

UNIT – I**PROPERTIES OF MATTER****Introduction****Elasticity**

Elasticity is a branch of physics which deals with the elastic property of materials. When an external force is applied to a body, there will be some change in its length, shape and volume.

Perfectly elastic body

When the external force is removed, if the body regain its original shape and size.

Perfectly plastic body

If the body does not regains its original shape or size, after the removal of the applied force.

Stress and Strain

Stress is defined as the restoring force per unit area which bring back the body to its original state from the deformed state.

Type of Stress**Normal Stress**

When the force is applied perpendicular to the surface of the body.

Tangentile Stess or shearing stress

When the force is applied along the surface of the body.

Strain

The change in dimension produced by the external force on the body.

Strain = Change in dimension/Original dimension

Types of Strain**i) Longitudinal or Tensile Strain**

Ratio between the change in length to the original length, without any change in its shape, after the removal of the external forces.

Longitudinal strain = l/L

ii) Shearing strain

The angular deformation produced on the body due to the application of external tangential forces on it.

iii) Volume strain

The ratio between the change in volume to the original volume, without any change in its shape.

Hooke's Law

Stress is directly proportional to the strain produced, within the elastic limit.

$$E = \text{Stress/Strain } \text{Nm}^{-2}$$

Classification of elastic modulus

There are 3 types of elastic modulus based on the 3 types of strain

- i) Young's modulus(Y)
- ii) Bulk modulus(K)
- iii) Rigidity modulus(n)

Youngs modulus

The ratio between the longitudinal stress to the longitudinal strain, within the elastic limits.

$$\text{Young's modulus}(Y) = \text{Longitudinal stress/Longitudinal strain } \text{Nm}^{-2}$$

Bulk modulus(K)

The ratio between the volume stress to the volume strain within the elastic limits

$$\text{Bulk modulus}(K) = \text{Bulk stress/ Bulk strain } \text{Nm}^{-2}$$

RIGIDITY MODULUS (n)

Definition: It is defined the ratio between the tangential stress to the shearing strain with in the elastic limits.

$$(i.e) \text{ Rigidity modulus } (n) = \text{Tangential stress / Shearing strain } \text{Nm}^{-2} \Phi$$

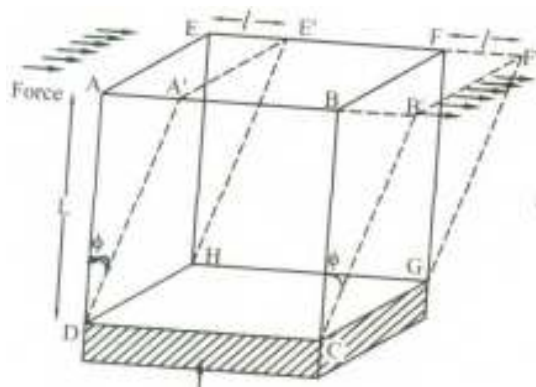


Fig. 2.4

Explanation: Let us consider a solid cube ABCDEFGH. Whose lower face CDHG is fixed as shown in fig 2.4. A tangential force 'F' is applied over the upper face ABEF. The result is that the cube gets deformed into a rhombus shape A'B'CDE'F'GH. (i.e) The lines joining the two faces are shifted to an angle ϕ . If 'L' is the original length and 'l' is the relative displacement of the upper face of the cube with respect to the lower fixed face, then

We can write tangential stress = F/A

The shearing strain (ϕ) can be defined as the ratio of the relative displacement between the two layers in the direction of stress, to the distance measured perpendicular to the layers.

We know, Rigidity modulus(n) = Tangential stress/ Shearing strain

$$n = F/A\phi$$

Rigidity Modulus(n) = $F/A\phi$ Nm^{-2}

POISSON'S RATIO(σ)

DEFINITION: It is defined as the ratio between the lateral strain per unit stress (β) to the longitudinal strain per unit stress (α), within the elastic limits.

(i.e) Poisson's ratio(σ) = lateral strain/longitudinal strain

(or) $\sigma = \beta/\alpha$

Explanation: Let us consider a wire, fixed at one end and is stretched along the other end as shown in fig 2.5.

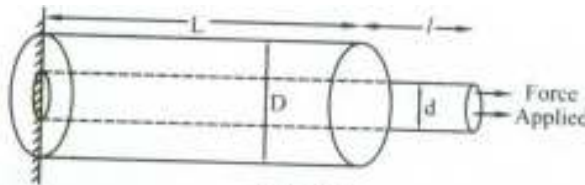


Fig. 2.5

Due to the force applied the wire becomes longer but it also becomes thinner (i.e) although there is an increase in its length, there is a decrease in its diameter as shown in fig. 2.5. therefore the wire elongates freely in the direction of tensile force and contracts laterally in the direction perpendicular to the force. Let 'L' be the original length and 'D' be the original diameter of the diameter decreases from D to d, then

Longitudinal strain = l/L

and Lateral strain = $(D-d)/D$

$$\sigma = -(D-d)/D/l/L$$

(-Ve sign indicates the decrease in length)

$$(or) \quad \sigma = -L(D-d)/lD$$

The negative sign indicates longitudinal strain and lateral strain are opposite to each other.

RELATIONSHIP BETWEEN THREE MODULI OF ELASTICITY

There are many relations connecting the lateral strain(β), strain (α), Poisson's ratio(σ) and the three elastic moduli. Some of the relations are given below.

- i. Relation between α and young's modulus is $\alpha = 1/Y$
- ii. Relation between α and β with the bulk modulus is $(\alpha - 2\beta) = 1/3K$
- iii. Relation between α and β with the Rigidity modulus is $(\alpha + \beta) = 1/2n$
- iv. Relation between Y , n , and K is $Y = 9Kn/3K + n$
- v. Relation between n , K and σ is $\sigma = 3K - 2n/6K + 2n$
- vi. Relation between Y , n and σ is $\sigma = Y/2n - 1$

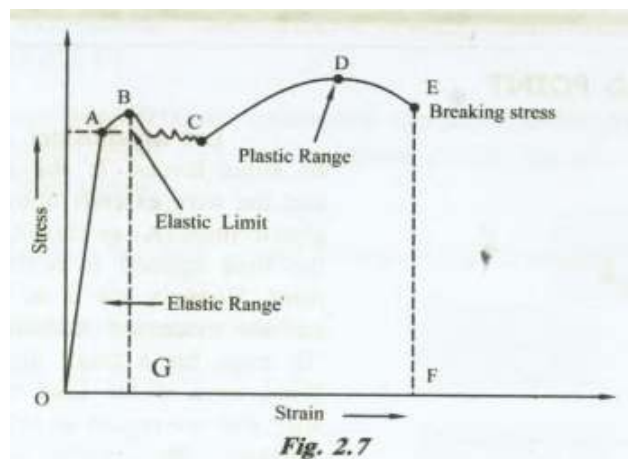
ELASTIC LIMIT

When forces are applied to bodies, each and every body has a tendency to oppose the forces and try to regain its original position after the removal of the force. When the applied force is increased beyond the maximum value, the body does not regain its original position completely, even after the removal of the external forces. Hence the maximum stress up to which a body can recover its original shape and size, after removing the external forces is called as elastic limit.

STRESS AND STRAIN DIAGRAM

Let us consider a body which is subjected to an uniformly increasing stress. Due to application of the stress, the change in dimension of the body takes place(i.e.) the strain is developed. If we plot a graph between stress and strain we get a curve as shown in fig.2.7. and is called as STRESS-STRAIN Diagram.

- i. From the fig.2.7, it is found that the body obeys Hooke's law up to the as region OA called as elastic range.
- ii. As soon as the maximum elastic limit (i.e) yield point 'B' is crossed, the strain increases rapidly than the stress.



- iii. Now, even if a small external force is applied, the body will take a new path CD and remains as plastic called as plastic range, where D is called as ultimate strength.
- iv. After this, the body will not come to its original state and the body acquires a permanent residual strain and it breaks down at a point as breaking stress, indicated by dotted line EF.

FACTORS AFFECTING ELASTICITY

It is found that some bodies lose their elasticity even within the elastic limit, due to elastic fatigue. Therefore the manufacturer should choose the material in such a way it should regain its elastic property even when it is subjected to large number of cycles of stresses.

For example substance like Quartz, phosphor, bronze etc., may be employed in the manufacturing of galvanometers, electrometers etc., after knowing their elastic properties.

Apart from elastic fatigue some materials will have change in their elastic property because of the following factors.

1. Effect of stress
2. Effect of annealing
3. Change in temperature
4. Presence of impurities
5. Due to the nature of crystals.

Effect of stress:

We know that when a material is subjected to large number of cycles of stresses. It loses its elasticity property even within the elastic limit.

Effect of annealing:

Annealing is a process by which the material is heated to a very high temperature and then it is slowly cooled.

Effect of temperature:

The elastic property of the materials changes with the temperature. Normally the elasticity increases with the decrease in temperature and vice versa.

Effect of impurities:

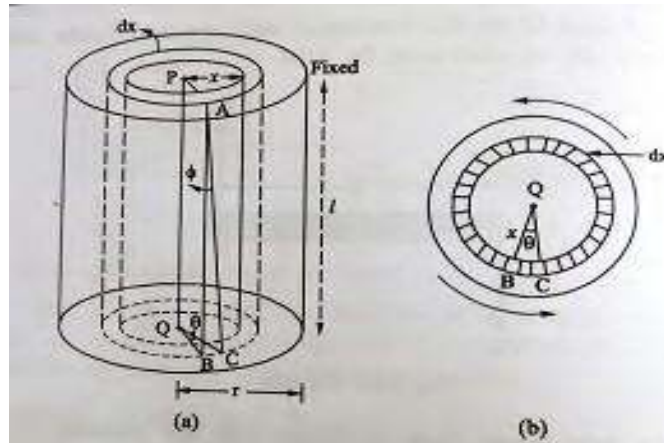
The addition of impurities produces variation in the elastic property of the materials. The increase and decrease of elasticity depends on the type of impurity added to it.

Effect of nature of crystals:

The elasticity also depends on the type of the crystals, whether it is a single crystal or poly crystals. For a single crystal the elasticity is more and for a poly-crystal the elasticity is less.

TWISTING COUPLE ON A WIRE

Consider a cylindrical wire of length l and radius r fixed at one end. (fig. 1.11a). It is twisted through an angle θ by applying couple to its lower end. Now, the wire is said to be under torsion.



Due to elastic property of the wire, an internal restoring couple is set up inside the wire. It is equal and opposite to the external twisting couple (applied). The cylinder is imagined to consist of a large number of thin hollow coaxial cylinders.

Consider one such cylinder of radius x and thickness dx (fig. 1.11b)

AB is a line parallel to PQ on the surface of this cylinder. As the cylinder is twisted, the line AB is shifted to AC through an angle $BAC = \Phi$

Shearing strain or Angle of shear = Φ

Angle of twist at the free end = θ

From the figure (1.11 (b))

$$BC = x\theta = l$$

$$= \frac{x\theta}{l}$$

$$\text{Rigidity modulus } n = \frac{\text{Shearing stress}}{\text{Shearing strain}}$$

$$\therefore \text{Shearing stress} = n \times \text{Shearing strain} = n$$

$$= \frac{nx\theta}{l}$$

$$\text{Shearing stress} = \frac{\text{Shearing force}}{\text{Area over which the force acts}}$$

$$\begin{aligned}
 \text{Area over which the force acts} &= n(x + dx)^2 - nx^2 \\
 &= n(x^2 + 2xdx + dx^2) - nx^2 \\
 &= nx^2 + 2nxdx + ndx^2 - nx^2
 \end{aligned}$$

(dx^2 term is neglected since it is very small)

$$= 2nxdx$$

$$\text{Hence, Shearing force } F = \frac{nx\theta}{l}$$

$$= \frac{2\pi n\theta}{l} x^2 dx$$

\therefore Moment of this force about the axis PQ of the cylinder.

$$= \text{Force} \times \text{perpendicular distance}$$

$$= \frac{2\pi n\theta}{l} x^2 dx \times x$$

$$= \frac{2\pi n\theta}{l} x^3 dx$$

The moment of the force acting on the entire cylinder of radius r is obtained by integrating the expression (3) between the limits $x=0$ and $x=r$.

Hence, twisting couple

$$C = \int_0^r \frac{2\pi n\theta}{l} x^3 dx$$

$$\frac{2\pi n\theta}{l} \int_0^r x^3 dx = \frac{2\pi n\theta}{l} \left[\frac{x^4}{4} \right]_0^r$$

Applying the limits, we have

$$= \frac{2\pi n\theta}{l} \left[\frac{r^4}{4} - 0 \right] = \frac{2\pi n\theta r^4}{4l}$$

$$C = \frac{\pi n\theta r^4}{2l}$$

In the above eqn if $\theta = 1$ radian, then we get.

$$\text{Twisting couple per unit twist } C = \frac{\pi n r^4}{2l}$$

This twisting couple required to produce a twist of unit radian in the cylinder is called torsional rigidity for material of the cylinder.

Hollow cylinder

For a hollow cylinder of the same length l and of inner radius r_1 and outer radius r_2

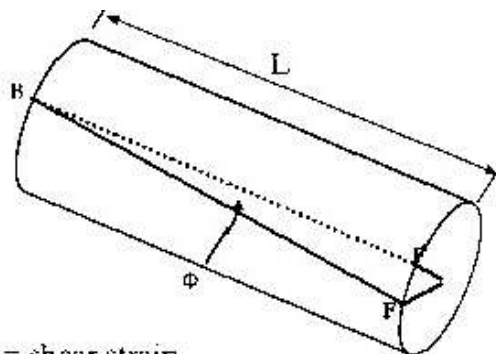
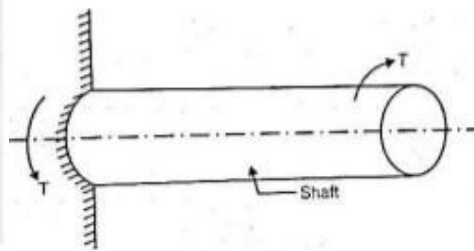
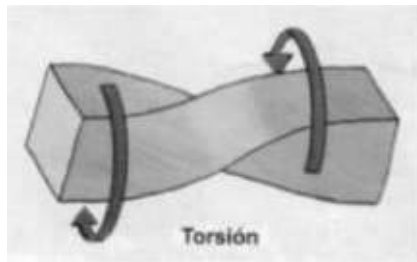
$$\begin{aligned} \text{Twisting couple of the cylinder } C &= \int_{r_1}^{r_2} \frac{2\pi r \theta}{l} r^2 dx \\ &= \frac{\pi \theta}{2l} [r_2^4 - r_1^4] \end{aligned}$$

Twisting couple per unit twist of the cylinder

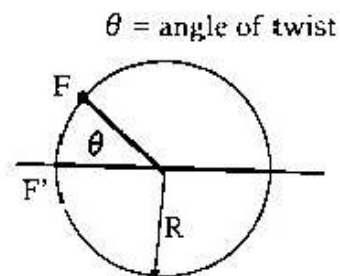
$$C = \frac{\pi \theta}{2l} [r_2^4 - r_1^4]$$

TORSIONAL STRESS AND DEFORMATIONS

The shear stress set up in the shaft when equal and opposite torques (twisting moments) are applied to the ends of a shaft about its axis, is called torsional stress.



Φ = shear strain



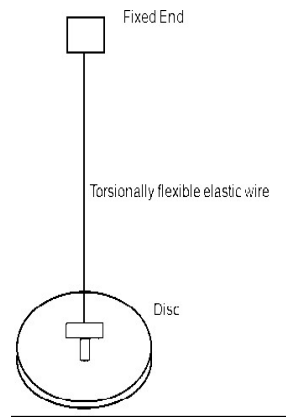
θ = angle of twist

Note that shear strain does not only change with the amount of twist, but also varies along the radial direction such that it is zero at the center and increases linearly towards the outer periphery.

TORSION PENDULUM – THEORY AND EXPERIMENT

A circular metallic disc suspended using a tin wire that executes torsional oscillation is called torsional pendulum.

It executes torsional oscillation whereas a simple pendulum executes linear oscillations.



Description

A torsional pendulum consists of a metal wire suspended vertically with the upper end fixed. The lower end of the wire is connected to the centre of a heavy circular disc.

When the disc is rotated by applying a twist, the wire is twisted through an angle θ . Then, the restoring couple set up in the wire

$$= C\theta \quad \text{---(1)}$$

Where C – couple per unit twist.

If the disc is released, it oscillates with angular velocity $d\theta/dt$ in the horizontal plane about the axis of the wire. These oscillations are known as torsional oscillations.

If $d^2\theta/dt^2$ is the angular acceleration produced in the disc and I its moment of inertia of the disc about the axis of the wire then,

$$\text{Applied couple} = I \frac{d^2\theta}{dt^2} \quad \text{---(2)}$$

In equilibrium, applied couple = restoring couple

$$I \frac{d^2\theta}{dt^2} = C\theta$$

$$\frac{d^2\theta}{dt^2} = \frac{C}{I}\theta \quad \text{----- (3)}$$

This equation represents simple harmonic motion which shows that angular acceleration ($\frac{d^2\theta}{dt^2}$) is proportional to angular displacement θ and is always directed towards the mean position.

Hence, the motion of the disc being simple harmonic motion, the time period of the oscillation is given by

$$T = 2\pi \sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

$$= 2\pi \sqrt{\frac{\theta}{\frac{C}{I}\theta}}$$

$$T = 2\pi \sqrt{\frac{I}{C}} \quad \text{----- (4)}$$

Determination of Rigidity Modulus of the wire

The rigidity modulus of the wire is determined by the following eqn

$$T = 2\pi \sqrt{\frac{I}{C}} \quad \text{----- (1)}$$

Experiment

A circular disc is suspended by a thin wire, whose rigidity modulus is to be determined. The top end of the wire is fixed firmly in a vertical support.

The disc is then rotated about its centre through a small angle and set it free. It executes torsional oscillations.

The time taken for 20 complete oscillations is noted. The experiment is repeated and the mean time period (T) of oscillation is determined.

The length l of the wire is measured. This length is then changed by about 10 cm and then experiment is repeated. The readings for five or six different lengths of wire are measured.

The disc is removed and its mass and diameter are measured.

