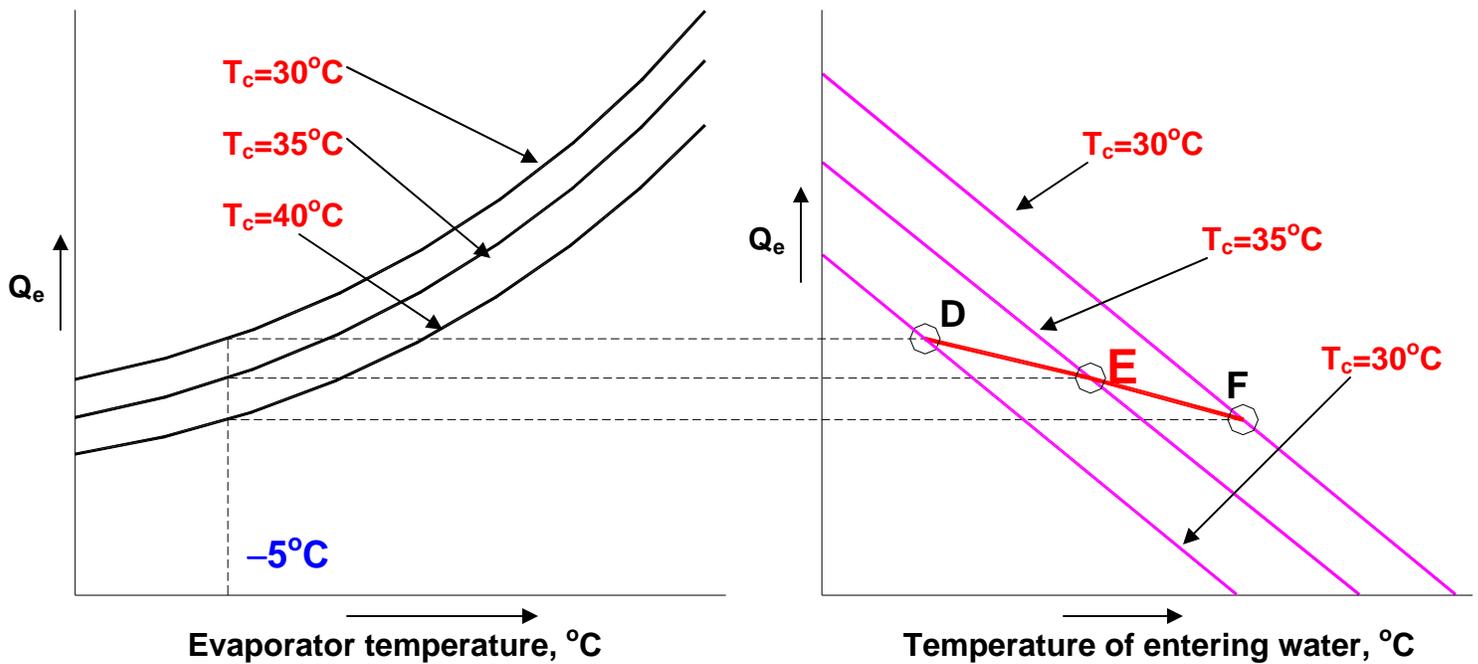
Fig.25.8: *Performance of the condensing unit as a function of water temperature at condenser inlet*

These characteristics can also be obtained by simultaneous solution of Eqns. (25.3) and (25.9) for constant water temperature at condenser inlet and constant water flow rate. For example, we wish to find the condenser temperature and capacity for a given evaporator temperature of say 10°C. An iterative procedure may be devised as follows:

(i)   For $T_e = 10^{\circ}$C assume a condensing temperature $T_c = 35^{\circ}$C
(ii)   Find Q_e from Eqn.(25.3)
(iii) Substitution of $T_e = 10^{\circ}$C and Q_e in Eqn.(25.9) will yield a quadratic equation for T_c. The value of T_c is found and checked against the assumed value of T_c (35°C being the first iterate) and iteration is continued until the calculated value matches with the assumed value of condenser temperature.

