GEARS

Gears – General Gearing Arrangements

Gear trains and gearing linkage exists in many aspects of aerospace mechanisms. Gearing systems are also part of any electromechanical actuator design. Gearing systems at their simplest consist of 2 gears. When 2 gears are not sufficient due to size, volume or strength constraints, a multi-gear train or a compound gear train is required. When a large gear ratio is desired within a small volume, then a planetary gear arrangement is required. Designs of these types of gear trains are discussed below. Other gearing arrangements – rack & pinion, bevel and worm gears are also presented below. The same principles for spur gears apply to other gear arrangements.

When designing gear trains a preliminary structural guideline regarding gear teeth size is given by the relationship

$$0.2 \le \frac{N_1}{N_2} \le 5 \qquad (1.)$$

This is a basic guideline on gear ratio limitations that is driven by gear teeth size and ensuring gear teeth have sufficient structural strength. The smaller gear is usually limited by physical size limitation and by ensuring gear teeth have sufficient strength. The larger gear is usually limited by physical size and ensuring gear teeth are not too small. A textbook rule of thumb for the face width of a gear tooth is that the face width should be 3 to 5 times the circular pitch of the gear. More specifically, sizing of gears from a stress and fatigue point of view should take into account (i) heat generated during operation, (ii) static strength margin of the teeth, (iii) fatigue capability, (iv) abrasive wear characteristics of the gear material over the expected life, and (v) noise/vibration affects during operation. Gear teeth can be analyzed structurally treating each tooth has a cantilever beam. Special formulas are available in various gear design handbooks and AGMA standards.

Two Gear Train

A two gear train is shown in Figure 1.

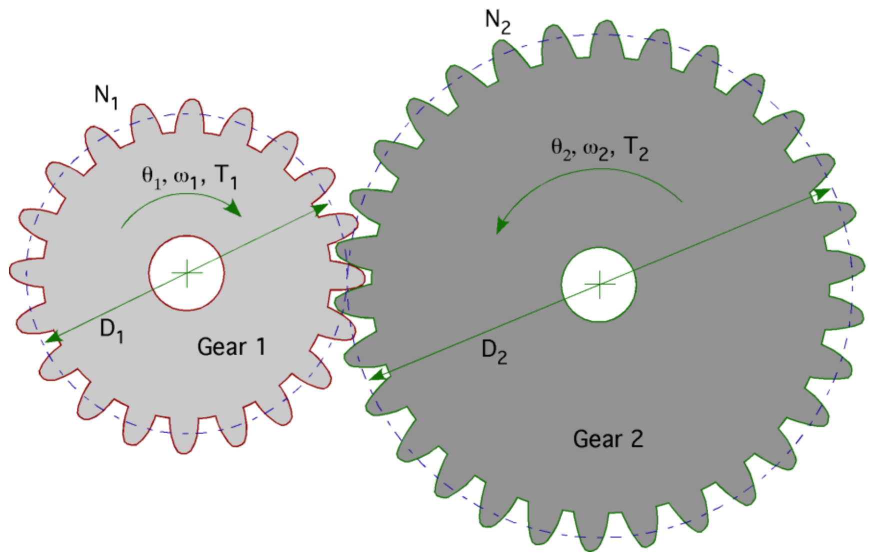

Figure 1 Two Gear Train

For a desired velocity ratio, gears are selected using the relationship

$$\frac{\omega_2}{\omega_1} = -\frac{D_1}{D_2} = -\frac{N_1}{N_2} \qquad (2.)$$

However, N_1 and N_2 must be integers and therefore ω_2/ω_1 can only be adjusted in discrete increments. If N_1 and N_2 can be found such that equation (1) is satisfied, then a workable gear design is achievable. If this relationship is not satisfied, then a multiple or compound gear train may be required. If a gear relationship exists that satisfies the constraints, then the pitch diameter can be chosen using N_1 and N_2, followed by the remaining gear parameters. Stress analysis must also be performed using expected loads in service to validate gear teeth sizing.

Results can be used to select gears from vendor catalogs or gear drawings can be created for manufacture. Using non-standard gears, it is possible to have non-integer values of gear teeth per inch.

Multi Gear Train Arrangement

Multi-gear trains are used when input and output shafts are far apart and space/design considerations do not allow for 2 large gears. Multi-gear trains are also used when more than 1 shaft must be driven at different speeds. Multi-gear trains are required to provide a + or – direction of the output gear relative to the input gear (2 gear trains always have input and output gears rotating in opposite directions). For a design example, consider a 3 gear train shown in Figure 2.