The time period of oscillation is

$$T = 2\pi \sqrt{\frac{I}{C}} \quad \text{----- (2)}$$

Squaring on both sides, we have

$$T^2 = 2^2 \pi^2 \left(\sqrt{\frac{I}{C}} \right)^2 \quad \text{----- (3)}$$

$$T^2 = \frac{4\pi^2 I}{C} \quad \text{----- (4)}$$

Substituting couple per unit twist $C = \frac{\pi n r^4}{2l}$ in eqn(4)

$$T^2 = \frac{4\pi^2 I}{\frac{\pi n r^4}{2l}} = 2l \times \frac{4\pi^2 I}{\pi n r^4}$$

Rearranging the eqn (5)

The rigidity modulus of the material of the wire

$$T^2 = \frac{4\pi^2 I}{\frac{\pi n r^4}{2l}} = 2l \times \frac{4\pi^2 I}{\pi n r^4}$$

$$n = \frac{8\pi I}{r^4} \left(\frac{l}{T^2} \right)$$

I – moment of inertia of circular disc = $MR^2/2$

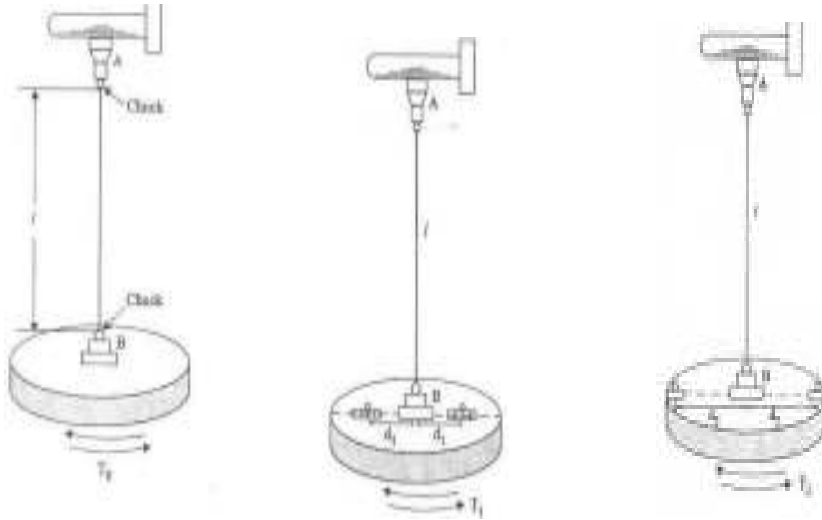
M – Mass of the circular disc

R – Radius of the disc

RIGIDITY MODULUS BY TORSION PENDULUM (DYNAMIC TORSION METHOD)

The torsion pendulum consists of a steel or brass wire with one end fixed in an adjustable chuck and the other end to the centre of a circular disc as shown in fig.

The experiment consists of three parts.



First the disc is set into torsional oscillations without any cylindrical masses on the disc. The mean period of oscillation T_0 is found out.

$$T_0 = 2\pi \sqrt{\frac{I_0}{C}}$$

Where I_0 – moment of inertia of the disc

$$T_0^2 = \frac{4\pi^2 I_0}{C} \quad \text{----- (1)}$$

Two equal cylindrical masses are placed symmetrically along a diameter of the disc at equal distance d_1 on the two sides of the centre of the disc.

Mean time period of oscillation T_1 is found.

$$\text{Then, } T_1 = 2\pi \sqrt{\frac{I_1}{C}}$$

$$T_1^2 = \frac{4\pi^2 I_1}{C} \quad \text{----- (2)}$$

Then, by the parallel axis theorem, the moment of inertia of the whole system is given by,

$$I_1 = I_0 + 2i + 2m d_1^2 \quad \text{----- (3)}$$

Subs I_1 in eqn (2)

$$T_1^2 = \frac{4\pi^2}{C} (I_0 + 2i + 2m d_1^2) \quad \text{----- (4)}$$

Now, two cylindrical masses are placed symmetrically at equal distances d_2 from the axis of the

wire

$$T_2 = 2\pi \sqrt{\frac{I_2}{C}}$$

$$T_2^2 = \frac{4\pi^2 I_2}{C}$$

$$T_2^2 = \frac{4\pi^2}{C} (I_0 + 2i + 2m d_2^2) \text{ --- (5)}$$

$$\therefore I_2 = I_0 + 2i + 2m d_2^2$$

Now,

$$I_2 - I_1 = I_0 + 2i + 2m d_2^2 - I_0 - 2i - 2m d_1^2 = 2m (d_2^2 - d_1^2)$$

Subtracting (4) from (5)

$$T_2^2 - T_1^2 = \frac{4\pi^2}{C} (I_0 + 2i + 2m d_2^2 - I_0 - 2i - 2m d_1^2) = \frac{4\pi^2}{C} 2m (d_2^2 - d_1^2)$$

$$T_2^2 - T_1^2 = \frac{4\pi^2}{C} (I_2 - I_1) \text{ --- (6)}$$

Dividing eqn (1) by (6)

$$\frac{T_0^2}{T_2^2 - T_1^2} = \frac{I_0}{I_2 - I_1} \text{ --- (7)}$$

Substituting the value of $(I_2 - I_1)$ in this equations (7) we have

$$\frac{T_0^2}{T_2^2 - T_1^2} = \frac{I_0}{2m (d_2^2 - d_1^2)}$$

$$I_0 = \frac{2m (d_2^2 - d_1^2) T_0^2}{T_2^2 - T_1^2}$$

Thus moment of inertia of the disc about the axis of rotation is calculated substituting the values of T_0, T_1, T_2, d_1 and d_2 in the above formula.

Calculation of rigidity modulus of the wire

We know that restoring couple per unit twist

$$C = \frac{\pi r^4}{2l}$$

Subs value of C in eqn (6)

$$T_2^2 - T_1^2 = \frac{4\pi^2}{\pi n r^4} 2m (d_2^2 - d_1^2)$$

$$T_2^2 - T_1^2 = \frac{4\pi^2 \times 2l}{\pi n r^4} 2m (d_2^2 - d_1^2)$$

$$T_2^2 - T_1^2 = \frac{16\pi l m (d_2^2 - d_1^2)}{n r^4}$$

$$\text{Or } n = \frac{16\pi l m (d_2^2 - d_1^2)}{(T_2^2 - T_1^2) r^4} \text{ Nm}^{-2}$$

Using the above relation the rigidity modulus of wire is determined.

STRESS DUE TO BENDING IN BEAMS

Fig shows a beam under the action of a bending moment M . In a particular segment of the beam, bending occurs with center of curvature at O , radius of curvature R , included angle θ and neutral surface MN .

If the longitudinal stress at a filament AB at distance x from the neutral surface MN' is σ , then the strain in AB is given as

$$\text{Strain } \epsilon = \frac{\text{change in length}}{\text{original length}} = \frac{P_1Q_1 - PQ}{PQ}$$

$$\text{Strain } \epsilon = \frac{(R+x)\theta - R\theta}{R\theta}$$

$$= \frac{R\theta + x\theta - R\theta}{R\theta} = \frac{x\theta}{R\theta} = \frac{x}{R}$$

$$\text{But } \epsilon = \frac{\sigma}{Y} = \frac{\text{stress}}{\text{Young's modulus}}$$

$$\text{Thus } \frac{\sigma}{Y} = \frac{x}{R}$$

Here x/R is constant for a particular cross section of the beam. Thus, the bending stress (σ) at a particular cross section is proportional to the distance from the neutral axis (x).

For filaments above neutral axis (i.e. negative values of x), bending stress is compressive (i.e., σ is negative). For filaments below neutral axis (for positive values of x) bending stress is tensile (i.e., σ is

positive)

DEPRESSION OF A CANTILEVER LOADED AT ITS END

Theory: let l be the length of the cantilever OA fixed at O . let w be the weight suspended at the free end of the cantilever. Due to the load applied the cantilever moves to a new position OA' as shown in fig. 2.16.

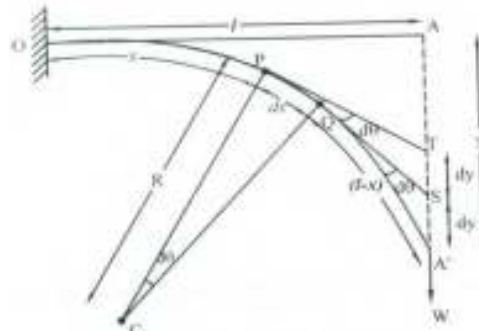


Fig. 2.16

Let us consider an element PQ of the beam of length dx , at a distance $OP = x$ from the fixed end. Let C be the centre of curvature of the element PQ and let R be the radius of curvature.

Due to the applied load at the free end of the cantilever, an external couple is created between the load W at A and the force of reaction at Q . here, the arm of the couple is $(l-x)$.

$$\text{The external bending moment} = W \cdot (l-x) \dots \dots \dots (1)$$

We know the

$$\text{Internal bending moment} = YI/R \dots \dots \dots (2)$$

We know under equilibrium condition

$$\text{External bending moment} = \text{internal bending moment}$$

$$\text{Therefore we can write equ (1) = equ (2)}$$

$$\text{(i.e) } W(l-x) = YI/R$$

$$R = YI/W (l-x) \dots \dots \dots (3)$$

Two tangents are drawn at points P and Q , which meet the vertical line AA' at T and S respectively.

Let the smallest depression produced from T to $S = dy$

And let the angle between the two tangents = $d\theta$

Then we can write

The angle between CP and CQ is also $d\theta$ (i.e) $PCQ = d\theta$

Therefore we can write the arc of the length $PQ = R d\theta = dx$

$$\text{Or} \quad d\theta = dx/R \dots\dots\dots(4)$$

Sub, equ (3) in equ (4)

$$d\theta = dx/[YI/W (1-x)]$$

$$\text{or} \quad d\theta = W/YI(1-x) \dots\dots\dots(5)$$

From the QA'S we can write $\sin d\theta = dy/(1-x)$

If $d\theta$ is vey small then we can write,

$$dy=(1-x)d\theta \dots\dots\dots (6)$$

sub, equ (5) in equ(6), we have

$$dy =W/YI(1-x)^2d\theta \dots\dots\dots (7)$$

Therefore the depression at the free end of the cantilever can be derived by integrating the equ (7) within the limits 0 to 'l'

Therefore $y= W/YI (1-x)^2 dx$

$$= W/YI (l^2+x^2- 2lx) dx$$

$$= W/YI (l^2+x^2- 2lx^2/2 + x^3/3)$$

$$= W/YI (l^3- l^3 + l^3/3)$$

$$y= W/YI(l^3/3)$$

therefore the cantilever at free end

$$y= W l^3/3YI \dots\dots\dots (8)$$

Special cases

- (i) Rectangular cross section

If 'b' be the breadth and 'd' is the thickness of the beam then we know

$$I = bd^3/12$$

Sub, the value of I^e in equ (8), we can write

The depression produced at free end for a rectangular cross section

$$Y = Wl^3/3Y(bd^3/12)$$

$$Y = 4Wl^3/Ybd^3$$

(ii) Circular cross section

If 'r' is the radius of the circular cross section, then

We know, $I = \pi r^4/4$

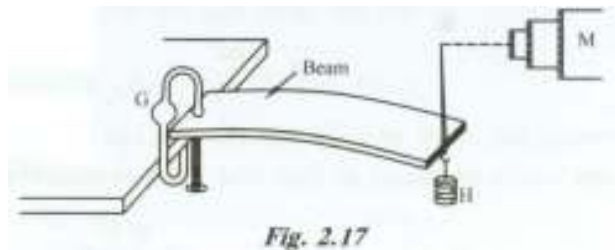
Sub, the value of I_g in equ (8), we can write

The depression produced $y = Wl^3/3Y (\pi r^4/4)$ or

$$y = 4Wl^3/3\pi r^4 Y$$

EXPRIMENTAL DETERMINATION OF YOUNGS MODULUS BY CANTILEVER DPRESSION

Description: it consists a beam clamped rigidly at one end on the table by the use of a G-clamp. The weight hanger (H) is suspended at the other end of the beam, through a small groove on the beam as shown in fig. 2.17. A pin is fixed at the free end of the beam by means of a wax. A microscope (M) is placed in front of this arrangement for measuring the variation of height of the pin.



Procedure: The weight hanger is kept hanged in a dead load position (W) (i.e) without any slotted weights. The microscope is adjusted and the tip of the pin is made to coincide with the horizontal cross wire. The reading in the vertical scale of the microscope is noted.

s.no.	Load(W)	Microscope reading			Depression (y)	M/y
		Increasing load	Decreasing load	mean		
Unit	Kg	10^{-2} m	10^{-2} m	10^{-2} m	Meter	Kg m ⁻¹
1.	W					
2.	W + 50					
3.	W + 100					
4.	W + 150					
5.	W + 200					
6.	W + 250					

Then the weights m, 2m, 3m etc. are increased in steps to the weight hanger. Each time the microscope is adjusted to coincide the tip of the pin to the horizontal cross wire and the readings are noted from the vertical scale of the microscope.

The experiment is repeated by decreasing the loads and the respective readings are noted from the vertical scale of the microscope and are tabulated in the tabular column as shown. The mean depression 'y' for a load M kg is found.

Theoretically, we know the depression produced for a load of M kg is

$$Y = \frac{Wl^3}{3YI} \dots\dots\dots(1)$$

Where 'l' is the length of the beam (i.e) the distance between the clamped end and the loaded end.

If 'b' be the breadth of the beam and 'd' is the thickness of the beam then

The geometrical moment of inertia for a rectangular cross sectional bar

$$I = \frac{bd^3}{12} \dots\dots\dots(2)$$

Also the weight $W = Mg \dots\dots\dots(3)$

Sub, equ (2) and (3) in equ (1), we have

$$Y = \frac{4Mgl^3}{Ybd^3} \dots\dots\dots(4)$$

Rearranging equ(4) we get

Young's modulus

$$Y = \frac{4Mgl^3}{bd^3y}$$

Or $Y = \frac{4gl^3}{bd^3} \left(\frac{M}{y}\right) \text{ Nm}^{-2} \dots\dots\dots(5)$

UNIFORM BENDING – ELEVATION AT THE CENTR OF THE BEAM LOADED AT BOTH ENDS

Theory:

Let us consider a beam of negligible mass, supported symmetrically on the two knife edges A and B as shown in fig.2.19. let the length between A and B be 'l'. let equal weights W, be added to either end of the beam C and D.

Let the distance $CA = BD = a$

Due to the load applied the beam bends from position F to E into an arc of a circle and produces as elevation 'x' from the position F to E. let 'W' be the reaction produced at the points A and B which acts vertically upwards as shown in fig. 2.19.

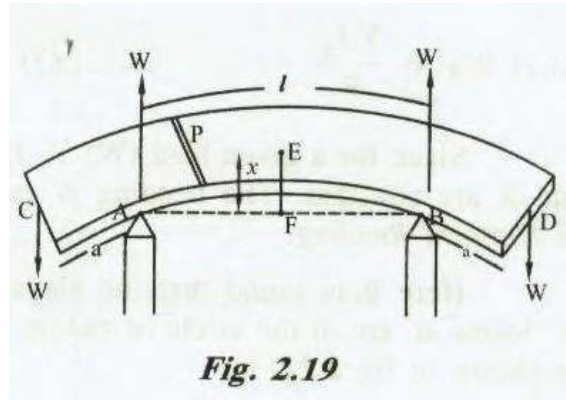


Fig. 2.19

Consider a point 'P' on the cross section of the beam. Then the forces acting on the part PC of the beam are

- (i) Forces W at 'C' and
- (ii) Reaction W at A as shown in fig. 2.20

Let the distance PC = a_1 and PA = a_2 , then

The external bending moment about 'P' is

$$M_p = W \times a_1 - W \times a_2$$

Here, the clockwise moment is taken as negative and anticlockwise moment is taken as positive.

External bending moment about P can be written as

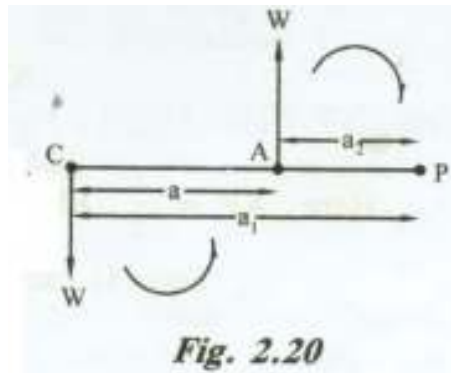


Fig. 2.20

$$M_p = W \cdot (a_1 - a_2)$$

$$M_p = Wa \dots \dots \dots (1)$$

We know the internal bending moment = $YI/R \dots \dots \dots (2)$

Under the equilibrium condition

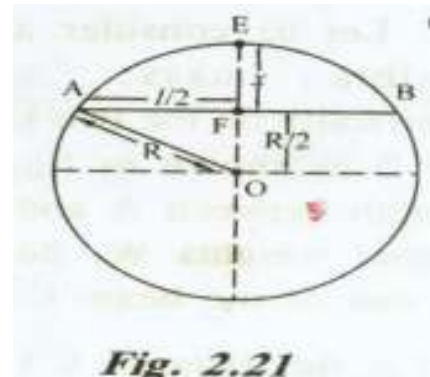
External bending moment = internal bending moment

We can write equ(1) = equ (2)

$$Wa = YI/R \dots \dots \dots (3)$$

Since for a given load (W) Y, I_g, a and R are constant. The bending is called as uniform bending.

Here it is found that the elevation 'x' forms an arc of the circle of radius 'R' as shown in fig. 2.21.



arc

From the Δ AFO we can write

$$OA^2 = AF^2 + FO^2$$

Since OF = FE,

Therefore we can write

$$OA^2 = AF^2 + FE^2 \text{ or } AF^2 = OA^2 - FE^2$$

Rearranging we can write

$$AF^2 = FE [OA^2 / FE - FE] \dots \dots \dots (4)$$

Here, AF = l/2 ; FE = x = R/2; OA = R

Therefore equ (4) can be written as

$$(l/2)^2 = x [R^2 / (R/2) - x]$$

$$(l^2/4) = X [2R - x]$$

$$(l^2/4) = 2Rx - x^2$$

If the elevation 'x' is very small, then the term x² can be neglected.

Therefore we can write l²/4 = 2xR

Or $x = l^2/8R$

Radius of curvature R = l²/8x..... (5)

Sub, the value of 'R' value in equ (3) we have

Or W.a = YI/(l²/8x)

Or W.a = 8YIx/l²

Rearranging the equation (6),

The elevation of point 'E' above 'A' is

$$X = \frac{W a l^2}{8 Y I}$$

EXPERIMENTAL DETERMINATION OF YOUNG'S MODULUS BY UNIFORM BENDING

Statistical method

Description:

It consists of a beam, symmetrically supported on the two knife edges A and B. Two weight hangers are suspended on either side of the beam at the position C and D. The distance between AC and BD are adjusted to be equal. A pin is fixed vertically at the centre of the beam. A travelling microscope is placed in front of the whole set up for finding the position of the pin.