25.7. Performance of complete system - condensing unit and evaporator:

In steady state, a balance condition must prevail between all the components, that is, between condensing unit and evaporator assuming that the expansion valve will provide appropriate mass flow rate. This confluence will represent the performance of complete single-stage vapour compression refrigeration system. The combined curves will also give insight into the off-

design performance of the system and operational problems. Superimposing Fig.25.6 for the evaporator characteristics and Fig.25.7 for condensing unit characteristics yields the balance point of the system. This is shown in Fig.25.9. The characteristic curve shown in Fig.25.9 is for constant water temperature at condenser inlet, constant flow rate to the condenser, constant compressor speed and constant brine temperature at the inlet to the evaporator. The point of intersection of the two curves gives the refrigeration capacity and the evaporator
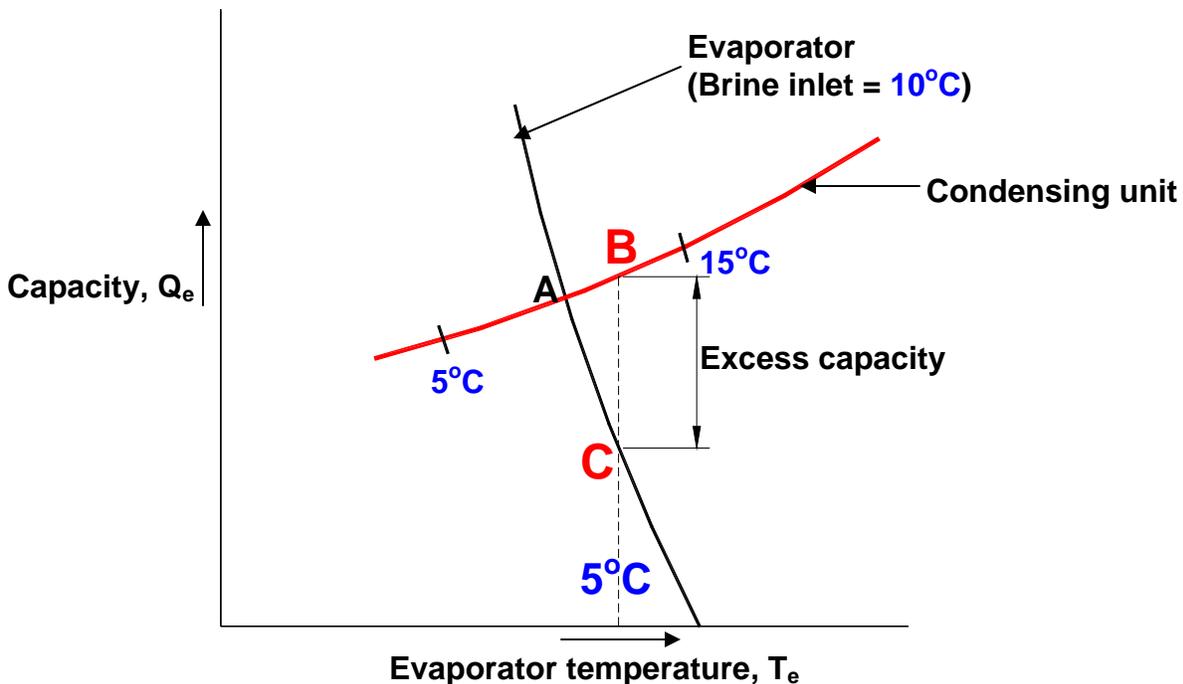
Fig.25.9: *Performance of the complete system as an intersection of evaporator and condensing unit characteristics at a brine inlet temperature* **of 10°C**

temperature that the system will achieve.

One can study the response of the system in transient state also by this figure. In a transient state, say the evaporator temperature is 5°C. The figure shows that at this point the condensing unit has a capacity corresponding to point B while the evaporator has capacity corresponding to a lower value at C. Hence the condensing unit has excess capacity. The excess capacity will reduce the temperature of refrigerant and the metallic wall of the evaporator. This will continue until the balance point of 3°C is reached at point A.

Figure 25.10 shows the effect of brine mass flow rate compared to that at the balance point. If the brine flow rate is increased, it is observed that cooling capacity increases to point D. At higher flow rate the overall heat transfer coefficient increases while ($T_{b,i}-T_{b,o}$) decreases permitting a larger mean temperature difference between refrigerant and brine. Therefore with increase in mass flow rate of brine, the cooling capacity increases. The pump power also