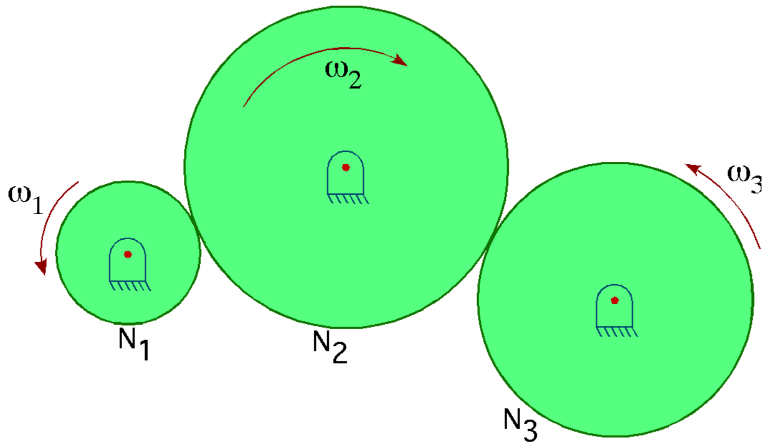

Figure 2 Multi Gear Arrangement

At each gear mesh,

$$\frac{\omega_2}{\omega_1} = -\frac{N_1}{N_2} \quad , \quad \frac{\omega_3}{\omega_2} = -\frac{N_2}{N_3} \tag{3.}$$

and for the gear train

$$\frac{\omega_2}{\omega_1}\frac{\omega_3}{\omega_2} = (-1)^2 \frac{N_1}{N_2}\frac{N_2}{N_3} \tag{4.}$$

or

$$\frac{\omega_3}{\omega_1} = \frac{N_1}{N_3} \tag{5.}$$

Velocity ratios for a multi-gear train are computed in the same manner as a two gear train. However, the only 2 gears that matter when computing the overall gear ratio is the first and last gear [see equation (5)]. Intermediate gears do not have any effect on the overall velocity ratio and are called "idler gears". If an intermediate gear was used to drive a 2^{nd} shaft, the 2^{nd} shaft would have a different velocity ratio. Design principles for each pair of gears in a multi-gear train are the same as a two gear train where tooth ratios should be within the constraints of equation (1).

Compound Gear Train Arrangement

A compound gear is a gear where 2 or more gears are rigidly attached, such that they rotate at the same speed. Compound gear trains are used when the velocity ratio is outside of the 0.2 – 5 range and space is limited. An example compound gear train is shown in Figure 3.

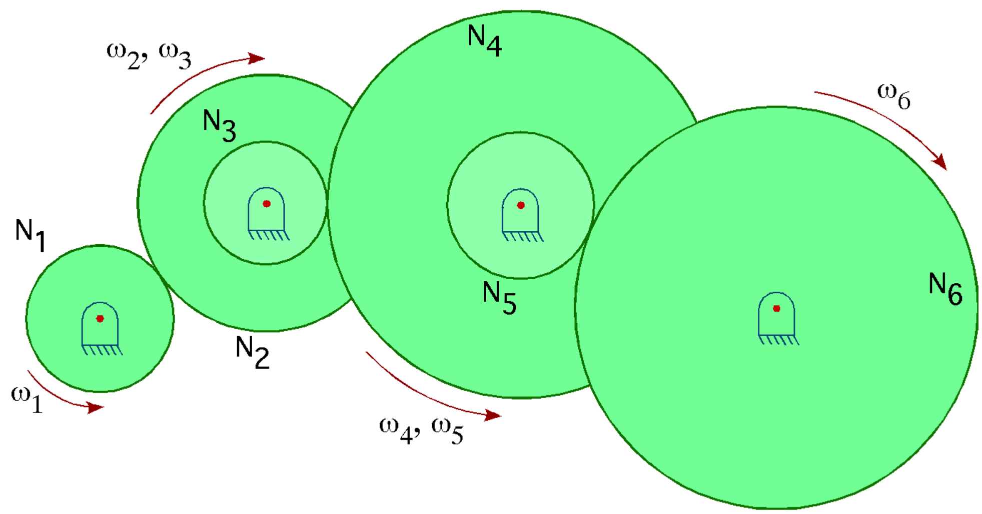

Figure 3 Compound Gear Train Arrangement

$$\frac{\omega_6}{\omega_1} = \left(\frac{\omega_2}{\omega_1}\right)\left(\frac{\omega_3}{\omega_2}\right)\left(\frac{\omega_4}{\omega_3}\right)\left(\frac{\omega_5}{\omega_4}\right)\left(\frac{\omega_6}{\omega_5}\right) = \left(-\frac{N_1}{N_2}\right)(1)\left(-\frac{N_3}{N_4}\right)(1)\left(-\frac{N_5}{N_6}\right)$$

$$= (-1)^3 \frac{N_1 N_3 N_5}{N_2 N_4 N_6} \qquad (6.)$$

An iterative process is required to design a compound gear train. To design a compound gear train, the general steps are as follows:

- Select number of stages, n_s. A stage is the number of gear meshes between compound gears. For example, for the compound gear train shown in Figure 3, the number of stages is 3.
- Using the desired overall velocity ratio $\left(\omega_1/\omega_{ns} = p_t\right)$, compute the velocity ratio for each stage using

$$p_s = \left(p_t\right)^{1/n_s} \qquad (5)$$

If the p_s ratio is outside of the 0.2 to 5 range, increase n_s and recompute p_s

- Choose a ratio for the number of teeth for each stage such that the ratio is close to p_s. Referring to Figure 3, the teeth ratios (N_1/N_2), (N_3/N_4), and (N_5/N_6) would need to be chosen.