Procedure:

Taking the weight hanger as the dead load (W), the microscope is adjusted and the tip of the pin is made to coincide with the vertical cross wire. The reading is noted from the vertical scale of the microscope.

Now the load on each hanger is increased in equal steps of m, 2m, 3m etc, kilogram and the corresponding readings are noted from the vertical scale of the microscope. The same procedure is repeated during unloading. The readings are noted from the vertical scale of the microscope. The readings are tabulated in the tabular column as shown.

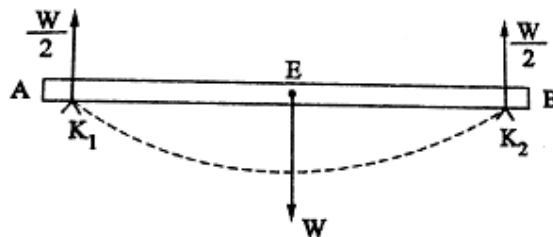
The mean elevation 'x' of the centre for M kg is found. The distance between the two knife edges is measured as 'l' and the distance from the point of suspension of the load to the knife edge is measured as 'a'

S.No.	Load(W)	Microscope reading			Elevation (x)	M/x
		Increasing load	Decreasing load	Mean		
Unit	Kg	$\times 10^{-2}$ m	$\times 10^{-2}$ m	$\times 10^{-2}$ m	Meter	Kg m ⁻¹
1.	W					
2.	W + 50					
3.	W + 100					
4.	W + 150					
5.	W + 200					
6.	W + 250					
7.	W + 300					
8.	W + 350					

NON- UNIFORM BENDING

If the beam is loaded at its mid-point, the depression produced does not form an arc of a circle. This type of bending is called non-uniform bending.

Consider a uniform cross sectional beam (rod or bar) AB of length l arranged horizontally on two knife edges K₁ and K₂ near the ends A and B.



A weight W is applied at the midpoint 'O' of the beam. The reaction force at each knife edge is equal to $W/2$ in the upward direction. y is the depression at the midpoint O.

The bent beam is considered to be equivalent to two inverted cantilevers, fixed at O each length $(l/2)$ and each loaded at K_1 and K_2 with a weight $W/2$.

In the case of a cantilever of length $(l/2)$ and load $(W/2)$, depression = $Wl^3/3Iy$

Hence, for cantilever of length $(l/2)$ and load $(W/2)$, depression is

$$y = \frac{\left(\frac{W}{2}\right)\left(\frac{l}{2}\right)^3}{3IY} = \frac{\left(\frac{W}{2}\right)l^3}{3IY} = \frac{Wl^3}{2 \times 3IY}$$

$$y = \frac{Wl^3}{48IY}$$

$$Y = \frac{Wl^3}{48Iy}$$

If M is the mass, the corresponding weight W is $W = Mg$

If the beam is a rectangular, $I = bd^3/12$, where b is the breadth and d is the thickness of the beam.

Hence,

$$Y = \frac{Mgl^3}{48y \frac{bd^3}{12}}$$

$$y = \frac{Mgl^3 \times 12}{48bd^3Y}$$

$$Y = \frac{Mgl^3}{4bd^3y} \text{ Nm}^{-2}$$

The value of Y can be determined by the above equation.

Experiment

The given beam AB of rectangular cross section is arranged horizontally on two knife edges K_1 and K_2 near the ends A and B.

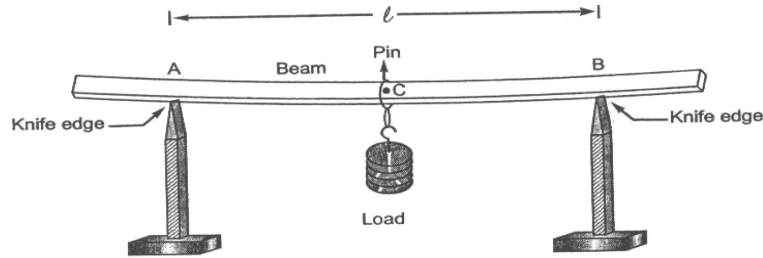


FIG 2.1 Young's Modulus - Non Uniform Bending

A weight hanger is suspended and a pin is fixed vertically at mid-point O. A microscope is focused on the tip of the pin.

The initial reading on the vertical scale of the microscope is taken. A suitable mass M is added to the hanger. The beam is depressed. The cross wire is adjusted to coincide with the tip of the pin. The reading of the microscope is noted.

The depression corresponding to the mass M is found.

The experiment is repeated by increasing and decreasing the mass step by step. The corresponding readings are tabulated. The average value of depression y is found from the observation.

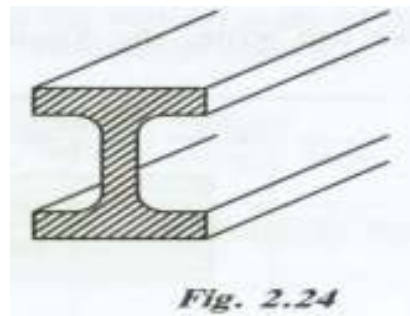
S. No.	Load(W)	Microscope reading			Mean depression (y) for a load of M Kg.cm Meter
		Increasing load	Decreasing load	Mean	
Unit	Kg	$\times 10^{-2}$ m	$\times 10^{-2}$ m	$\times 10^{-2}$ m	
1.	W				
2.	W + 50				
3.	W + 100				
4.	W + 150				
5.	W + 200				
6.	W + 250				

The breadth b , thickness d and length l of the beam are measured. The value of Young's modulus of the beam is found by the relation.

$$Y = \frac{Mgl^3}{4bd^3y} \text{ Nm}^{-2}$$

I SHAPE GIRDERS

Definition: The girders with upper and lower section broadened and the middle section tapered, so that it can withstand heavy loads over its called as I shape girders. Since the girders look like letter I as shown in fig 2.24 they are named as I shape girder



EXPLANATION: In general any girder supported at its two ends as on the opposite walls of a room, bends under its own weight and a small depression is produced at the middle portion. This may also be caused when loads are applied to the beams.

Due to depression produced, the upper parts of the girder above the neutral axis contracts, while, the lower parts below the neutral axis expands. i.e the stresses have a maximum value at the top and bottom. The stresses progressively decreases as it approaches towards the neutral axis. Therefore the upper and lower surfaces of the girder must be stronger than the intervening part. Thus the girders are made of I shape girders. We know the depression produced in the case of a rectangular section.

$$Y = \frac{4Wl^3}{Ybd^3}$$

Therefore for stability, the upper part and the lower part is made broader than the centre part and hence forming an I shape called as I shape girders. The depression can also be reduced by properly choosing the materials of high young modulus.

APPLICATIONS

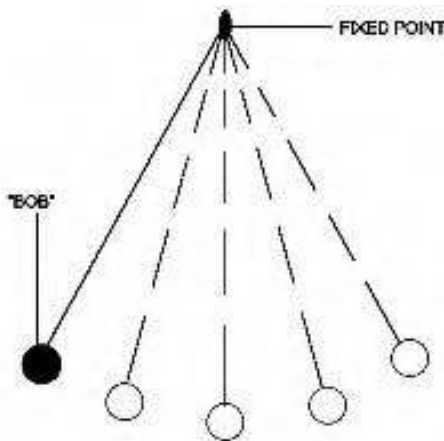
- They are used in the construction of bridges over the rivers.
- They are very much useful in the production of iron rails which are employed in railway tracks.
- They are used as supporting beams for the ceilings in the construction of buildings.
- They are used in the construction of iron beams to support the bridges for the heavy vehicles and also in the construction of dams.

ADVANTAGES

- More stability
- More stronger
- High durability

UNIT – 2**WAVES AND OPTICS****Introduction:**

The oscillatory motion:

**The simple pendulum**

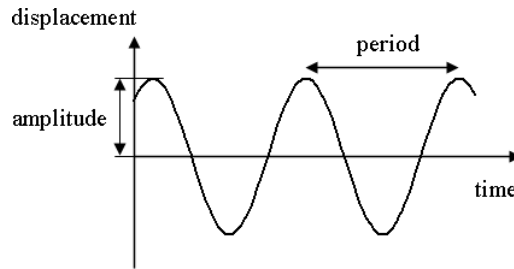
The oscillatory motion is the motion of the oscillating body around its rest point, where the motion is repeated through the equal intervals of the time.

Examples of the oscillatory motion:

- The clock, The tuning fork, The spring, The stretched string, The motion of the swing, The rotary bee.
- The movement of the Earth's crust during the earthquakes.
- The movement of the atoms in the molecules.

Some concepts related to the oscillatory motion:

The amplitude: You should know that the amplitude is the maximum displacement done by the oscillating body away from its original position. The measuring unit of the amplitude is meter or centimeter.



The periodic time is taken by an oscillating body to make one complete oscillation.

The complete oscillation: It is very important to know that the complete oscillation is the motion of an oscillating body when it passes by a fixed point on its path two successive times in the same direction .

The periodic time : You should know that the periodic time is the time taken by an oscillating body to make one complete oscillation , The measuring unit of the periodic time is the second .

The frequency: It is very important to know that the frequency is the number of the complete oscillations made by an oscillating body in one second. The relation between the periodic time and the frequency is an inverse relation.

Equation of Damped harmonic oscillation:

Damped harmonic oscillators are vibrating systems for which the amplitude of vibration decreases over time. Since nearly all physical systems involve considerations such as air resistance, friction, and intermolecular forces where energy in the system is lost to heat or sound, accounting for damping is important in realistic oscillatory systems. Examples of damped harmonic oscillators include any real oscillatory system like clock pendulum, or guitar string: after starting the clock, or guitar string vibrating, the vibration slows down and stops over time, corresponding to the decay of sound volume or amplitude in general.

Mathematically, damped systems are typically modeled by simple harmonic oscillators with **viscous damping** forces, which are proportional to the velocity of the system and permit easy solution of Newton's second law in closed form. These are second-order ordinary differential equations which include a term proportional to the first derivative of the amplitude. As described below, the magnitude of the proportionality describes how quickly the vibrations of the damped oscillator damp down to nothing.

Damping forces are often due to motion of an oscillatory system through a fluid like air or water, where interactions between the molecules of the fluid (e.g. air resistance) become important. At low velocities in non-turbulent fluid, the damping of a harmonic oscillator is well-modeled by a viscous damping force $F_d = -bx'$. Adding this term to the simple harmonic oscillator equation

given by Hooke's law gives the equation of motion for a viscously damped simple harmonic oscillator.

$$ax^2 + bx + c = 0 \quad (1)$$

where a is a constant sometimes called the **damping constant**.

Solutions should be oscillations within some form of damping envelope:

$$x(t) = Ae^{-gt}e^{iat} = Ae^{t(ai-g)} = Ae^{rt} \quad (2)$$

where A is some constant and $r = ai-g$ will be found. Plugging this into the equation of motion yields:

$$(nr^2 + br + k)Ae^{rt} = 0 \quad (3)$$

$$r^2 + \frac{b}{n}r + \frac{k}{n} = 0 \quad (4)$$

which is a quadratic equation in r with solutions:

$$r = \frac{-b \pm i\sqrt{\left(\frac{k}{n}\right) - (b^2/4n^2)}}{2n} \quad (5)$$

Note that there are two roots r to the quadratic as long as the imaginary part is nonzero, corresponding to the two general solutions:

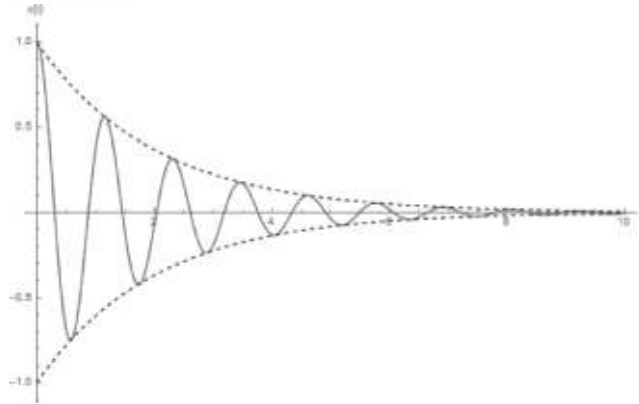
$$x(t) = Ae^{(-\frac{b}{2n})t} e^{i\sqrt{\left(\frac{k}{n}\right) - (b^2/4n^2)}t} + Be^{\frac{-b}{2n}t} e^{-i\sqrt{\left(\frac{k}{n}\right) - (b^2/4n^2)}t} \quad (6)$$

These solutions in general describe oscillation at frequency $m = \sqrt{\left(\frac{k}{n}\right) - (b^2/4n^2)}$ within a decay envelope of time-dependent amplitude $e^{-(\frac{b}{2n})t}$.

Depending on the values of m , k , y and the solution exhibits different types of behavior

(i) $b^2 < 4kn$: under damping

Under damped solutions oscillate rapidly with the frequency and decay envelope described above. For objects with very small damping constant (such as a well-made tuning fork), the frequency of oscillation is very close to the undamped natural frequency $m_0 = \sqrt{\frac{k}{n}}$



Underdamped oscillations within an exponential decay envelope

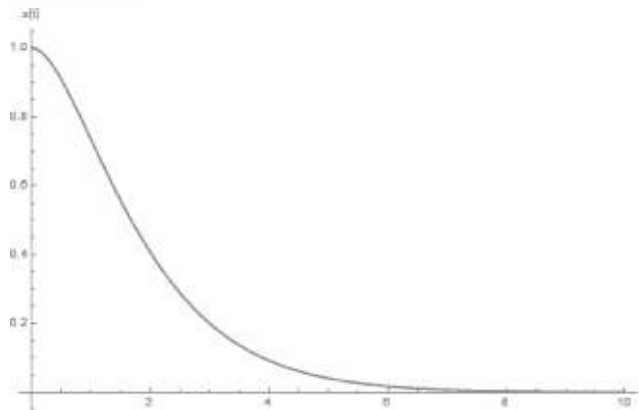
(ii) $b^2 = 4kn$: Critical damping

This case corresponds to the vanishing of the frequency $\sqrt{\frac{k}{n} - \frac{b^2}{4n^2}}$ described previously.

At this level of damping, the solution $x(t)$ most rapidly approaches the steady-state amplitude of zero. Larger amounts of damping (see overdamping) cause the solution to more slowly approach zero as it moves slowly through the damping fluid, whereas smaller amounts of damping cause the solution to oscillate more rapidly around zero. Notably, solutions at critical damping do not oscillate.

If the frequency $\sqrt{\frac{k}{n} - \frac{b^2}{4n^2}}$ vanishes, the two linearly independent solutions can be written:

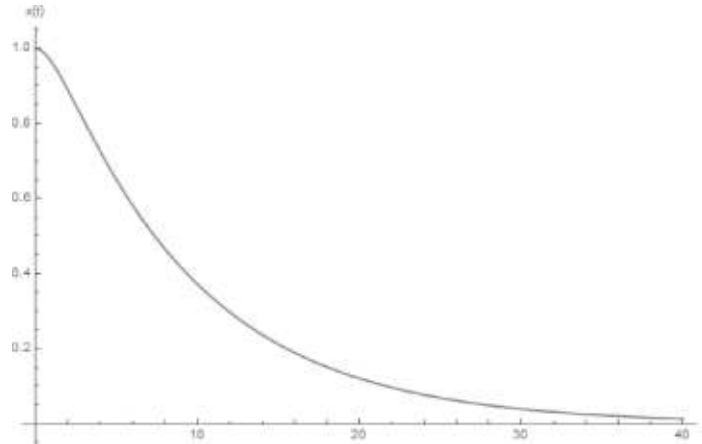
$$x(t) = Ae^{-\frac{b}{2n}t} + Bte^{-\frac{bt}{2n}} \quad (7)$$



Time evolution of the amplitude of a critically damped harmonic oscillator

(iii) $b^2 > 4kn$: Over damping

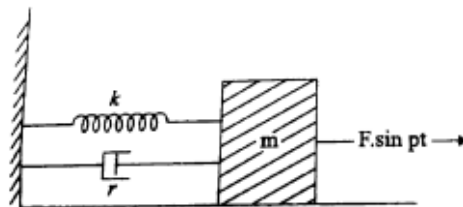
In the overdamped case, the frequency $\sqrt{\frac{k}{m} - (b^2/4m^2)}$ becomes imaginary. As a result, the oscillatory terms e^{imt} and e^{-imt} become growing and decaying exponentials and e^{lmt} and e^{-lmt} . Overdamped solutions do not oscillate and instead slowly decay towards equilibrium.



An overdamped harmonic oscillator approaching equilibrium slowly

Differential equation of Forced Harmonic Oscillation:

Consider a particle of mass m connected to a spring. The particle is driven by a periodic force.



Mechanical forced oscillator with force $F \sin pt$

The oscillations are started and the forces acting on the particles are,

- (i) a restoring force proportional to the displacement acting in the opposite direction. It is given by $-ky$ where k is known as the restoring force constant.
- (ii) a frictional force proportional to velocity but acting on the opposite direction. It is given $-r \frac{dy}{dt}$ where r is the frictional force constant.
- (iii) The external periodic force, $F \sin Pt$ where F is the maximum value of the force and P is its angular frequency. This force opposes the restoring force as well as the frictional force and helps in motion.