increases for the increased brine mass flow rate. Hence one has to make a compromise between increased capacity and increased cost of pump power. Figure 25.10 shows the condition for lower brine flow rate when the heat transfer coefficient on brine side decreases and temperature difference $(T_{b,i}-T_{b,o})$ increases. This is referred to as starving of evaporator.
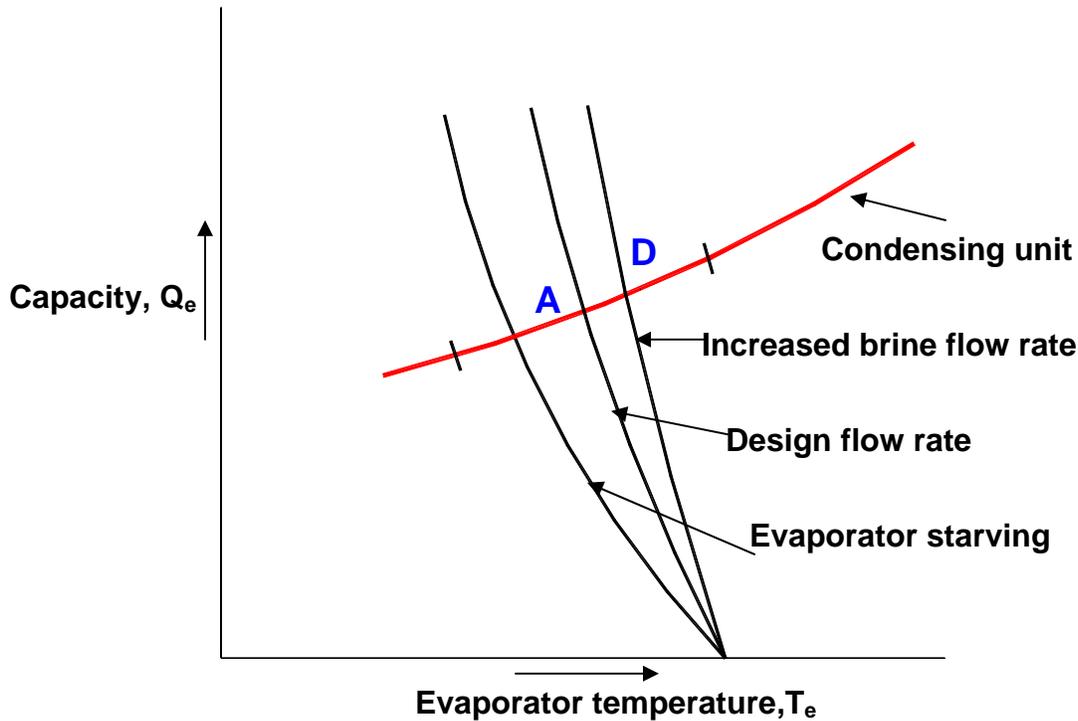
Fig.25.10: *Influence of brine flow rate on system cooling capacity*

25.8. Effect of expansion valve:

So far we have considered the balance between compressor, condenser and evaporator assuming that expansion valve can feed sufficient refrigerant to the evaporator so that heat transfer surface of the evaporator is wetted with refrigerant. Thermostatic expansion valve meets this requirement. Automatic expansion valve and capillary tube as observed in Chapter 24, result in a condition where sufficient quantity of refrigerant could not be supplied to evaporator. This condition was referred to as *starving of evaporator*. Starving reduces the heat transfer coefficient in evaporator since there is not sufficient refrigerant to wet the heat transfer surface consequently the cooling capacity reduces. There are other conditions also which may lead to this situation. These are as follows:

(i)   Expansion valve is too small,

(ii)   Some vapour is present in the liquid entering the expansion valve, and

(iii)   Pressure difference across the expansion valve is small

       If the refrigerant charge in the system is small then condition (ii) is likely to occur. Also if the frictional pressure drop in the liquid line is large or the valve is located at higher elevation than condenser then this condition may occur. During winter months the ambient temperature is low hence in air-cooled condenser the condenser pressure is low and the difference between evaporator and condenser pressure is small, as a result the starving condition (iii) is likely to occur. In this condition the expansion valve does not feed sufficient refrigerant to the evaporator since the driving force; the pressure difference across the expansion valve is small. The evaporator pressure also decreases in response to drop in condenser pressure. The evaporator pressure may become so low that mass flow rate through compressor may decrease due to lower volumetric efficiency. Hermetic compressor depends upon the mass flow rate of refrigerant for cooling on motor and compressor. This may be adversely affected under starved condition.