- Choose gear tooth numbers for other stages such that the gear tooth ratio is close to p_s and the total velocity ratio is equal to p_t
- Choose a diametrical pitch for each gear pair (preferably the same for all gear pairs). Compute the pitch diameters using

$$P_d = \frac{N}{D} = \frac{\pi}{P_c} \qquad (7.)$$

- Iterate as required until results are satisfactory

Bevel Gears

A bevel gear arrangement is shown in Figure 4. Bevel gears provide a change in direction between the input and output shafts. Figure 4 shows a 90 degree change in direction, but other angles are possible.

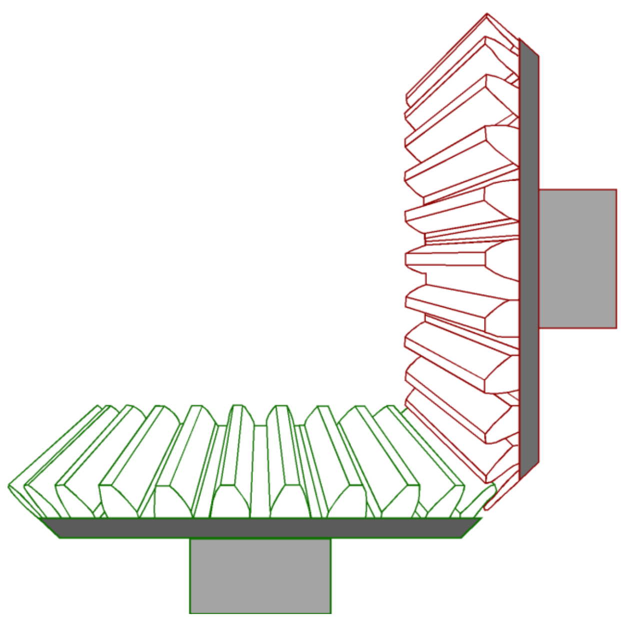

Figure 4 Bevel Gear Arrangement

The kinematic nomenclature for a bevel gear arrangement is shown in Figure 5. This figure shows a bevel gear arrangement that has a shaft angle different than 90°.

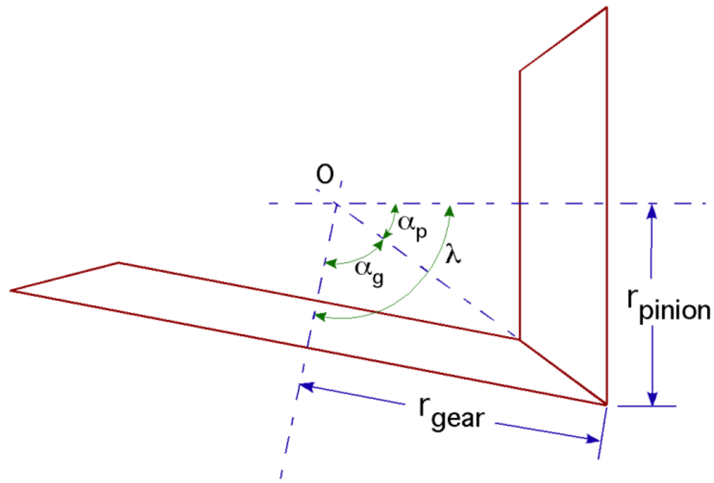

Figure 5 Bevel Gear Kinematics

In Figure 5, α_p and α_g are the pitch angles for the pinion and output gear, respectively. The shaft angle, λ, is the sum of , α_p and α_g. The pitch radiuses are given by r_{pinion} and r_{gear}. For bevel gears, the pitch radius is normally specified at the larger end of the teeth. The pitch cone for each gear represents the area between the root and end of the teeth. Similar to spur gears, the velocity ratio is computed as

$$\frac{\omega_{pinion}}{\omega_{gear}} = \frac{r_{gear}}{r_{pinion}} = \frac{N_{gear}}{N_{pinion}} \qquad (8.)$$

When designing bevel gears, the shaft angle and number of teeth on each gear are usually known up front. The corresponding pitch angles can be computed using

$$\tan\alpha_p = \frac{\sin\gamma}{\left(\frac{N_{gear}}{N_{pinion}} + \cos\gamma\right)} \qquad (9.)$$

and

$$\tan\alpha_g = \frac{\sin\gamma}{\left(\frac{N_{pinion}}{N_{gear}} + \cos\gamma\right)} \qquad (10.)$$

Rack & Pinion Gears

A rack and pinion gear arrangement converts rotary motion from a pinion to linear motion of a rack. Figure 6 shows a picture of a rack and pinion.