Therefore, net force F' acting on the particle

$$F = -ky - r \frac{dy}{dt} + F \sin Pt \quad (1)$$

By Newton's second law of motion, the resultant force acting on the particle

$$F = \text{mass} \times \text{acceleration} = ma$$

$$F = m \frac{d^2y}{dt^2} \quad (2)$$

From the equ. 1 & 2 we have,

$$m \frac{d^2y}{dt^2} = -ky - r \frac{dy}{dt} + F \sin Pt$$

$$\text{Or } m \frac{d^2y}{dt^2} + r \frac{dy}{dt} + ky = F \sin pt \quad (3)$$

$$\frac{d^2y}{dt^2} + \frac{r}{m} \frac{dy}{dt} + \frac{ky}{m} = \frac{F}{m} \sin pt$$

$$\frac{d^2y}{dt^2} + 2b \frac{dy}{dt} + \omega^2 y = f \sin pt \quad (4)$$

Where $r/m = 2b$, $k/m = \omega^2$ and $F/m = f$

The equ (4) is differential equation of motion of the forced oscillation of the particle

The solution of differential equation (4)

$$y = A \sin (Pt - \theta) \quad (5)$$

Where A is the steady amplitude of vibration

differentiating equ (5)

$$\frac{dy}{dt} = A p \cos(pt - \theta)$$

$$\text{and } \frac{d^2y}{dt^2} = -A p^2 \sin(pt - \theta)$$

substituting y , $\frac{dy}{dt}$ and $\frac{d^2y}{dt^2}$ in equ (4), we have

$$\begin{aligned} -A p^2 \sin (pt - \theta) + 2bA p \cos (pt - \theta) + \omega^2 A \sin (pt - \theta) \\ = f \sin pt = f \sin \{(pt - \theta) + \theta\} \end{aligned} \quad (6)$$

Or

$$A(\omega^2 - p^2) \sin (pt - \theta) + 2bA p \cos (pt - \theta)$$

$$= f \sin (pt - \theta) \cos \theta + f \cos (pt - \theta) \sin \theta \quad (7)$$

This relation holds well for all values of t . The coefficients of $\sin (pt - \theta)$ and $\cos (pt - \theta)$ terms on both sides of this equation must be equal.

$$A(\omega^2 - p^2) = f \cos \theta \quad (8)$$

$$2bAp = f \sin \theta \quad (9)$$

Squaring the equ. (8) and (9) and then adding, we have

$$A^2(m^2 - p^2)^2 + 4b^2A^2p^2 = f^2\cos^2\theta + f^2\sin^2\theta$$

$$A^2[(m^2 - p^2)^2 + 4b^2A^2p^2] = f^2$$

$$A^2 = \frac{f^2}{[(m^2 - p^2)^2 + 4b^2A^2p^2]}$$

Taking square root, and on dividing equ (9) by (8),

$$\tan\theta = \frac{2bAp}{A(m^2 - p^2)} = \frac{2bp}{(m^2 - p^2)}$$

$$\theta = \tan^{-1} \frac{2bp}{(m^2 - p^2)} \quad (10)$$

The equation (9) gives the amplitude of forced vibration and equ. (10) gives the phase.

Wave Equation for a Plane Progressive Wave:

The simplest type of wave is the one in which the particles of the medium are set into simple harmonic vibrations as the wave passes through it. The wave is then called a simple harmonic wave.

Consider a particle O in the medium. The displacement at any instant of time is given by

$$y = A \sin \omega t \quad \text{--- (1.3)}$$

Where A is the amplitude, ω is the angular frequency of the wave. Consider a particle P at a distance x from the particle O on its right. Let the wave travel with a velocity v from left to right. Since it takes some time for the disturbance to reach P, its displacement can be written as

$$y = A \sin(\omega t - \phi) \text{ ----- (1.4)}$$

Where ϕ is the phase difference between the particles O and P.

We know that a path difference of l corresponds to a phase difference of 2π radians. Hence a path

difference of x corresponds to a phase difference of $\frac{2\pi}{\lambda} \cdot x$

$$\therefore \phi = \frac{2\pi x}{\lambda} \text{ -----(1.5)}$$

Substituting equation (1.5) in equation (1.4)

We get,

$$y = A \sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$$

But $\omega = \frac{2\pi}{T}$

$$\therefore y = A \sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right)$$

$$\text{or } y = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \text{ -----(1.6)}$$

But $v = \frac{\lambda}{T}$ or $T = \frac{\lambda}{v}$

$$\therefore y = A \sin 2\pi\left(\frac{vt}{\lambda} - \frac{x}{\lambda}\right)$$

$$\text{or } y = A \sin \frac{2\pi}{\lambda} (vt - x) \text{ -----(1.7)}$$

Differential equation for wave motion:

Differentiating equation (1.7) with respect to x ,

We get,

$$\frac{dy}{dx} = -A \frac{2\pi}{\lambda} \cos \frac{2\pi}{\lambda} (vt - x) \text{ ----- (1.8)}$$

The velocity of the particle whose displacement y is represented by equation (1.7), is obtained by differentiating it with respect to t , since velocity is the rate of change of displacement with respect to time.

$$\therefore \frac{dy}{dt} = A \frac{2\pi v}{\lambda} \cos \frac{2\pi}{\lambda}(vt-x) \text{ ----- (1.9)}$$

Comparing equations (1.8) and (1.9) we get,

$$\frac{dy}{dt} = -v \frac{dy}{dx} \text{ ----- (1.10)}$$

\therefore Particle velocity = wave length \times slope of the displacement curve or strain

Differentiating equation (1.8),

$$\frac{d^2y}{dx^2} = -A \frac{4\pi^2}{\lambda^2} \sin \frac{2\pi}{\lambda}(vt-x) \text{ ----- (1.11)}$$

Differentiating equation (1.9)

$$\frac{d^2y}{dt^2} = -A \frac{4\pi^2}{\lambda^2} v^2 \sin \frac{2\pi}{\lambda}(vt-x) \text{ ----- (1.12)}$$

Comparing equation (1.11) and (1.12) we get,

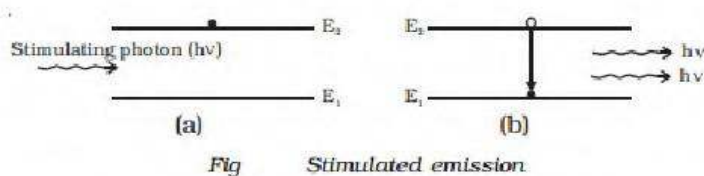
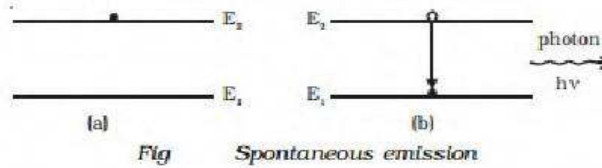
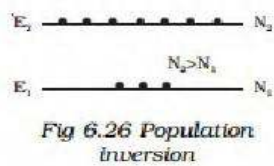
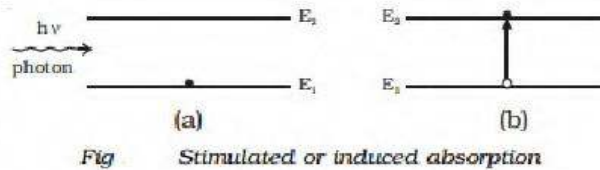
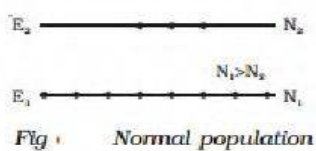
$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2} \text{ ----- (1.13)}$$

This is the differential equation of wave motion.

Laser

Introduction:

The light emitted from an ordinary light source is incoherent, because the radiation emitted from different atoms has no definite phase relationship with each other. For interference of light, coherent sources are essential. Two independent sources cannot act as coherent sources. For experimental purposes, from a single source, two coherent sources are obtained. In recent years, some sources have been developed, which are highly coherent known as LASER. The word 'Laser' is an acronym for Light Amplification by Stimulated Emission of Radiation. The difference between ordinary light and laser beam is pictorially depicted in Fig .



Characteristics of laser:

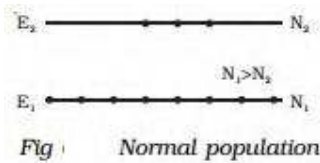
The laser beam (i) is monochromatic. (ii) is coherent, with the waves, all exactly in phase with one another, (iii) does not diverge at all and (iv) is extremely intense

Population of Energy levels:

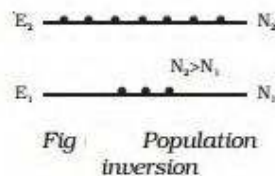
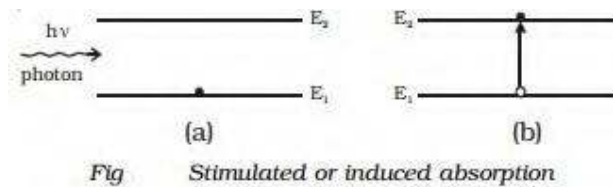
Spontaneous and stimulated emission:

An atom may undergo transition between two energy states E_1 and E_2 , if it emits or absorbs a photon of the appropriate energy $E_2 - E_1 = hv$.

In a system of thermal equilibrium, the number of atoms in the ground state (N_1) is greater than the number of atoms in the excited state (N_2). This is called **normal population** (Fig). Consider a sample of free atoms, some of which are in the ground state with energy E_1 and some in the excited energy state with energy E_2 . If photons of energy $h\nu = E_2 - E_1$ are incident on the sample, the photons can interact with the atoms in the ground state and are taken to excited state. This is called stimulated or induced absorption (Fig.) The process by which the atoms in the ground state is taken to the excited state is known as pumping.



If the atoms are taken to the higher energy levels with the help of light, it is called optical pumping. If the atoms in the ground state are pumped to the excited state by means of external agency, the number of atoms in the excited state (N_2) becomes greater than the number of atoms in the ground state (N_1). This is called **population inversion** (Fig).



The life time of atoms in the excited E_2 state is normally 10^{-8} second. Some of the excited energy levels have greater life times for atoms (10^{-3} s). Such energy levels are called as the metastable states. If the excited energy level is an ordinary level, the excited atoms return to the lower (or) ground energy state immediately without the help of any external agency. During this transition (Fig), a photon of energy $E_2 - E_1 = h\nu$ is emitted. This is called spontaneous emission.

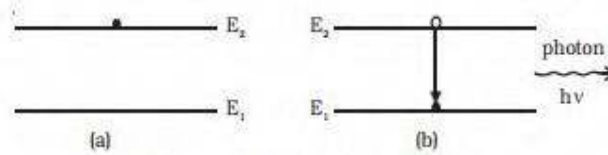


Fig Spontaneous emission

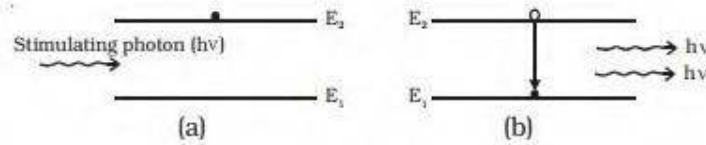


Fig Stimulated emission

If the excited state is a metastable state, the atoms stay for some time in these levels. The atoms in such metastable state can be brought to the lower energy levels with the help of photons of energy $h\nu = E_2 - E_1$. During this process, a photon of energy $E_2 - E_1 = h\nu$ is emitted. This is known as stimulated emission (or) induced emission (Fig.). A photon produced by stimulated emission is called secondary photon (or) stimulated photon. The secondary photon is always in phase with the stimulating photon. These photons in turn stimulate the emission further and the process continues to give a chain - reaction. This is called laser action and by this action all the emitted photons having same energy and same frequency are in phase with each other. Hence, a highly monochromatic, perfectly coherent, intense radiation is obtained in laser.

Conditions to achieve laser action:

- (i) There must be an inverted population i.e. more atoms in the excited state than in the ground state.
- (ii) The excited state must be a metastable state.

The emitted photons must stimulate further emission. This is achieved by the use of the reflecting mirrors at the ends of the system.

Resonant cavity & Optical amplification:

An optical cavity, resonating cavity or optical resonator is an arrangement of mirrors that forms a standing wave cavity resonator for light waves. Optical cavities are a major component of lasers, surrounding the gain medium and providing feedback of the laser light. They are also used in optical parametric oscillators and some interferometers. Light confined in the cavity reflects

multiple times producing standing waves for certain resonance frequencies. The standing wave patterns produced are called modes; longitudinal modes differ only in frequency while transverse modes differ for different frequencies and have different intensity patterns across the cross section of the beam

Einstein's A and B Coefficient derivation and discussion:

Einstein showed the interaction of radiation with matter with the help of three processes called stimulated absorption, spontaneous emission and stimulated emission. He showed in 1917 that for proper description of radiation with matter, the process of stimulated emission is essential. Let us first derive the Einstein coefficient relation on the basis of above theory:

Let N_1 be the number of atoms per unit volume in the ground state E_1 and these atoms exist in the radiation field of photons of energy $E_2 - E_1 = h\nu$ such that energy density of the field is E .

Let R_1 be the rate of absorption of light by $E_1 \rightarrow E_2$ transitions by the process called stimulated absorption

This rate of absorption R_1 is proportional to the number of atoms N_1 per unit volume in the ground state and proportional to the energy density E of radiations.

That is $R_1 \propto N_1 E$

$$R_1 = B_{12} N_1 E \quad (1)$$

Where B_{12} is known as the Einstein's coefficient of stimulated absorption and it represents the probability of absorption of radiation. Energy density e is defined as the incident energy on an atom as per unit volume in a state.

Now atoms in the higher energy level E_2 can fall to the ground state E_1 automatically after 10^{-8} sec by the process called spontaneous emission.

The rate R_2 of spontaneous emission $E_2 \rightarrow E_1$ is independent of energy density E of the radiation field.

R_2 is proportional to number of atoms N_2 in the excited state E_2 thus

$R_2 \propto N_2$

$$R_2 = A_{21} N_2 \quad (2)$$

Where A_{21} is known as Einstein's coefficient for spontaneous emission and it represents the probability of spontaneous emission.

Atoms can also fall back to the ground state E_1 under the influence of electromagnetic field of incident photon of energy $E_2 - E_1 = h\nu$ by the process called stimulated emission

Rate R_3 for stimulated emission $E_2 > E_1$ is proportional to energy density E of the radiation field and proportional to the number of atoms N_2 in the excited state, thus

$$R_3 \propto N_2 E$$

$$\text{Or} \quad R_3 = B_{21} N_2 E \quad (3)$$

Where B_{21} is known as the Einstein coefficient for stimulated emission and it represents the probability of stimulated emission.

In steady state (at thermal equilibrium), the two emission rates (spontaneous and stimulated) must balance the rate of absorption.

$$\text{Thus} \quad R_1 = R_2 + R_3$$

Using equations (1, 2, and 3), we get

$$N_1 B_{12} E = N_2 A_{21} + N_2 B_{21} E \quad \text{Or}$$

$$N_1 B_{12} E - N_2 B_{21} E = N_2 A_{21}$$

$$\text{Or} \quad (N_1 B_{12} - N_2 B_{21}) E = N_2 A_{21}$$

$$\text{Or} \quad E = N_2 A_{21} / (N_1 B_{12} - N_2 B_{21})$$

$$= N_2 A_{21} / N_2 B_{21} [N_1 B_{12} / N_2 B_{21} - 1]$$

[by taking out common $N_2 B_{21}$ from the denominator]

$$\text{Or} \quad E = A_{21} / B_{21} \{ 1 / (N_1 / N_2 (B_{12} / B_{21}) - 1) \} \quad (4)$$

Einstein proved thermodynamically, that the probability of stimulated absorption is equal to the probability of stimulated emission. thus

$$B_{12} = B_{21}$$

Then equation (4) becomes

$$E = A_{21} / B_{21} (1 / (N_1 / N_2 - 1)) \quad (5)$$

From Boltzmann's distribution law, the ratio of populations of two levels at temperature T is expressed as

$$N_1 / N_2 = e^{(E_2 - E_1) / KT}$$

$$N_1 / N_2 = e^{h\nu / KT}$$

Where K is the Boltzmann's constant and h is the Planck's constant.

Substituting value of N_1 / N_2 in equation (5) we get

$$E = A_{21}/B_{21}(1/e^{hv/KT}-1) \quad (6)$$

Now according to Planck's radiation law, the energy density of the black body radiation of frequency ν at temperature T is given as

$$E = 8\pi h\nu^3/c^3(1/e^{hv/KT}) \quad (7)$$

By comparing equations (6 and 7), we get

$$A_{21}/B_{21}=8\pi h\nu^3/c^3$$

This is the relation between Einstein's coefficients in laser.

Significance of Einstein coefficient relation: This shows that the ratio of Einstein's coefficient of spontaneous emission to the Einstein's coefficient of stimulated absorption is proportional to cube of frequency ν . It means that at thermal equilibrium, the probability of spontaneous emission increases rapidly with the energy difference between two states.

Semiconductor Laser:

Homojunction semiconductor laser:

Specifications

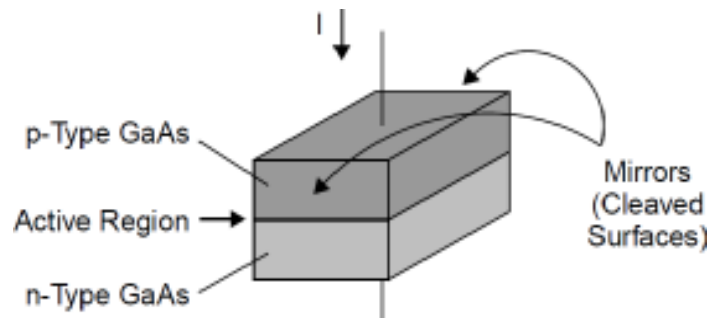
- × Type: Homojunction semiconductor
- × Active medium: p-n junction diode
- × Active centre: recombination of electrons and holes
- × Optical resonator: junction of diodes polished
- × Nature of output: pulsed or continuous
- × Power output: 1 mw
- × Wavelength of output: 8400 – 8600 Å
- × Band gap: 1.44 eV

Principle

- × The electrons in the conduction band (CB) combines with a hole in the valence band (VB)
- × Recombination of hole and electron produces energy in the form of light
- × This photon induces another electron in CB to VB stimulating another photon

Construction:

- × The active medium is made of pn junction diode made of GaAs
- × The P region is doped with Germanium
- × The N region is doped with tellurium
- × Thickness of the light is very narrow so that emitted laser has large divergence
- × The refractive index (μ) of GaAs is high, so it acts as optical resonator(no need of mirrors)
- × The upper and lower electrodes fixed on p and n region helps for the flow of current while biasing
- × Experimental set up of homojunction semiconductor laser :

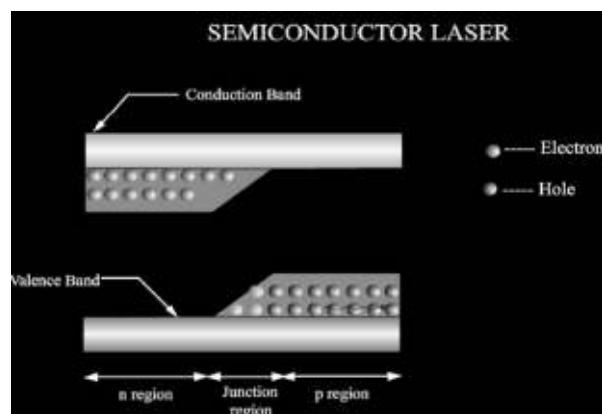


Working:

- × Population inversion is achieved by heavily doping p and n materials. Fermi level lies within CB of N type and VB of P type
- × Junction is forward biased with an applied voltage nearly equal to band gap voltage direct conduction takes place
- × Active region is generated near the depletion layer
- × Radiation with frequency ν is incident on the pn junction
- × Emitted photons increase the rate of recombination of electrons and holes

- × Frequency of the incident radiation should be in the range

$$(E_G / h) < \nu < (E_{fc} - E_{fv}) / h$$
- × Emitted photons are in the same phase as that of the incident photon
- × Energy level diagram of homo. Jn. Laser :