25.9. Conclusion:

       The methods presented in this chapter are useful when compressor, condenser, evaporator and expansion valve have been selected and the performance of combined system is desired. This analysis may not be useful in selecting the initial equipment. The techniques presented in this chapter are useful in predicting system performance for off-design conditions like a change in ambient temperature, condenser inlet water temperature and brine inlet temperature etc. The power requirement of the compressor has not been given due emphasis in the analysis. In fact, an equation similar to Eqn. (25.3) may be written for this also. This can also be found from known values of condenser and evaporator loads.

       An important aspect of refrigeration system performance is the sensitivity analysis which deals with % change in, say cooling capacity with % change in capacity of individual components like the compressor size, heat transfer area of evaporator and condenser etc. This can easily be done by mathematical simulation using the performance characteristics of the components given by empirical equations. It has been shown in Stoecker and Jones that compressor capacity has the dominant effect on system capacity and evaporator is next in importance. An increase in compressor capacity by 10% has the effect of 6.3% increase in system capacity. A 10% increase in evaporator gives 2.1% increase in system capacity, while 10% increase in condenser gives 1.3 % increase in system capacity. Such a data along with the relative costs of the components can be used for optimization of the first cost of the system. Table 25.1 taken from Stoecker and Jones illustrates the results of sensitivity analysis.

Ratio of component capacity to base capacity				
Compressor	Condenser	Evaporator	Refrigeration capacity, TR	% increase
1.0	1.0	1.0	95.6	-
1.1	1.0	1.0	101.6	6.3
1.0	1.1	1.0	96.8	1.3
1.0	1.0	1.1	97.6	2.1
1.1	1.1	1.1	1.1	10.0

Table 25.1: *Results of sensitivity analysis of a vapour compression refrigeration system (Stoecker and Jones, 1982)*

Questions and answers:

1. Which of the following statements are TRUE?

a) A graphical method generally considers two components at a time for system analysis
b) An analytical method can consider more than two components at a time for system analysis
c) Use of analytical method requires simultaneous solution of algebraic equations
d) All of the above

Ans.: d)

2. Which of the following statements are TRUE?

a) At a fixed RPM, the cooling capacity of a reciprocating compressor decreases as the evaporator temperature decreases and condensing temperature increases
b) At fixed water inlet temperature and flow rate, the capacity of a condenser increases as the condensing temperature and evaporator temperature increase
c) At fixed water flow rate and condensing temperature, the capacity of a condenser increases as the water inlet temperature increases
d) At fixed water flow rate and cooling capacity, the condensing temperature increases as the water inlet temperature increases

Ans.: a), b) and d)

3. Which of the following statements are TRUE?

a) At a fixed evaporator LMTD, the cooling capacity of a brine chilling evaporator increases with brine flow rate

b) At a constant brine flow rate and a given evaporator temperature, the cooling capacity of the evaporator increases as the brine temperature at evaporator inlet increases

c) At a constant brine flow rate and a given evaporator temperature, the cooling capacity of the evaporator increases as the brine temperature at evaporator inlet decreases

d) For constant cooling capacity and brine flow rate, the evaporator temperature has to decrease as the brine temperature at the inlet decreases

Ans.: a), b) and d)

4. Which of the following statements are TRUE?

a) The performance characteristics of a condensing unit are obtained by matching the characteristics of compressor and condenser

b) The performance characteristics of a condensing unit are obtained by matching the characteristics of evaporator and condenser

c) The performance characteristics of a condensing unit are obtained by matching the characteristics of expansion valve and condenser

d) The performance characteristics of a condensing unit are obtained by matching the characteristics of compressor and evaporator

Ans.: a)

5. Which of the following statements are TRUE?

a) At constant RPM, cooling water flow rate and inlet temperature, the balance point condensing temperature increases as evaporator temperature increases

b) At constant RPM, cooling water flow rate and inlet temperature, the balance point condensing temperature increases as evaporator temperature decreases

c) At constant RPM, cooling water flow rate and inlet temperature, the cooling capacity at balance point increases as evaporator temperature increases

d) At constant RPM, cooling water flow rate and inlet temperature, the cooling capacity at balance point increases as evaporator temperature decreases

Ans.: a) and c)

6. Starving of evaporator followed by reduction cooling capacity occurs when:

a) The capacity of expansion valve is larger than required
b) The inlet to the expansion valve is in two-phase region
c) The expansion valve is located at a higher elevation compared to condenser
d) There is a refrigerant leakage in the system

Ans.: b), c) and d)