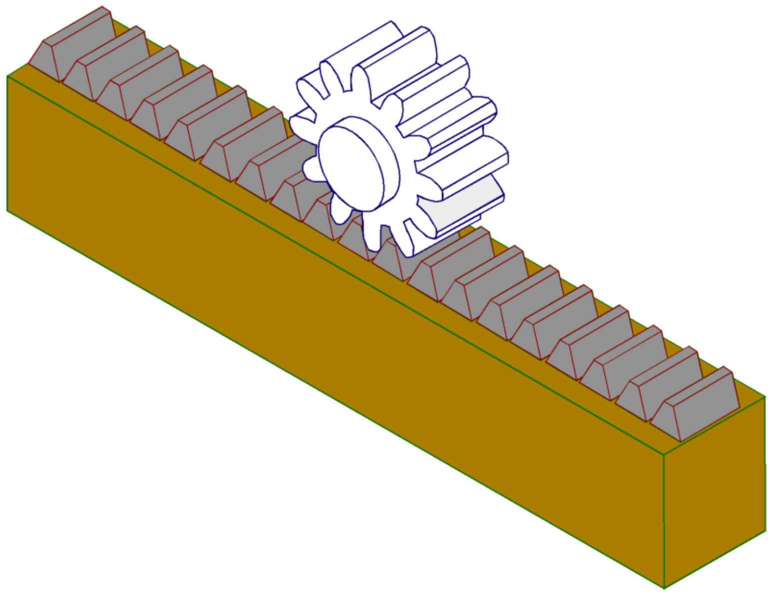

Figure 6   Rack and Pinion Gear Arrangement

The kinematic relationships for rack and pinion gear arrangements are similar to spur gears. Figure 7 shows the basic definitions for a rack and pinion gearset.

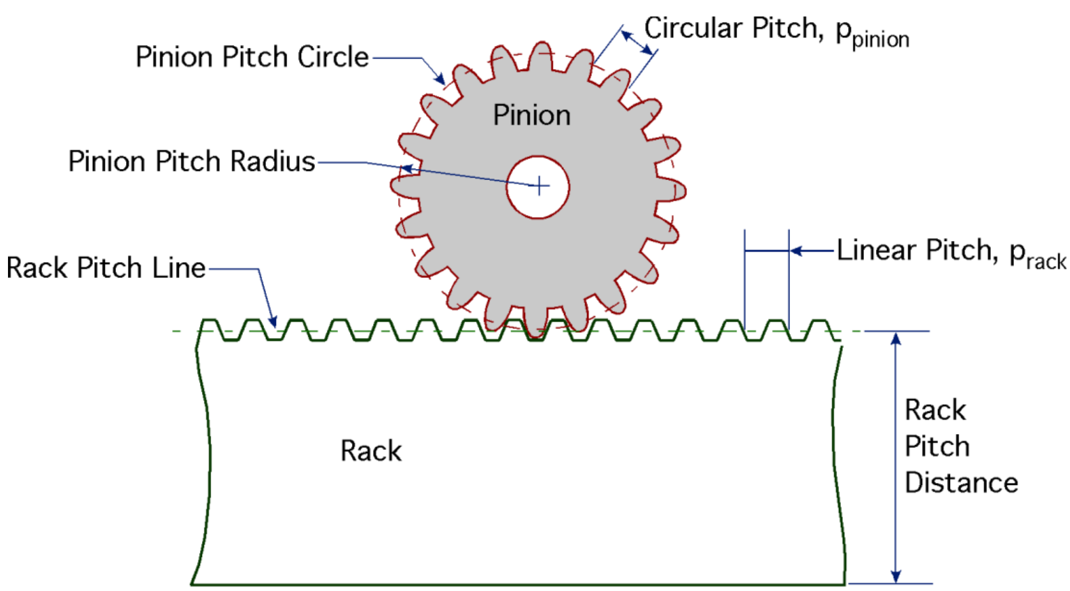

Figure 7   Kinematic Definitions for Rack and Pinion Gearset

For a rack and pinion to mesh together properly, the pitch of rack and pinion must be equal, i.e., $p_{pinion} = p_{rack}$. The gear ratio equation is given by

$$\frac{\omega_{pinion}}{V_{rack}} = \frac{1}{r_{pinion}} \qquad (11.)$$

Note that the number of teeth on the rack is not relevant to the velocity ratio. The linear speed of the rack is simply a function of the pinion pitch radius and the angular velocity of the pinion.

Worm Gears

A worm gear provides a 90 degree change in the direction of rotation while also providing a large gear ratio. A worm gear consists of a helical gear and a worm as shown in Figure 8. The worm gear threads are shaped like a power screw (see Acme thread screws in Power Screws – Description). The threads on the helical gear are not straight (like a standard spur gear) but have a helix angle that matches the lead on the worm gear threads. Worm gear arrangements are non-backdriving, as the helical gear will not be able to back drive the worm gear. Hence worm gears are self-locking.