Calculation of wavelength

- × Bandgap of gaas = 1.44 eV
- × $E_g = hc / \lambda = 1.44$
- × Sub. The values of c & h in above equation
- × The wavelength can be obtained
- × $\lambda = 8626 \text{ \AA}$

- × Wavelength is in IR region

Heterojunction semiconductor laser

Specifications:

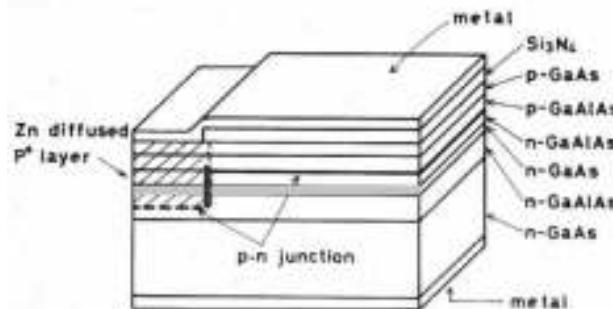
- Type: Heterojunction semiconductor laser
- Active medium: p-n junction
- Active center: recombination of electrons and holes
- Pumping: direct pumping
- Optical resonator: polished junctions of diode
- Power output: 10mw
- Nature of output: continuous wave form
- Wavelength of output: 8000 Å
- Band gap: 1.55eV

Principle:

- × Same as that of homojunction semiconductor laser
- × Electrons in the conduction band (CB) combines with a hole in the valence band (VB)
- × Recombination of hole and electron produces energy in the form of light
- × This photon induces another electron in CB to VB stimulating another photon

Construction:

- × There are five layers in this type. They are:
 - 1) first layer : a contact layer of GaAs (p type)
 - 2) Second layer : GaAlAs (p type) with wider band gap
 - 3) Third layer : a layer of GaAlAs (n type) with narrow band gap
 - 4) Fourth layer: GaAs (n type) with wider band gap
 - 5) Fifth layer: GaAlAs (n type)
 - 6) Sixth layer: GaAs (n type)
- × The first and fifth layers are polished well and they act as optical resonator
- × Upper and lower electrodes are connected to top and bottom. They help in forward biasing



Working:

- × Similar to homojunction semiconductor laser
- × Diode is forward biased with the help of upper and lower electrodes

- × Charge carriers are produced in wider band gap layers(2 & 4)
- × These charge carriers are injected to active region(3)
- × population inversion is achieved
- × Some of the injected carriers recombine and produce spontaneously photons
- × These photons stimulate more ejections
- × Photons are reflected back and forth by the polished layers
- × Intense and coherent beam emerges from layer 3 and 4

Calculation of wavelength:

- × Band gap of GaAlAs = 1.55eV
- × $E_g = hc / \lambda = 1.55 \text{ eV}$
- × Sub. The values of c & h in above equation,
- × The wavelength can be obtained
- × $\lambda = 8014 \text{ \AA}$
- × Wavelength is in IR region

Advantages:

- × Power output is high
- × Produces continuous wave
- × High directionality and coherence
- × Diodes are highly stable and has longer life time

Disadvantages:

- × Cost is higher than the other type
- × Practical difficulty in growing different layers of p – n junction

FIBRE OPTICS

Introduction:

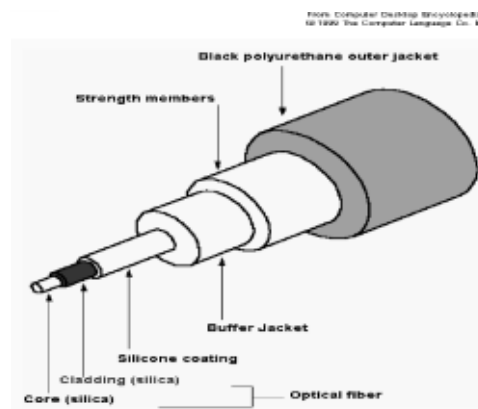
The development of lasers and optical fiber has brought about a revolution in the field of communication systems. Experiments on the propagation of information – carrying light waves through an open atmosphere were conducted. The atmospheric conditions like rain, fog etc affected the efficiency of communication through light waves. To have efficient communication systems, the information carried by light waves should need a guiding medium through which it can be transmitted safely.

Optical fiber:

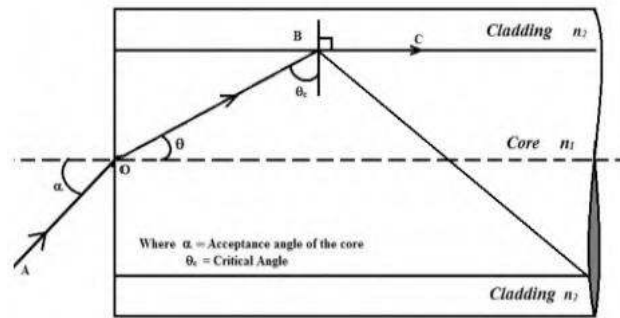
The optical fiber is a wave guide. It is made up of transparent dielectrics (SiO), (glass or plastics).

Fiber Construction:

It consists of an inner cylinder made of glass or plastic called core. The core has high refractive index n_1 . This core is surrounded by cylindrical shell of glass or plastic called cladding. The cladding has low refractive index n . This cladding is covered by a jacket which is made of polyurethane. It protects the layer from moisture and abrasion. The light is transmitted through this fiber by total internal reflection. The fiber guides light waves to travel over longer distance without much loss of energy. Core diameters range from 5 to $600\mu\text{m}$ while cladding diameters vary from 125 to $750\mu\text{m}$. Core transmits the light waves. The cladding keeps the light waves within the core by **total internal reflection**.



Numerical aperture Acceptance angle and fractional index change



Acceptance angle:

- Consider a cylindrical fiber as shown
- It consists of core and cladding
- Let n_0 be the refractive index of the medium
- It can be noted from the figure below
- Incident ray travels through AO
- Enters at an angle “i” to fiber axis
- Ray is refracted along OB at an angle θ in the core
- Further it also falls at a critical angle ϕ_c , the reflected ray grazes the surface
- For any angle of incidence more than critical angle, ray is total internally reflected ($\phi > \phi_c$)
- It is the acceptance angle
- Those rays which pass within the acceptance angle will be total internally reflected
- Light incident on core within this maximum external incident angle i_m can be propagated through the fiber
- This angle i_m is called waveguide acceptance angle
- Mathematical relation
- By applying Snell's law to AO;

$$n_0 \sin i = n_1 \sin \theta$$

$$\sin i = (n_1/n_0) \sin \theta = (n_1/n_0) \sqrt{1 - \cos^2 \theta} \quad (\text{since } \sin^2 \theta + \cos^2 \theta = 1) \quad (1)$$

Mathematical relation

- Applying the same to ‘OB’ interface

$$n_1 \sin \phi = n_2 \sin 90^\circ$$

$$\sin \phi = (n_2/n_1) \quad \{\text{since } \phi = 90^\circ - \theta\}$$

$$\sin(90^\circ - \theta) = (n_2/n_1)$$

$$\cos \theta = (n_2/n_1) \quad \longrightarrow (2)$$

Sub the value of $\cos \theta$ from (2) in (1) the value of angle of incidence is given by,

$$I_m = \sin^{-1} \left(\sqrt{n_1^2 - n_2^2} \right)$$

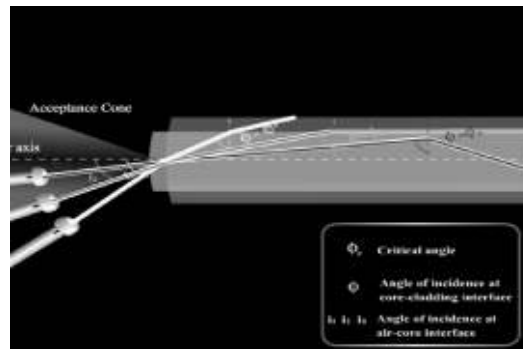
Definition:

- The maximum angle at which or below which the light suffers total internal reflection is called acceptance angle
- The cone which contains this angle is called acceptance cone

• Numerical aperture:

- Sine of acceptance angle is called numerical aperture
- $Na = \sin i_m$

$$= \sqrt{n_1^2 - n_2^2}$$



Fractional index change:

It is the ratio of refractive index difference in core and cladding to the refractive index of the core

$$\Delta = (n_1 - n_2) / n_1$$

$$Na = \sqrt{n_1^2 - n_2^2}$$

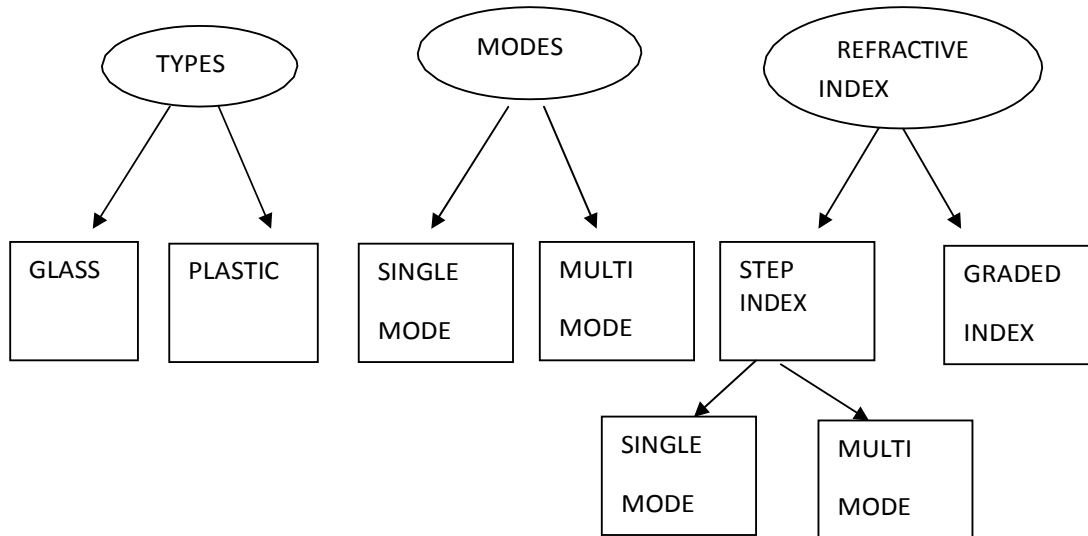
$$= \sqrt{(n_1 - n_2)(n_1 + n_2)}$$

$$= \sqrt{(n_1 + n_2)(n_1 \delta)}$$

$$= \sqrt{2n_1^2 \delta}$$

$$Na = n_1 \sqrt{2 \delta}$$

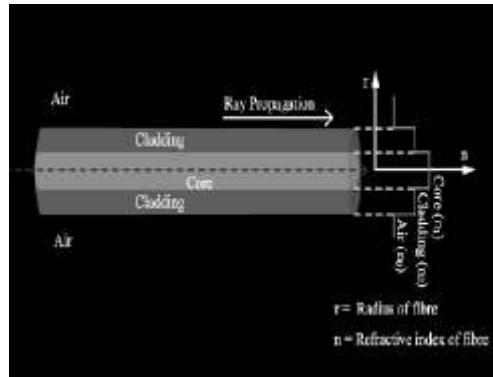
Types of Optical fibers:



- Glass : made of mixtures of metal oxides and silica. eg: core : SiO_2 ,
- Cladding : $\text{P}_2\text{O}_3 - \text{SiO}_2$
- Plastics: made of plastics, can be handled tough due to its durability. Core: polystyrene ,cladding: methyl methacrylate

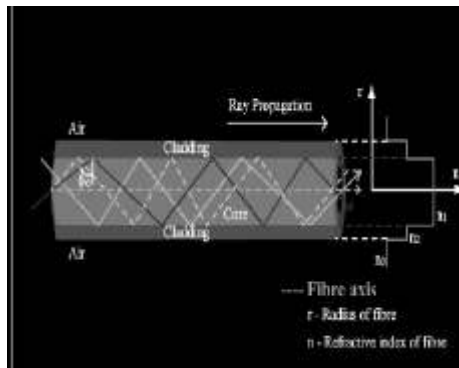
Classification based on modes:

- Mode describes the propagation of electromagnetic waves in the ave guides.
- Single mode :
 1. Core diameter is very small so that it allows only one mode of propagation
 2. Cladding diameter is very large
 3. Optical loss is reduced.



Multimode fiber:

1. Core diameter is very large compared to single mode
2. Allows many modes to propagate.
3. It is made up of glass-clad glass, silica –clad silica



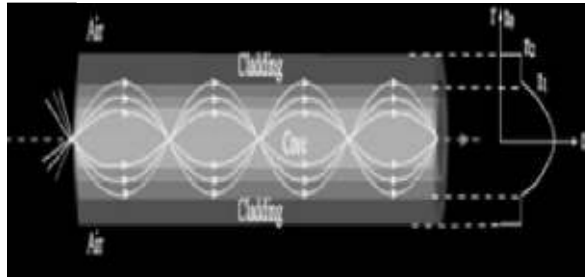
Classification based on refractive index:

Further classified into step index and graded index fiber

- Step index fiber: refractive index of air cladding and core varies step by step.
- This fiber is sub classified into single mode and multimode.

2. graded index fiber:

- Refractive index of the core varies radially from the axis of the fiber
- Refractive index of the core is maximum along fiber axis and gradually decreases



Refractive index is minimum at interface.

Losses in optical fiber:

- Attenuation is defined as the ratio of optical output power from fiber of length 'l' to input power
- Attenuation $\alpha = -(10 / l) \log(p_{in} / p_{out})$
- Three types of losses.
- They are a) absorption b) scattering and c) radiative losses

Absorption:

- Occurs due to imperfections of atomic structures(eg : missing molecules)
- There are three bands of wavelength where absorption increases drastically is 950 nm , 1250 nm , 1380 nm.

Scattering:

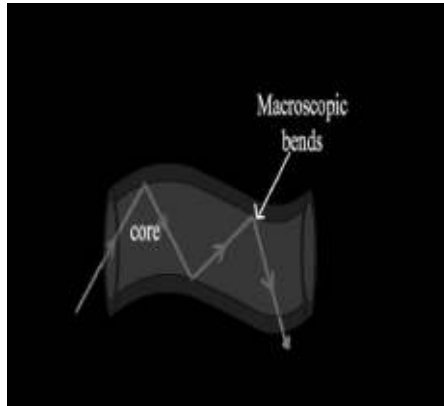
- Loss is dependant on wavelength
- Glass is used in manufacture of optical fiber
- Due to disordered structure of glass, there is variation in refractive index
- Some portion of light is scattered where as some travels through the fiber
- This is called Rayleigh scattering
- Rayleigh scattering $\propto (1 / \lambda^4)$

Radiative loss:

- Due to bending of finite radius of curvature in optical fiber
- There are two types of radiative losses
- They are due to macroscopic bends and microscopic bends in the fiber

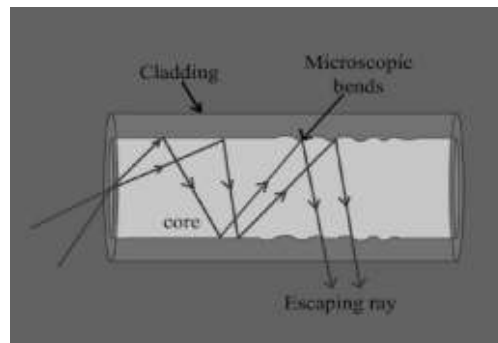
Macroscopic Bends:

- $R_{core} > R_{cladding}$
- This may cause large curvature at the position where fiber takes turn
- At these corners light do not satisfy total internal reflection conditions
- Therefore escapes out of optical fiber
- This loss is called macroscopic bending loss.



Microscopic bends:

- Caused due to micro bends or non-uniformities inside the fiber
- Micro bends are caused due to non uniform pressures created during manufacture
- Leads to leakage of light through optical fiber
- This loss can be minimized by extruding compressible jacket over optical fiber.



Distortion:

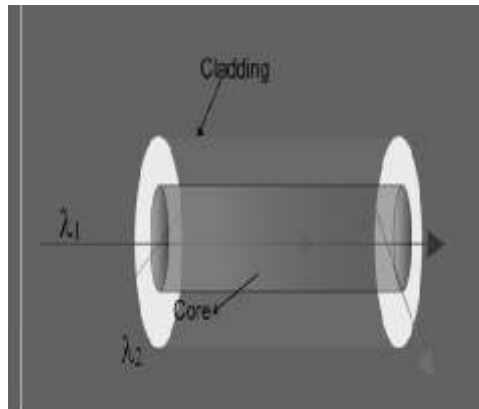
- When an optical signal is sent into optical fiber, the pulse spreads or broadens when it passes through optical fiber. this is called distortion
- This is clearly shown in the diagram below

Dispersion :

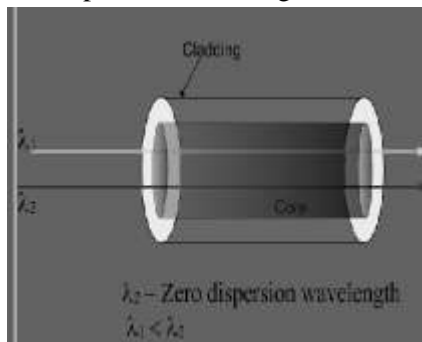
- Three types of dispersion
 1. Intermodal dispersion
 2. Material or chromatic dispersion
 3. Waveguide dispersion

Intermodal dispersion:

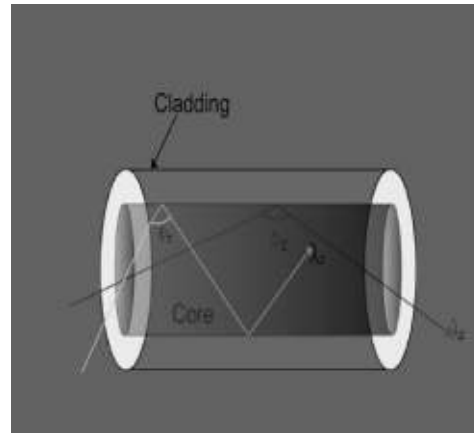
- Occurs when more than one mode propagates through optical fiber
- When ray is launched in optical fiber, pulse is dispersed in all paths
- Each mode is of different wavelength
- Hence light takes different time to propagate through the optical fiber
- Results in elongation of pulse
- Called intermodal dispersion

**Material dispersion:**

- Otherwise called as chromatic dispersion
- Occurs due to light waves travelling with different velocities
- This can be minimized when light travels with $\lambda = 870 \text{ nm}$.
- This wavelength is called zero dispersion wavelengths

**Waveguide dispersion:**

- This loss occurs due to guiding property of optical fiber
- Light is incident at different angles at core-cladding interface
- Due to this loss occurs.