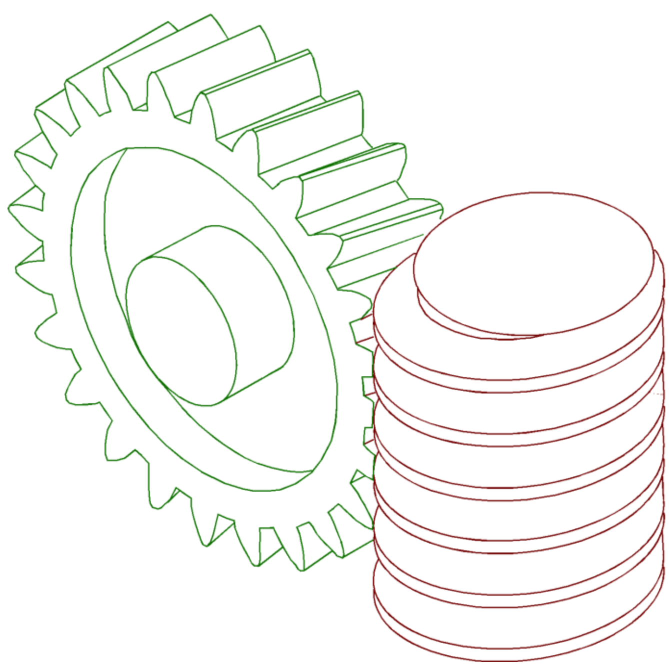

Figure 8 Worm Gear Arrangement

The kinematics of worm gear arrangement is shown in Figure 9.

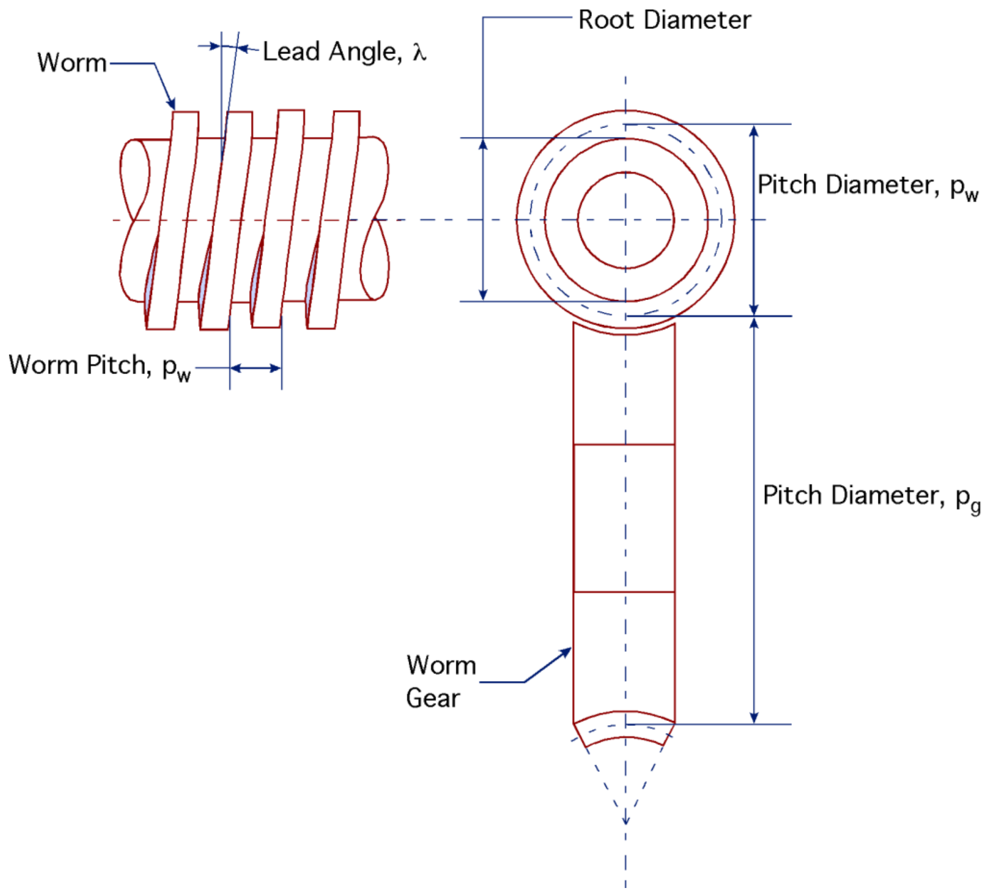

Figure 9 Worm Gear Kinematics

When specifying a worm and worm gear pair, the axial pitch of the worm and the circular pitch of the worm gear need to be specified. The gear ratio equation is given by

$$\frac{\omega_{worm\ gear}}{\omega_{worm}} = \frac{r_{worm}}{r_{worm\ gear}} = \frac{N_{worm}}{N_{worm\ gear}} \qquad (12.)$$

where the radius terms are ½ of the corresponding pitch dimensions. The pitch diameter of the worm gear is given by

$$d_g = \frac{N_{wg} p_{wg}}{\pi} \qquad (13.)$$

where N_{wg} is the number of teeth on the worm gear and p_{wg} is the circular pitch of the worm gear. The worm may have any pitch diameter since there is no relationship between the number of worm gear teeth and the worm diameter.