- In general:
 Intermodal Dispersion > material dispersion > waveguide dispersion

Fiber optic sensors:

- Device which converts any form of signal into optical signal in measurable form
- Optical fiber are used as guiding media and hence called as wave guides
- Optical source is LED / Laser

Types of sensors:

- There are two types of sensors. They are
 1. Intrinsic or active sensors
 2. Extrinsic or passive sensors

Active sensor:

- The physical parameter to be sensed directly acts on fiber itself to produce change in transmission characteristics
- Eg: temperature / pressure sensors

Passive sensors

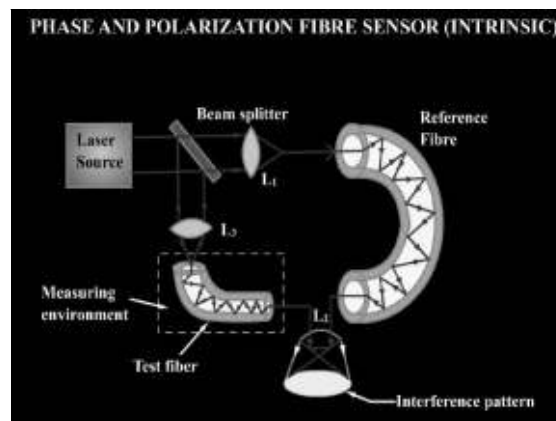
- In this type of sensors, separate sensing element is used
- Optical fiber acts as guiding media
- Eg: displacement sensor

Pressure sensor:**Principle:**

Based on interference between beams emerging from reference fiber and the fiber kept in measuring environment.

Description:

- consists of laser source to emit light
- Beam splitter is made of glass plate inclined at 45°
- There are two fibers : reference and test fiber
- Rf is isolated from the environment
- Tf is kept in the environment
- Separate lens systems are provided to split and collect the beam

**Working:**

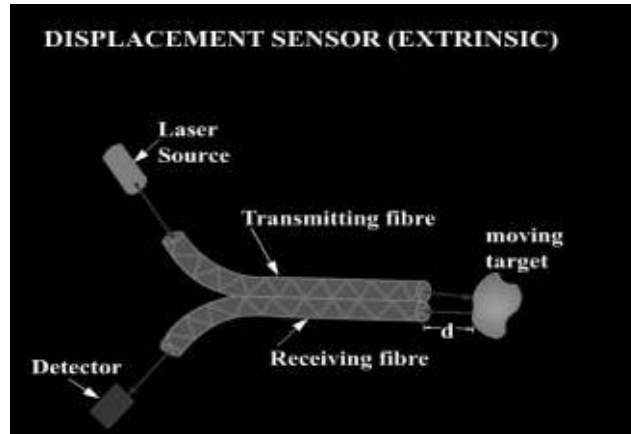
- Laser source emits laser
- Beam splitter is inclined at 45°
- Beam splitter divides the beam into main and splitted beam
- Main beam passes through L_1 and focussed onto the reference fiber
- The beam from rf is passed on to L_2
- splitted beam passes through L_3 and focussed onto test fiber
- Splitted beam is made to falls on L_2
- Both the beam produces path difference due to change in pressure
- Causes interference pattern. From this pattern, change in pressure can be measured accurately

Displacement sensors:**Principle:**

Light is sent to the transmitting fiber and made to fall on the moving target. The reflected light from the target is sensed by the detector. With respect to intensity of the light reflected from it the displacement of the target is measured.

Construction:

- Bundle of transmitting fibers are coupled to laser source
- Bundle of receiving fibers are connected to detector
- Axis of both fibers can be adjusted w.r.t to moving target to increase sensitivity of sensor

**Working**

- Light from source transmitting fiber is made to fall on the moving target.
- The light is reflected from the target is made to pass through the rf and detected by the detector.
- Based on intensity of light received displacement of target is measured
- Eg: if intensity is more target is moving towards the sensor

UNIT - 3**Thermal Physics****Introduction:**

Thermal Physics is a field of science that deals with heat and temperature. The study of thermal physics is important because of its applications in various fields of engineering. In designs of internal and external combustion engines, refrigeration and air-conditioning plants, heat-exchangers, coolers, condensers, furnaces, preheaters, cooling system for motors, transformers and generators, Control of heat transfer in dams, structures, building and tunnels, in the application of heat transfer is in freezing, boiling, evaporation and condensation processes, etc.

Thermal expansion of solids

It is a common observation that nearly all substances expand on heating and contract on cooling.

Types of expansion

A solid substance can undergo three types of expansion:

- (i) Expansion in length is known as linear expansion
- (ii) Expansion in area is known as superficial expansion
- (iii) Expansion in volume is known as cubical expansion.

Coefficients of linear, superficial and cubical expansion of a solid:

- (a) The coefficient of linear expansion of a solid is the increase in length of unit length of the solid when its temperature is raised by 1K. It is denoted by the letter α .

Thus if a rod having a length L_1 at T_1 K is heated to a temperature T_2 K its length increases to L_2 .

$$\alpha = \frac{L_2 - L_1}{L_1(T_2 - T_1)} = \frac{\text{Increase in length}}{\text{Original length} \times \text{Rise in temperature}}$$

- (b) The coefficient of superficial expansion of a solid is the increase in area produced per unit area of the solid when the temperature is raised by 1 K. It is denoted by the letter β .

Thus if a solid having area A_1 at T_1 K is heated to T_2 K, its area increases to A_2 .

$$\beta = \frac{A_2 - A_1}{A_1(T_2 - T_1)} = \frac{\text{Increase in area}}{\text{Original area} \times \text{Rise in temperature}}$$

- (c) Coefficient of cubical expansion of a solid is the increase in volume per unit volume of a solid for 1 K rise of temperature. It is denoted by the letter γ .

Thus if a solid having a volume V_1 at T_1 K is heated to a temperature T_2 K, let V_2 be its volume at T_2 K. Then,

$$y = \frac{V_2 - V_1}{V_1(T_2 - T_1)} = \frac{\text{Increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

The unit for coefficients of linear expansion, superficial expansion and cubical expansion of solids is per °C or per K.

Thermal Expansion of Liquids

Absolute and apparent expansion of a liquid:

A liquid has always been taken in a container and so when heat is given to the vessel, both the liquid and the container expand in volume.

The observed expansion of the liquid is only relative to the container. If we ignore the expansion of the container what we obtain is only apparent expansion.

To find the absolute expansion of the liquid we should take into account the expansion of the container.

(a) Coefficient of apparent expansion of a liquid:

It is the observed increase in volume of unit volume of the liquid per degree

Kelvin rise of temperature. It is denoted by γ_a . Thus if V_1 and V_2 be the observed volumes of a liquid at temperature T_1 K and T_2 K respectively then

$$\gamma_a = \frac{V_2 - V_1}{V_1(T_2 - T_1)} = \frac{\text{Increase in volume of liquid}}{\text{Original volume} \times \text{Rise in temperature}}$$

(b) Coefficient of real or absolute expansion of a liquid:

It is the real increase in volume of unit volume of a liquid per degree Kelvin rise of temperature. It is denoted by γ_r .

If V_1 and V_2 be the real volumes of a liquid at T_1 K and T_2 K, then

$$\gamma_r = \frac{V_2 - V_1}{V_2(T_2 - T_1)} = \frac{\text{Real increase in volume of liquid}}{\text{Original volume} \times \text{Rise in temperature}}$$

The units of coefficient of apparent and real expressions of the liquid is per °C or per K.

Applications of expansion of solids

Expansion of solids plays an important role in numerous engineering applications.

1. Thermal expansion joints with gaps are provided in building concrete highways and bridges to compensate for change in dimension with temperature variations.
2. The length of the electric wire between two electric poles is kept slightly larger to compensate for its contraction during winter days.
3. A gap is left at the joint of two rails.
4. One of the most important applications is expansion joints and bimetallic strip.

Expansion Joints

An expansion joint or movement joint is an assembly designed to safely absorb the heat induced expansion or contraction of a pipeline, duct or vessel. It helps to hold parts together.

Types of expansion joints

The expansion joints are classified as follows.

1. Metallic expansion joint
2. Wall expansion joint
3. Slip type expansion joint
4. Fabric expansion joint
5. Rubber expansion joint

Metal expansion joints

They are also called compensators. These expansion joints are provided for thermal expansion and relative movement in pipelines, containers and machines.

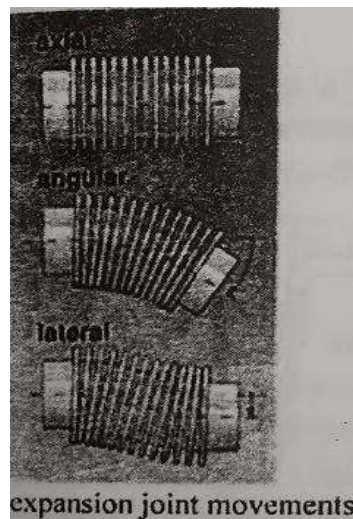
They consist of one or more metal bellows, connectors at both ends, and tie rods that depend on the application.

They are classified according to the three basic types of movement:

- (i) Axial
- (ii) Angular and
- (iii) Lateral expansion joints.

Axial

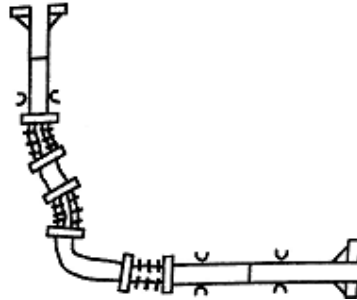
In axial compensation, the thermal expansion of a straight line section between two fixed points is absorbed by an axial expansion joint.



The distance between two fixed points defines the pipeline length requiring compensation, and thus determines the axial movement that must be achieved by the expansion joint.

Angular

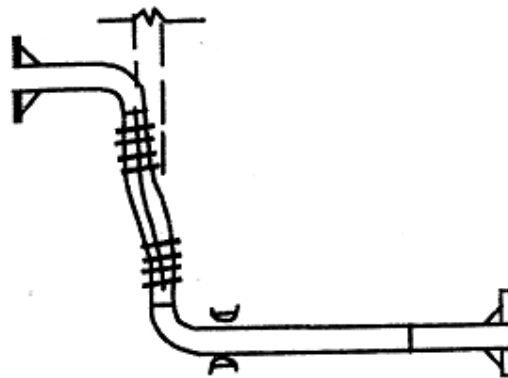
The angular compensation of thermal expansion requires at least two, and for full compensation even three, angular expansion joints. Angular expansion joints offer a wide variety of combination options in so called two hinge or three hinge systems.



Lateral

Lateral compensation is likewise associated with a redirection of flow by 90° within single plane or multi plane piping systems. Usually lateral expansion joints are installed in existing right angle redirections in the system.

The movement of a lateral expansion joint always consists of the desired lateral movement and a slight unavoidable axial movement that comes from the expansion joint itself.



Lateral expansion joints that are movable in all planes simultaneously absorb expansion from two pipe sections in different directions.

Applications

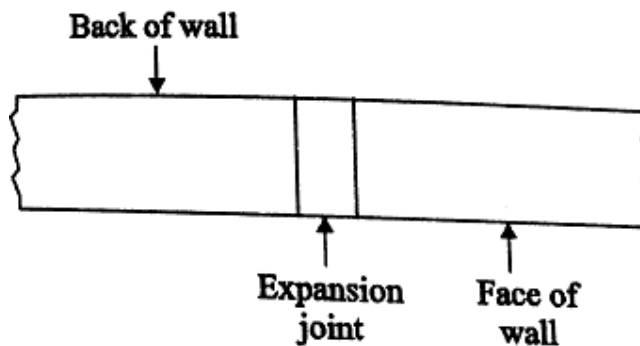
1. Heating and air conditioning installation
2. Hot water and fire protection system

3. Compressors inlet and outlet
4. Pipe with axial displacement by temperature or pressure changes
5. Thermal motors refrigeration circuit

Wall expansion joints

These vertical joints are provided in large retaining walls to allow for the expansion of concrete due to temperature changes. They are usually extended from top to bottom of the wall.

These joints may be filled with flexible joint fillers. Horizontal reinforcing steel bars running across the stem are continuous through all joints.



- In building construction, an expansion joint is a mid-structure separation designed to relieve stress on building materials caused by building movement induced by thermal expansion and contraction caused by temperature changes, sway caused by wind, seismic events, etc.
- Bridges are made of steel, which expands and contracts with temperature. A span long enough to need expansion joints may expand or contract several inches in the course of a year.
- Without those joints either the bridge butt joints would crush and buckle in hot weather, or the bridge would pull apart in cold weather.

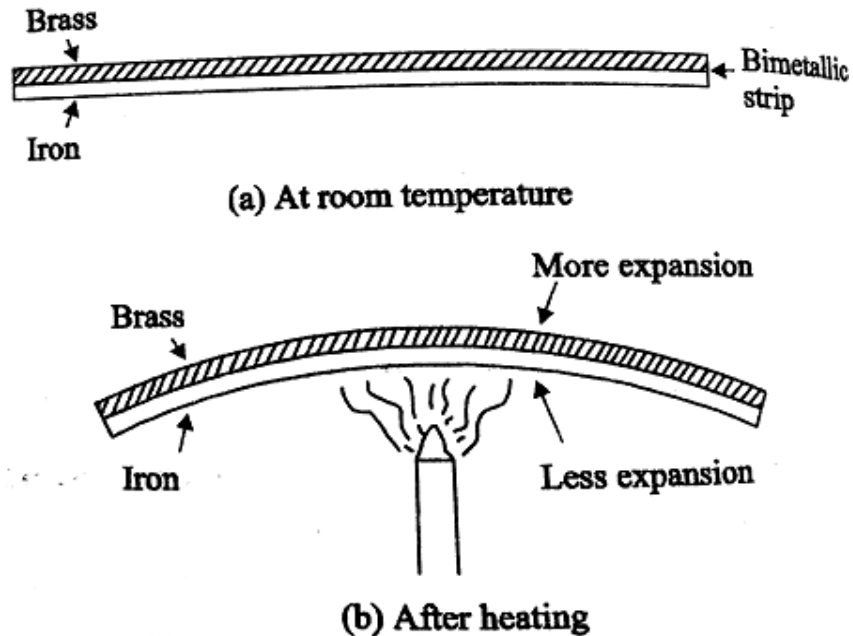
Bimetallic strip

Definition

A bimetallic strip means a strip made of two metals of different expansion coefficients joined together. It is like compound bar.

Principle

It operates on the principle that different metals have different coefficients of expansion.



Description & working

Two strips of brass and iron of equal lengths are joined together to form a bimetallic strip.

This bimetallic strip is straight at room temperature.

Now, it is heated by using a burner. On heating, the metallic strip bends to form a curve in such a way that brass remains on the outside of the curve.

This means that brass strip has become longer than the iron strip after heating, that is, brass expands more than iron for the same rise in temperature.

It is obvious that a bimetallic strip bends due to the unequal expansion rate of the two metals.

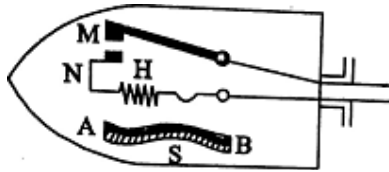
In fact on heating, the bimetallic strip bends towards iron side. If, however, the bimetallic strip shown in figure is cooled in freezing mixture, then it bends the other way showing that brass contracts more than iron.

Applications of bimetallic strips

They are used as thermo-switches in automatic electrical heating appliances like thermostats, electric ovens, electric irons, fire alarms, and bimetallic thermometers. A thermo switch is a switch which works by the action of heat.

(i) Bimetallic thermostat

This is a simple device to maintain the temperature constant. This consists of a bimetallic strip AB, made by welding together brass and iron. The bimetallic strip is wound into a small spring and it is connected by a rod to the contact maker M of the circuit of an electric heater.

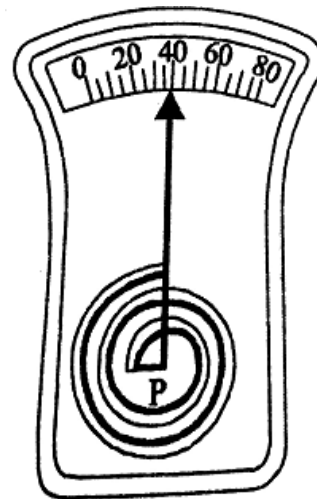


When the temperature increases, the expansion of the brass of the strip causes the spring to unwind thus breaking the electric circuit.

When the temperature decreases the contraction of the brass strip to bend in the opposite direction making the spring to wind and make the electric circuit again.

(ii) Bimetallic Thermometer

It shows a bimetallic thermometer. It is a rugged portable thermometer. It is made using a bimetallic strip which is in the form of a spring centered at point 'P' and carries a pointer.



It uses the principle of differential expansion to activate the pointer. When the temperature is raised, the strip bends and hence the pointer is moved over a graduated scale in temperature.

Advantages of bimetallic thermometers

1. They are simple, robust and inexpensive.
2. Their accuracy is between + or - 2% to 5% of the scale.
3. They can withstand 50% over range in temperatures.

Limitations of bimetallic thermometers

1. They are not recommended for temperature above 400° C.
2. When regularly used, the bimetallic may permanently deform, which in turn will introduce errors.

(iii) Bimetallic automatic fire alarms

A bimetallic strip is also used for making automatic fire alarms. When fire breaks out in a building, heat is produced. This heat bends the bimetallic strip in such a way that on bending, it completes the circuit of an electric bell. The bell starts ringing and gives the warning of a fire.

(iv) Bimetallic Sensors

A bi-metallic strip is used to convert a temperature change into mechanical displacement and thus acts as a temperature sensor.

Transfer of heat energy

(i) CONDUCTION: In this process heat is transmitted from one point to the other through the substance without the actual motion of the particles. When one end of a brass rod is heated in flame the other end gets heated in course of time. In this case the molecules at the hot end vibrate with higher amplitude (K.E) and transmit the heat energy from one particle to the next and so on. Heat is said to be conducted through the rod. However, particles in the body remain in their position and so not move. Thus conduction is the transference of heat from the hotter part of a body to the colder part without the motion of the particles in the body.

Metals are good conductors of heat and wood, glass, brick, cotton, wool, rubber are bad conductors of heat. For example thick brick walls are used in the construction of a cold storage. Brick is a bad conductor of heat and does not allow outside heat to flow inside the cold storage. Also a steel blade appears colder than a wooden handle in winter. Steel is a good conductor of heat. As soon as a person touches the blade heat flows from the hand (higher temperature) to the blade to low temperature. Therefore it appears colder. Since wood is a bad conductor of heat, does not allow heat to flow to the handle.