Planetary Gear Trains

A sample planetary gear train is shown in Figure 10. This planetary gear consists of 2 gears and an arm. Gear F is called the sun gear and gear L is called a planet gear. The input gear is gear F. The output rotation is the arm rotation. The planetary gear arrangement shown in Figure 10 has 2 degrees of freedom (DOFs) since the arm can rotate with or without input from gear F. Either spur or bevel gears can be used in planetary gear arrangement.

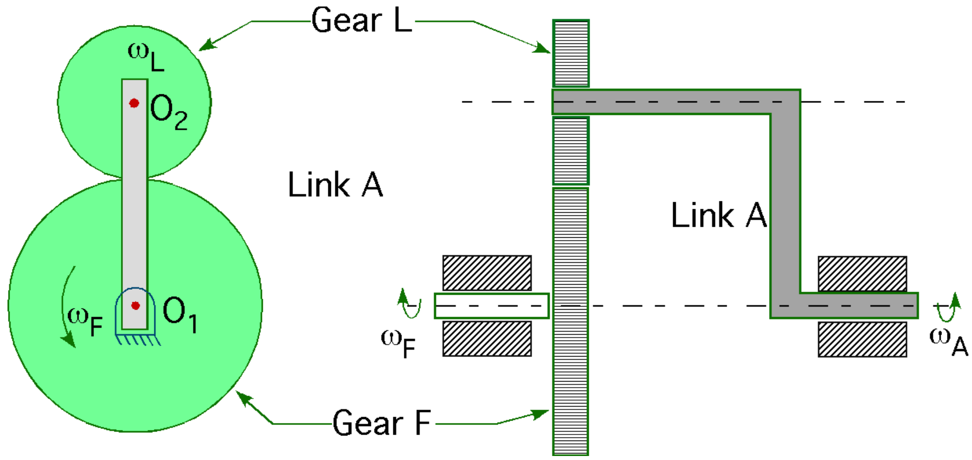

Figure 10 Planetary Gear Arrangement

Another example of a planetary gear train is shown in Figure 11. The input gear is gear 2 and the output is the arm rotation. This arrangement is similar to the arrangement of Figure 10 except that an intermediate gear (gear 3) has been added and gear 4 is connected to a ring gear (gear 5), which is fixed (non-moving). This arrangement has 1 DOF since the output arm will only move when the input gear (gear 2) is rotating. All realistic planetary gear arrangements will have 1 DOF.

Figure 11 also provides some insight into the operation of planetary gear arrangements. Note that 1 rotation of the output arm occurs when gear 4 travels one full rotation around the ring gear. For this to happen the input gear, gear 2, will have to rotate many, many times. Hence a high-speed input is geared down significantly at the output. An example is provided later to show this gearing down affect.