(ii) CONVECTION: It is the process in which heat is transmitted from one place to another by the actual movement of the heated particles. It is prominent in the case of liquids and gases. Land and sea breezes and trade winds are formed due to convection. Suppose water in a container is heated from below. The layer of water in the bottom gets heated, its density decreases and it comes up transferring heat. In this case, the transference of heat from the bottom of the vessel to the top of the vessel is by convection. It is used in ventilation. Rooms are provided with ventilators near the ceiling. Air in the room gets warmer due to respiration of persons in the room. Warm air containing more of CO₂ (gas) and water vapour has less density and moves upwards. Fresh air from outside enters the room through the doors and windows. The impure air moves outside through the ventilators. This phenomenon is continuous.

(iii) RADIATION: It is the process in which heat is transmitted from one place to the other directly without the necessity of the intervening medium. We get heat radiations directly from the sun without affecting the intervening medium. Heat radiation can pass through vacuum. Also it forms a part of electromagnetic spectrum.

Applications of heat radiations:

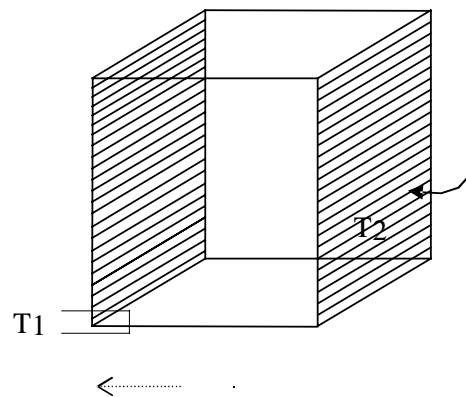
- (1) White cloths are preferred in summer and dark colored clothes in winter.

Reason: When heat radiations fall on white clothes, they are reflected back. No heat is absorbed by the clothes and a person does not get heat from outside in summer. Dark clothes in winter will absorb the heat radiations falling on them and keep the body warm.

- (2) Polished reflectors are used in electric heaters to reflect maximum heat in the room.

Co-efficient of thermal conductivity:

Fig. 2.1



Suppose there is a slab of material of area of cross-section A. Let the opposite faces be maintained at temperatures $T_1^{\circ}\text{C}$ and $T_2^{\circ}\text{C}$ ($T_1 > T_2$). Let x be the distance between the faces. It is found that the quantity of Q of heat conducted in a time t is directly proportional, to A to $(T_1 - T_2)$ to t (time) and inversely proportional to x . Therefore,

$$Q \propto \frac{A(T_1 - T_2)t}{x}$$

$$Q = \frac{kA(T_1 - T_2)t}{x}$$

Where K is called the co-efficient of thermal conductivity of the material of the slab.

Definition: (Thermal Conductivity)

It is defined as the quantity of heat conducted in one second from one face to the opposite

face of a slab of area of cross-section 1sq.Cm, when the distance between the faces is equal to 1cm and the difference in temperature between the faces is equal to 1°C.

Temperature gradient:

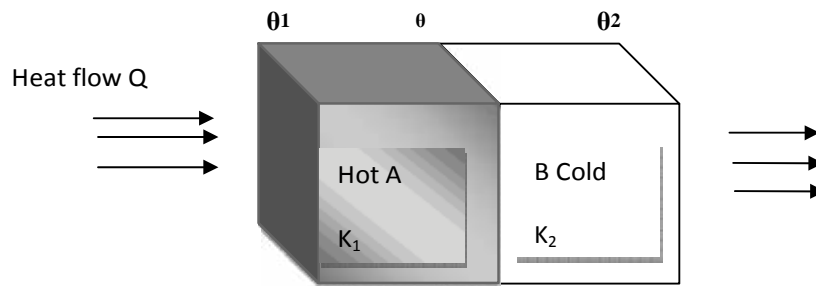
The quantity $(T_1-T_2)/X$ represents the rate of fall of temperature with respect to distance. The quantity (dT/dx) represents the rate of change of temperature with respect to the distance. As temperature decreases with inc rease in distance from the hot end, the quantity (dT/dx) is negative and is called the temperature gradient.

HEAT CONDUCTION THROUGH A COMPOUND MEDIA (SERIES AND PARALLEL)

Bodies in series:

Let us consider a compound media of two different materials A and B with thermal conductivities K_1 and K_2 and thickness x_1 and x_2 (Fig.).

The temperatures of the outer faces of A and B are θ_1 and θ_2 . The temperature of the surfaces in contact is θ . When the steady state is reached, the amount of heat flowing per second (Q) through every layer is same.



Amount of heat flowing through the material (A) per second

$$Q = \frac{K_1 A (\theta_1 - \theta)}{X_1} \dots\dots\dots (1)$$

Amount of heat flowing through the material (B) per second

$$Q = \frac{K_2 A (\theta - \theta_2)}{X_2} \dots\dots\dots (2)$$

The amount of heat flowing through the materials A and B is equal in steady state conditions.

Hence, the eqn (1) and (2) are equal

$$\frac{K_1 A (\theta_1 - \theta)}{X_1} = \frac{K_2 (\theta - \theta_2)}{X_2} \dots\dots(3)$$

Rearranging the eqn (3), we have

$$\begin{aligned}
 K_1 A (\theta_1 - \theta) x_2 &= K_2 A (\theta - \theta_2) x_1 \\
 K_1 \theta_1 x_2 - K_1 \theta x_2 &= K_2 \theta x_1 - K_2 \theta_2 x_1 \\
 K_1 \theta_1 x_2 + K_2 \theta_2 x_1 &= K_2 \theta x_1 + K_1 \theta x_2 \\
 K_1 \theta_1 x_2 + K_2 \theta_2 x_1 &= \theta (K_2 x_1 + K_1 x_2) \\
 \theta &= \frac{K_1 \theta_1 x_2 + K_2 \theta_2 x_1}{K_2 x_1 + K_1 x_2} \dots \dots \dots (4)
 \end{aligned}$$

This is the expression for interface temperature of two composite slabs in series.
 Sub, for θ in equation (1), we get

$$\begin{aligned}
 Q &= \frac{K_1 A}{s_1} \theta_1 - \left(\frac{K_1 \theta_1 s_2 + K_2 \theta_2 s_1}{K_2 s_1 + K_1 s_2} \right) \\
 &= \frac{K_1 A}{s_1} \left[\frac{K_2 \theta_1 s_1 + K_1 \theta_1 s_2 - K_1 \theta_1 s_2 - K_2 \theta_2 s_1}{K_2 s_1 + K_1 s_2} \right] \\
 &= \frac{K_1 A}{s_1} \left[\frac{K_2 \theta_1 s_1 - K_2 \theta_2 s_1}{K_2 s_1 + K_1 s_2} \right] \\
 &= \frac{K_1 K_2 A (\theta_1 - \theta_2)}{K_2 s_1 + K_1 s_2} \\
 &= \frac{A (\theta_1 - \theta_2)}{\frac{s_1}{K_1} + \frac{s_2}{K_2}} \dots \dots \dots (5)
 \end{aligned}$$

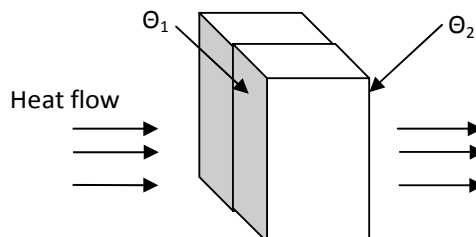
‘Q’ is the amount of heat flowing through the compound wall of two materials. This method can also be extended to composite slab with more than two slabs. In general for any number of slabs, the amount of heat conducted per sec is given by

$$Q = \frac{A(\theta_1 - \theta_2)}{s\left(\frac{x}{K}\right)}$$

Bodies in parallel

Consider a composite media of two different materials A and B with thermal conductivities K_1 and K_2 and thickness x_1 and x_2 . They are arranged in parallel as shown in fig.

The faces of the material A and B are at temperature θ_1 and the other end faces of A and B are at temperature θ_2 . A_1 and A_2 are the area of cross section of the materials.



Amount of heat flowing through the first material (A) in one second

$$Q_1 = \frac{K_1 A_1 (\theta_1 - \theta_2)}{X_1} \dots \dots \dots (1)$$

Similarly

Amount of heat flowing through the second material (B) in one second

$$Q_2 = \frac{K_2 A_2 (\theta_1 - \theta_2)}{X_2} \dots \dots \dots (2)$$

The total heat flowing through these materials per second is equal to the sum of Q_1 and Q_2

$$\begin{aligned} Q &= Q_1 + Q_2 \\ &= \frac{K_1 A_1 (\theta_1 - \theta_2)}{X_1} + \frac{K_2 A_2 (\theta_1 - \theta_2)}{X_2} \end{aligned}$$

Amount of heat flowing through per second

$$Q = (\theta_1 - \theta_2) \left(\frac{K_1 A_1}{s_1} + \frac{K_2 A_2}{s_2} \right)$$

In general, the net amount of heat flowing per second parallel to the composite slabs is given by

$$\Sigma Q = (\theta_1 - \theta_2) \Sigma \frac{KA}{x}$$

Methods to determine thermal conductivity

The thermal conductivity of a material is determined by various methods.

1. Searle's method – for good conductors like metallic rods.
2. Forbe's method – for determining the absolute conductivity of metals.
3. Lee's disc method – for bad conductors.
4. Radial flow method – for bad conductors.
- 5.

FORBES METHOD – THEORY AND EXPERIMENT

This is one of the earliest method to find the absolute thermal conductivity of metals.

Theory of the experiment

Consider a long rod. This rod is heated at one end and a steady state is reached after some time.

Amount of heat flowing per second across the cross-section A at the point B

$$= K A \left(\frac{d\theta}{ds}\right)_B \quad \text{-----(1)}$$

Where K - thermal conductivity

A - cross sectional area

$\left(\frac{d\theta}{ds}\right)_B$ - temperature gradient at B.

This amount of heat flowing across the section B is equal to the heat lost by radiation by the rod beyond the section B.

Consider an element of thickness dx of the rod.

Mass of the element = (A dx)ρ

Where ρ is density of the rod

Heat lost by the element per second = Mass x specific heat capacity x rate of fall of temp.

$$= (A dx) \rho \times S \times \frac{d\theta}{dt} \quad \text{-----(2)}$$

Where $\frac{d\theta}{dt}$ - rate of fall of temperature of the element

S - specific heat capacity of the rod

Total heat lost by the portion of the rod between section B and the end C

$$= \int_B^C (A dx) \rho \times S \times \frac{d\theta}{dt} \quad \text{-----(3)}$$

Amount of heat flowing per second across the cross section at the point B = Heat lost by radiation by radiation by the rod beyond the section B.

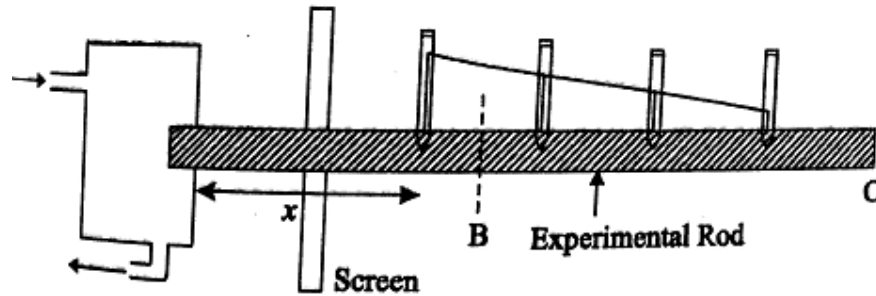
$$K A \left(\frac{d\theta}{ds}\right)_B = \int_B^C (A dx) \rho \times S \times \frac{d\theta}{dt} \quad \text{-----(4)}$$

$$K = \frac{\int_B^C \rho S \frac{d\theta}{dt} ds}{\left(\frac{d\theta}{ds}\right)_B} \quad \text{--- (5)}$$

Experiment consists of two parts

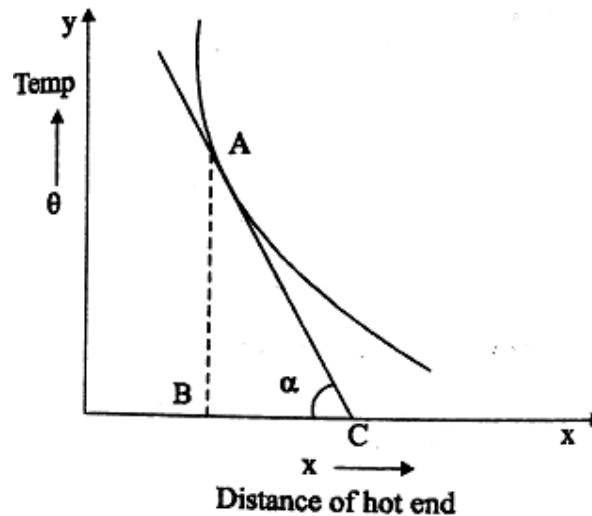
1. Static experiment to find $\left(\frac{d\theta}{ds}\right)_B$
 2. Dynamic experiment to find $\frac{d\theta}{dt}$ and $\int_B^C \frac{d\theta}{dt} dx$
1. Static experiment

The specimen metal is taken in the form of a long rod. One end of this rod is heated by a steam chamber. The rod has a series of holes into which thermometers are fitted. These thermometers record temperatures at different points along the rod.



When the steady state is reached, the temperature shown by the thermometers of the rod and their respective distances from the hot end are noted.

A graph is drawn between the temperature and distance from the hot end.



The value of $\left(\frac{d\theta}{dx}\right)_B$ is obtained by drawing a tangent to the curve at a point B.

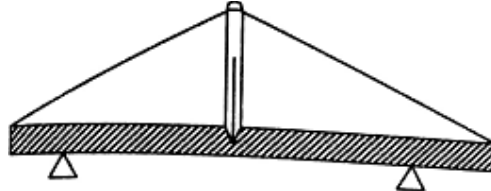
If this tangent makes an angle α with the x axis, then from the graph

$$\left(\frac{d\theta}{dx}\right)_B = \frac{AB}{BC} = \tan\alpha$$

2. Dynamic experiment

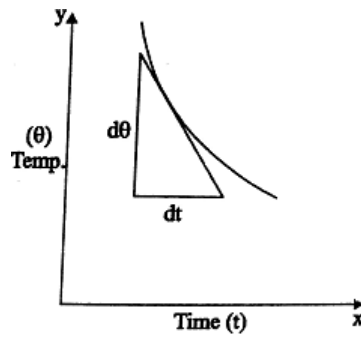
A piece of the original rod is heated to the same temperature as that of the hot end in the static experiment. The heated piece of the rod is suspended in air.

Now, it is allowed to cool. Its temperature is noted at regular intervals of time by a thermometer placed in a hole at the centre.



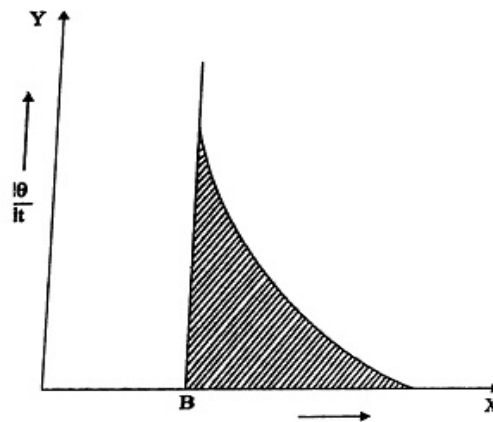
Forbe's method – Dynamic experiment

A graph is drawn between temperature and time.



From this graph, the value of $\frac{d\theta}{dt}$ for various values of θ are determined by drawing tangents at various points of the cooling curve.

From the graph between temperature θ and the distances of hot end x for various values of temperature θ are obtained. Now, third graph is drawn between $\frac{d\theta}{dt}$ and the corresponding values of x .



The curve is extended to meet the x axis corresponding to the point B, a point is located on the curve. The area bounded by the curve, x axis and the coordinate passing through B is

$$\int_B^C \frac{d\theta}{dt} dx.$$

The area of the shaded portion is determined.

$$\text{We know that } K = \frac{qS \int_B^C \frac{d\theta}{dt} ds}{\left(\frac{d\theta}{dx}\right)_B}$$

Substituting the values in the above equating, we have

$$K = \frac{qS \times (\text{Area of the shaded portion})}{\tan \alpha}$$

Hence, K is determined.

Merits

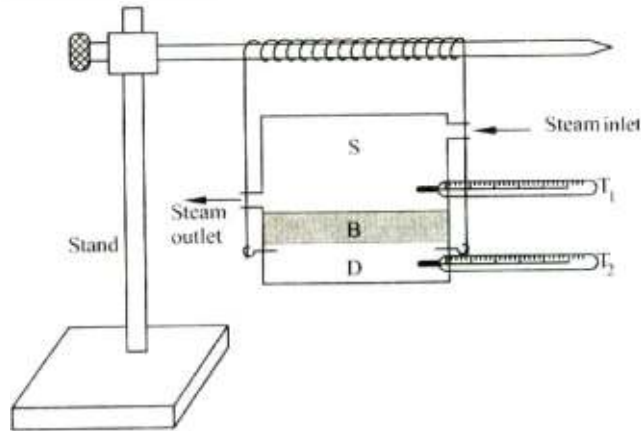
1. It is one of the earliest method to determine the absolute thermal conductivity of the material.
2. This method is based on the fundamental relation which defines thermal conductivity.

Demerits

1. It is tedious method and requires a lot of time for the completion of the experiment and drawing the three graphs.
2. The specific heat capacity of the material of the rod does not remain constant at different temperatures as assumed.
3. The distribution of heat is not uniform along the bar in the two experiments. Therefore, this experiment is not accurate.

Determination of thermal conductivity (K), of a bad conductor (card board) by Lee's disc method.

Description: The given bad conductor (B) is shaped with the diameter as that of the circular slab (or) disc 'D'. The bad conductor is placed in-between the steam chamber (S) and the disc (or) slab (D), provided the bad conductor, steam chamber and the slab should be of same diameter. Holes are provided in the steam chamber (S) and the discs (or) slab (D) in which thermometer are inserted to measure the temperatures. The total arrangement is hanged over the stand as shown in fig.



Fig

Working: Steam is passed through the steam chamber till the steady state is reached (i.e) the thermometer show constant temperature. Let the temperature of the steam chamber (hot end) and the disc or slab (cold end) be θ_1 and θ_2 respectively.