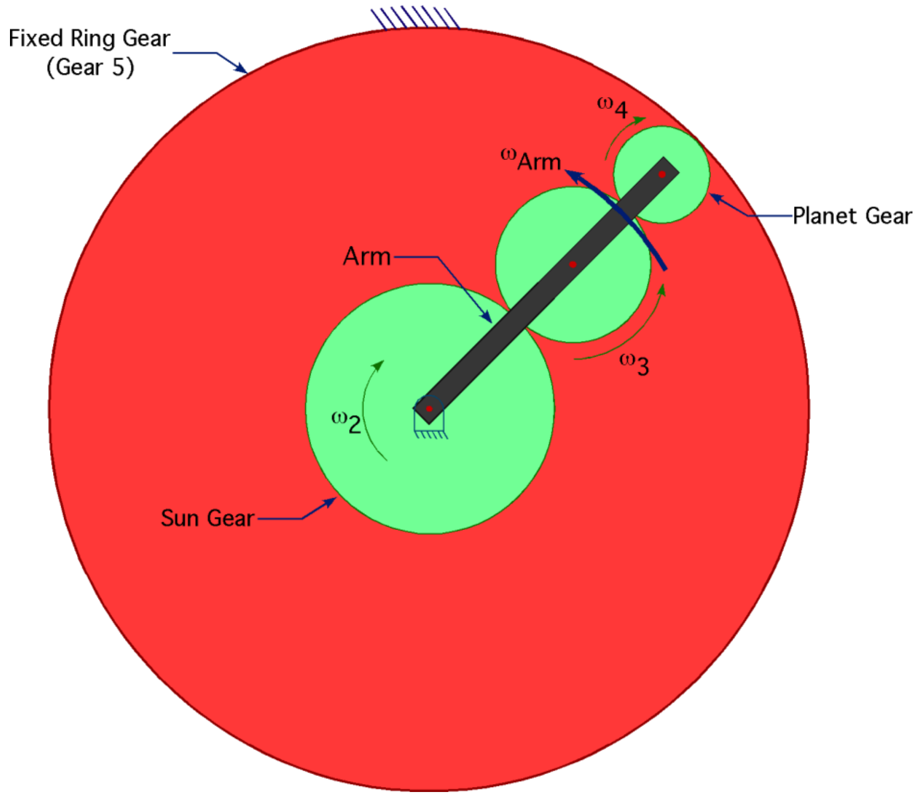

Figure 11   Planetary Gear Arrangement, 2nd Example

The number of degrees of freedom in a planetary gear train is computed using

$$F = 3(L-1) - 2B - M \qquad (14.)$$

where

F = number of degrees of freedom
L = number of rigid bodies (gears, arms and ground)
B = number of revolute joints (bearings)
M = number of meshes between gears

For the planetary gear train shown in Figure 10

L = 4
B = 3
M = 1

so that

$$F = 3(4-1) - (2)(3) - 1 = 2 \qquad (15.)$$

which verifies the planetary gear train in Figure 10 has 2 degrees of freedom.

Torque Summing and Speed Summing in Planetary Gear Arrangements

For input redundancy purposes, gear trains may be driven by separate drive shaft (motors). When two separate drive shafts are used, the gear train will either be a torque summing arrangement or a speed summing arrangement. In a torque summing arrangement, the torques are added (at the same speed) and in a speed summing the speeds are added (at a constant torque). Thus a torque summed gear arrangement cannot increase speed over a single drive motor speed and a speed summed gear arrangement cannot increase torque over the torque of a single drive motor.

A torque summing arrangement is shown in Figure 12.

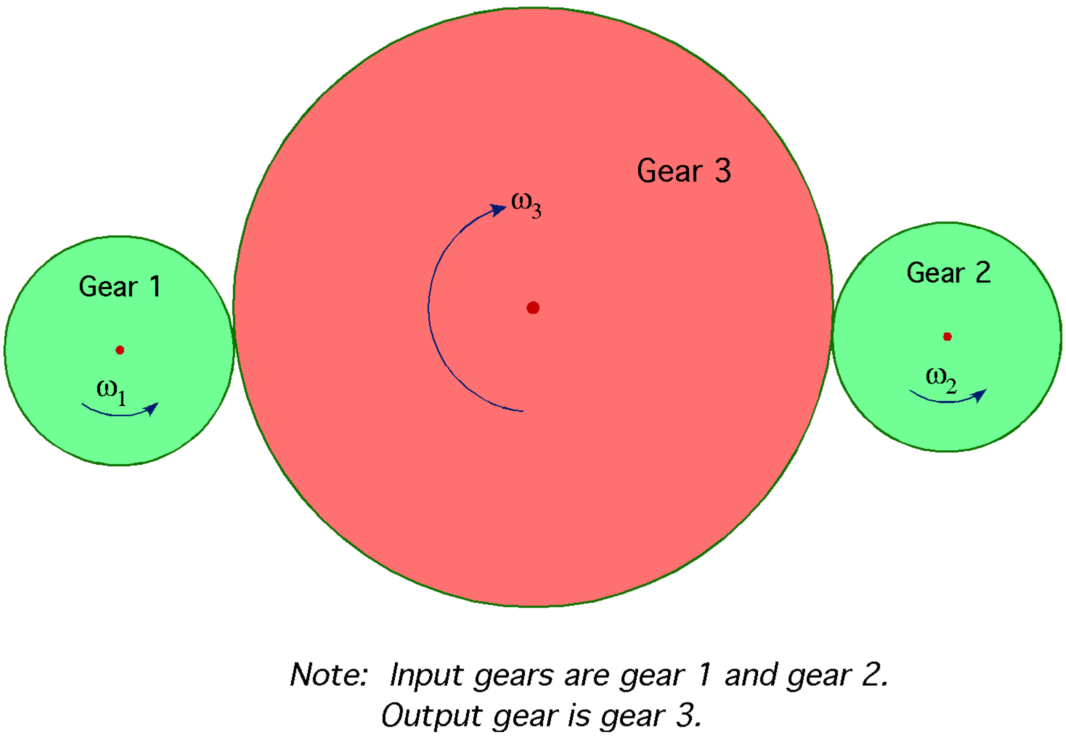

Figure 12 Torque Summing Gear Arrangement

In Figure 12, either gear 1 or gear 2 or both could drive the output gear, gear 3. If gear 1 is the driving gear, then gear 2 must be freewheeling otherwise the gear train will be lockup up. Likewise, if gear 2 is driving, then gear 1 must be freewheeling. If gear 1 and gear 2 are driven by an electric motor with a brake then this implies the brake must be open and the motor not powered. The driving motor must then overcome the residual friction in the opposite gear train and motor plus the back electromotive force in the opposite motor. If both gear 1 and gear 2 are driving gear 3, then gear 1 and gear 2 must be going at the same speed. In this case the output torque on gear 3 will be the sum of the applied torques at gear 1 and gear 2 (adjusted by the gear ratio).)

A speed summing arrangement is shown in Figure 13.

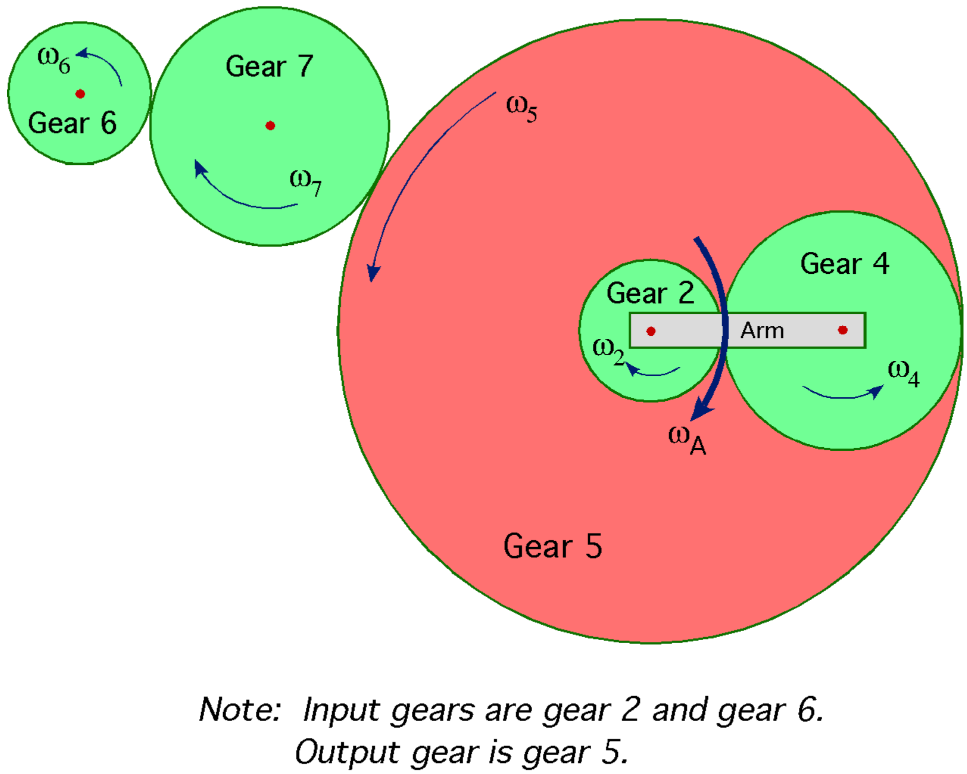

Figure 13 Speed Summing Gear Arrangement

In a speed summing arrangement the speed of the output arm is twice the speed of both input gears multiplied by the corresponding gear ratios. Generally the gear ratio from gear 2 to the arm is equal to the gear ratio from gear 6 to the arm. Like a torque summed gear arrangement, either input gear in a speed summed gear arrangement can drive the output gear. However, unlike a torque summed arrangement, when one gear is driving a speed summed gear arrangement the other input gear must be held fixed. Otherwise, the single driving gear will backdrive the other input gear and there will be little or no motion of the output gear (in effect, the gear train would have 2 DOFs). If motors having a brake drive the input gears, then the brake would need to be closed to hold the motor fixed when the other motor is driving. So, the brake control logic between a torque summed and speed summed arrangement is opposite. Assuming equal gear ratios from each motor, when one motor is driving the output gear the gear moves at ½ of full speed. Full speed is obtained when both motors are driving.

The output torque is equal to the torque of a single drive motor multiplied the corresponding gear ratio. For example, consider a speed summed gear arrangement with equal gear ratios of 10 (from each motor to the output arm). Let the motor torque be 2 in-lbs with a nominal speed of 300 rad/sec. When a single motor is driving, the output torque will be equal to 20 in-lbs with an output speed of 30 rad/sec. When both motors are driving, the output torque will be 20 in-lbs and the output speed will be 60 rad/sec.

Power analysis for either torque summed or speed summed gear arrangements are based on the relationship

$$Power = T\omega \qquad (16.)$$

where T is the torque and ω is the angular velocity. Obviously the power can be measured at any gear in the gear train. At the output shaft (arm) of the planetary gear train the total power output would be the output torque multiplied by the angular velocity of the arm.

In any gear train, there will be torque (power) losses due to friction at occurring at each gear mesh (approximately a 3% loss for each gear mesh for hardened steel gears). The basic equation is

$$Efficiency = \eta = \frac{Power\ Out}{Power\ In} = \frac{Torque\ Out}{Torque\ In} \quad (17.)$$

As stated above, η is approximately 0.97 for a single gear mesh.