Calculation:

Let 'x' be the thickness of the bad conductor (B), 'm' is the mass of the slab's' be the specific heat capacity of the slab , 'r' is the radius of the slab and 'h' be the height of the slab, then

$$= \frac{KA (\theta_1 - \theta_2)}{x}$$

Amount of heat conducted by the bad conductor per second

Since the area of the cross section is $= \pi r^2$

$$= \frac{K \pi r^2 (\theta_1 - \theta_2)}{x} \text{ Amount of heat conducted per second} \dots(1)$$

The amount of heat lost by the slab per second

$$= nsR_c \dots\dots\dots(2)$$

Under steady state

The amount of heat conducted by the bad conductor (B) per second = Amount of heat lost by the slab (D) per second

Therefore, we can write eqn.(1) = eqn.(2)

$$\frac{K \pi r^2 (\theta_1 - \theta_2)}{x} = nsR_c$$

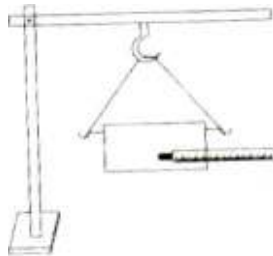
$$K = \frac{n c s R_c}{n r^2 (\theta_1 - \theta_2)} \dots\dots(3)$$

To find the rate of cooling (R_c):

In Eqn.(3) R_c represents the rate of cooling of the slab along with the steam chamber. To find the rate of cooling for the slab alone, the bad conductor is removed and the steam chamber is directly placed over the slab and heated. When the temperature of the slab attains 5°C higher than θ_2 , the steam chamber is removed. The slab is allowed to cool, as shown in fig.

A graph is plotted taking time along 'x' axis and temperature along 'y' axis, the rate of cooling for the slab alone (i.e) $\left(\frac{d\theta}{dt}\right)$ is found from graph, as shown in fig.

The rate of cooling is directly proportional to the surface area exposed.



Case (i)

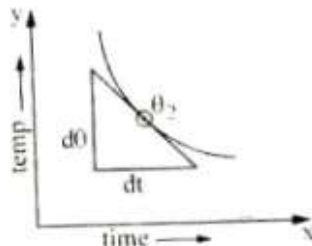
Steam chamber and bad conductor are placed over slab, in which radiation takes place from the bottom surface of area (πr^2) of the slab and the sides of the slab of area $(2\pi r h)$.

$$R_c = 2\pi r^2 + 2\pi r h$$

$$R_c = \pi r(r + 2h) \dots\dots\dots(4)$$

Case (ii)

The heat is radiated by the slab alone, (i.e) from the bottom of area (πr^2) , top surface of the slab of area πr^2 and also through the sides of the slab of area $2\pi r h$.



$$\left(\frac{d\theta}{dt}\right)_{\theta_2} = nr^2 + nr^2 + 2nrh$$

$$\left(\frac{d\theta}{dt}\right)_{\theta_2} = 2nr^2 + 2nrh$$

$$\left(\frac{d\theta}{dt}\right)_{\theta_2} = 2nr(r + h)$$

..... (5)

From eqn.(4) and eqn.(5)

$$\frac{R_c}{\left(\frac{d\theta}{dt}\right)_{\theta_2}} = \frac{nr(r + 2h)}{2nr(r + h)}$$

$$R_c = \frac{(r+2h)}{2(r+h)} \left(\frac{d\theta}{dt}\right)_{\theta_2} \dots\dots\dots (6)$$

Substituting eqn.(6) in eqn.(3) we have

$$K = \frac{ncs \left(\frac{d\theta}{dt}\right)_{\theta_2} (r+2h)}{nr^2(\theta_1 - \theta_2) 2(r+h)} \text{ Wm}^{-1}\text{K}^{-1}$$

Hence, thermal conductivity of the given bad conductor can be determined from the above formula.

Thermal insulation

Thermal insulation is to resist the flow of heat to and from a body. It is a material that reduces the rate of heat flow.

General principles of thermal insulation

1. The thermal resistance of an insulating material is directly proportional to its thickness.
 2. The provision of an air gap is a very important insulating agent.
 3. The thermal resistance of a building depends on its orientation also.
 4. Heat is energy that flows from one region to another because of a difference in temperature between the two regions. The heat is transferred by conduction, convection, radiation or any combination of them.
- (i) Reducing heat transfer by conduction

- In a flat wall made of any solid material, if one face is at a higher temperature than the other, heat will flow through the wall by conduction.
- The rate at which heat will flow through the wall depends on the thermal conductivity of the wall material.
- The higher the thermal conductivity of a material, the poorer its use as thermal insulation.

(ii) Reducing heat by convection

The mode of carrying heat from the warm to the cold side of the air space is called convection, and the air flows are called convection currents.

If the air space within the walls of a house is filled with a porous material, the air circulation will be impeded, and the rate of heat transfer due to convection will be greatly reduced.

(iii) Reducing heat transfer by radiation

- The transfer of heat by radiation does not require the presence of any matter.
- The rate at which heat is transmitted by radiation depends on various factors, including the temperatures of the surfaces and the kinds of surfaces involved.

Thermal insulating materials

The materials which are used to insulate thermally are known as thermal insulating materials.

Thermal insulating materials are classified as

- (a) Organic materials (b) Inorganic materials

(a) Organic materials

Cattle hair, Silk, Wool, Wood-pulp, Corkboard, Saw dust, Sugar-cane fiber, Cardboard, Paper, Leather.

(b) Inorganic insulating materials

Still air, Mineral wool, Slag-wool, Glass wool, Charcoal, Gypsum powder, Slag, Coke powder, Asbestos.

Types of Thermal insulation

There are three types of thermal insulation. They are

1. House thermal insulation
2. Industrial thermal insulation
3. Building thermal insulation

1. House thermal insulation

- In a warm house during the winter, insulation in the walls, ceilings, and floors reduces the loss of heat from the warm interior to the colder outdoor air.

- In a cool house during the summer, the insulation reduces the entry of heat from the warmer out-door air.
- Thermal insulation of house hold items are very essential.

1. Industrial thermal insulation

- In industry, thermal insulation is used for enclosing heating equipment, pipes that carry steam, and cold storage spaces.
- The insulation helps to conserve fuel or power or to maintain a uniform temperature in an enclosure.

2. Building thermal insulation

Thermal insulation of exposed doors and windows

The doors and windows which are exposed to outside world transmit heat to a considerable extent.

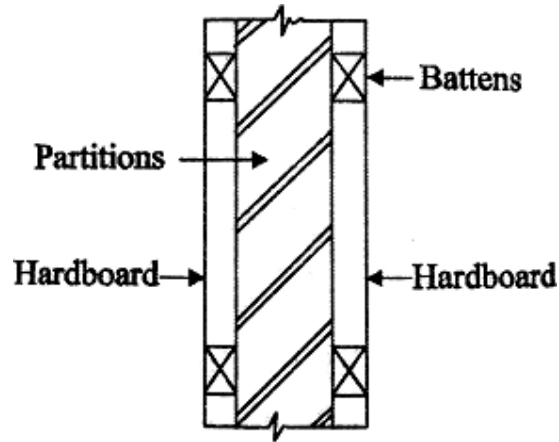
Following methods are used to ensure thermal insulation of the exposed doors and windows.

- The insulating glass or double glass with air space may be provided for glassed doors and windows. This will reduce heat transmission through doors and windows.
- In order to reduce incidence of solar heat, the protection in the form of sun breakers, weather sheds, projections curtains, may be provided on the exposed doors and windows.
- The flat roofs may be kept cool by water which may either be stored or sprayed at regular intervals. The surface temperature of the roof is reduced substantially by this method.
- The thermal insulation of flat roof may be provided by putting a layer of about 25 mm thickness of coconut pith or cement concrete.

Thermal insulation of exposed walls

The following methods are adopted for the thermal insulation of exposed wall.

- The suitable thickness of wall may be provided.
- The hollow wall or cavity wall construction may be adopted.
- For partitions, an air space may be created by fixing hardboards on battens, weather sheds, projections, curtains etc may be provided on the exposed doors and windows.



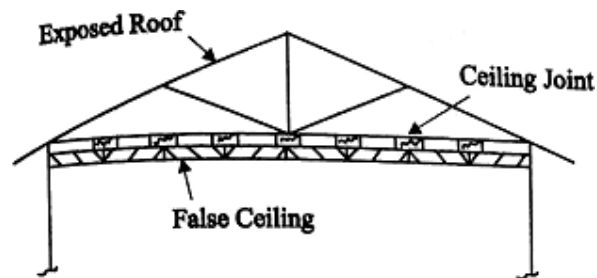
Thermal insulation of partition

Thermal insulation of exposed roofs

The thermal insulation of exposed roofs is achieved either by treating inside surface or outside surface.

Internal treatment:

- The false ceiling with an air gap may be provided. The ceiling is made of thermal insulating materials.
- The light insulating materials may be pasted by suitable adhesives to the inside surfaces of the exposed roof.



Internal Treatment for pitched roof

Heat Exchangers

Heat exchangers are devices used to transfer heat between two or more fluid streams at different temperatures.

Heat exchangers find widespread use in power generation, chemical processing, electronics, cooling, air-conditioning, refrigeration and automotive applications.

Examples for heat exchangers

- (i) Intercoolers and preheaters
- (ii) Condensers and boilers in steam plant
- (iii) Condensers and evaporators in refrigeration units
- (iv) Regenerators
- (v) Automobile radiators
- (vi) Oil coolers of heat engine
- (vii) Milk chiller of a pasteurizing plant

Types of heat exchangers

In order to meet the widely varying applications, several types of heat exchangers are developed. They are classified on the basis of nature of heat exchange process, relative direction of fluid motion, design and constructional features, and physical state of fluids.

1. Nature of heat exchange process

Heat exchangers, on the basis of nature of heat exchange process, are classified as follows:

- (i) Direct contact heat exchangers.
- (ii) Indirect contact heat exchangers
 - (a) Regenerator
 - (b) Recuperators

2. Relative direction of fluid motion

According to the relative directions of fluid motions the heat exchangers are classified into the following three categories:

- (i) Parallel flow or unidirectional flow
- (ii) Counter-flow
- (iii) Cross-flow

3. Design and constructional features

Based on design and constructional features, heat exchangers are classified as

- (i) Concentric tubes
- (ii) Shell and tube
- (iii) Multiple shell and tube passes
- (iv) Compact heat exchanger

4. Physical state of fluids

Based on the physical state of fluids, heat exchangers are classified as

- (i) Condensers
- (ii) Evaporators

Direct contact heat exchanger

In a direct contact or open heat exchanger, the exchange of heat takes place by direct mixing of hot and cold fluids and transfer of heat and mass takes place simultaneously.

Indirect contact heat exchanger

In this type of heat exchanger, the heat transfer between two fluids could be carried out by transmission through wall which separates the two fluids.

This type includes the following:

(a) Regenerators

In a regenerator type of heat exchanger the hot and cold fluids pass alternately through a space containing solid particles, these particles providing alternately a sink and a source for heat flow.

Examples

- (i) I.C. engines and gas turbines
- (ii) Open hearth and glass melting furnaces
- (iii) Air heaters or blast furnaces

(b) Recuperators

Recuperator is the most important type of heat exchanger in which the flowing fluids exchanging heat are on either side of dividing wall. These heat exchangers are used when two fluids cannot be allowed to mix.

Examples

- (i) Automobile radiators
- (ii) Oil coolers, inter coolers, air preheaters
- (iii) Milk chiller of pasteurizing plant
- (iv) Evaporator of an ice plant.

Refrigeration

Definitions:

Refrigeration: It is the process of continuous cooling or extraction of heat to below that of the atmosphere from a substance with a help of the external work.

Principle of refrigeration: It is based on the second law of thermodynamics that the heat can made to flow from cold body to a hot body with the help of external source.

Refrigerant: It is the working fluid used in the refrigerator. Eg. Ammonia, Methyl chloride.

Ton of refrigeration:

It is the standard unit of the refrigerator. It is the freezing capacity of the one American short ton of water from 0°C in 24 hrs.

One ton of the refrigeration = 3.5 kW = 3.5 kJ/s = 210 kJ/min.

Coefficient of performance: It is the measure of the performance of the refrigeration system. It is the ratio of the refrigerating effect to the input work required to produce the effect.

The value of the coefficient of performance may be less than the unity or greater than unity.

Relative Coefficient of performance: It is the ratio of the actual to the theoretical coefficient of performance.

Refrigerator

It is a machine which produces cold. It is used to remove heat from the refrigerated space and reject it to atmosphere. Hence, it maintains the temperature below the surrounding atmosphere.

Principle

A refrigerator works by passing a cool refrigerant gas around food items, which absorbs heat from them and then loses that heat to the relatively cooler surroundings on the outside.

Types of refrigeration systems

Mainly they are classified under two groups:

1. Based on the type of the external work

Using mechanical energy:

- (a) Cold air refrigeration
- (b) Vapors compression refrigeration

Using heat energy:

- (a) Simple vapour absorption refrigeration
- (b) Actual vapour absorption refrigeration

2. Based on the usage

- (a) Primary refrigeration
- (b) Secondary refrigeration

Parts and working of vapour compression refrigerator

A refrigerator consists of a few key components that play a vital role in the refrigeration process.

They are

- 1. Expansion valve
- 2. Compressor
- 3. Evaporator
- 4. Condenser
- 5. Refrigerant

Expansion valve

Also referred to as the flow control device, an expansion valve controls the flow of the liquid refrigerant into the evaporator. It is actually a very small device that is sensitive to temperature changes of the refrigerant.

Compressor

The compressor consists of a motor that sucks in the refrigerant from the evaporator and compresses it in a cylinder to make a hot, high-pressure gas.

Evaporator

This is the part that actually cools the stuff kept inside a refrigerator. It consists of finned tube that absorbs heat blown through a coil by a fan. The evaporator absorbs heat from the stuff kept inside, and as a result of this heat, the liquid refrigerant turns into vapor.

Condenser

The condenser consists of a coiled set of tubes with external fins and is located at the rear of the refrigerator. It helps in the liquefaction of the gaseous refrigerant by absorbing its heat and subsequently expelling it to the surroundings.

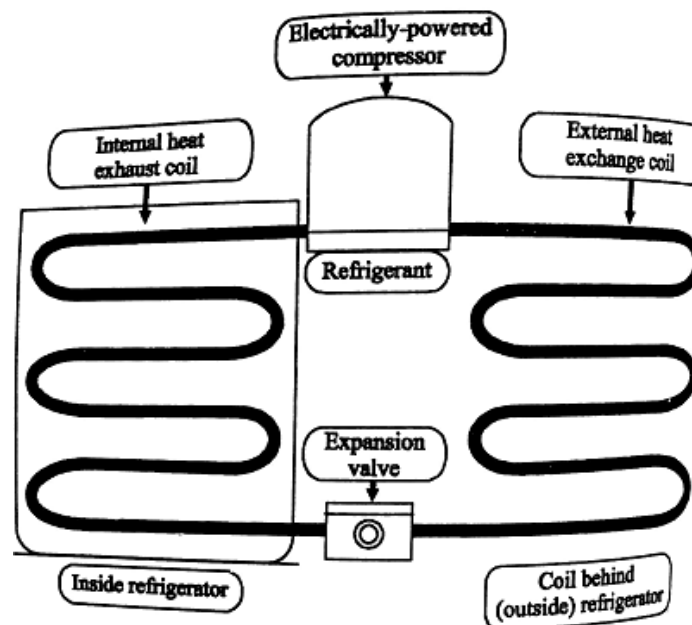
As the heat of the refrigerant is removed, its temperature drops to condensation temperature, and it changes its state from vapor to liquid.

Refrigerant

Also commonly referred to as the coolant, it's the liquid that keeps the refrigeration cycle going.

It's actually a specially designed chemical that is capable of alternating between being a hot gas and a cool liquid.

The refrigerants such as ammonia, R-290, R-600A are used.



Working

The refrigerant, which is now in a liquid state, passes through the expansion valve and turns into a cool gas due to the sudden drop in pressure.

As the cool refrigerant gas flows through the chiller cabinet, it absorbs the heat from the food items inside the fridge and vaporizes.

The refrigerant, which is now a gas, flows into the compressor, which sucks it inside and compresses the molecules together to make it into a hot, high-pressure gas.

Now, this gas transports to the condenser coils, where the coils help dissipate its heat so that it becomes cool enough to condense and convert back into its liquid phase.

After the condenser, the liquid refrigerant travels back to the expansion valve, where it experiences a pressure drop and once again becomes a cool gas. It then absorbs heat from the contents of the fridge and the whole cycle repeats.

Oven

An oven is a thermally insulated chamber used for heating, baking or drying of a substance and most commonly used for cooking

Kilns and furnaces are special purpose ovens, used in pottery and metalworking respectively.

Types of ovens

- Double oven
- Ceramic oven
- Gas oven
- Microwave oven
- Toaster oven
- Hot air oven

Hot Air Oven

The electrical device which is widely used in medical products industries, rubber industries, and for the process of sterilization using dry heat is known as hot air oven.

The instrument works on the basis of dry heat to sterilize the specimens and articles. The instrument can be operated at a temperature of 50°C to 250°C.

It is also known as the thermostat that controls the temperature. The device is provided with the digital panel to control the temperature digitally.

Operating Principle

It works on the principle of fine gravity air convection in a highly heated electrical chamber.

Description

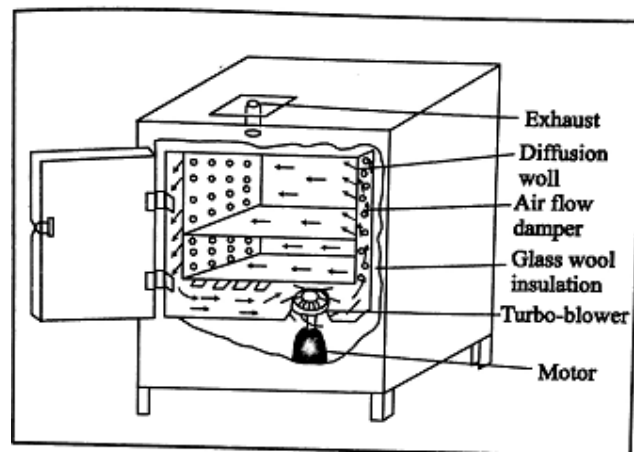
It consists of the following parts:

- An insulated chamber surrounded by an outer case containing electric heaters.
- A fan to circulate the air
- Shelves
- Thermocouples
- Temperature sensor
- Door locking controls

The apparatus consists of a large, rectangular, copper base and covered with asbestos sheets. It is also provided with a door and erected on a four-legged stand.

The roof is provided with a hole through which a thermometer is fitted inside for recording of temperature.

Dry of the instrument. There is a regulator of heater to control the inside temperature.



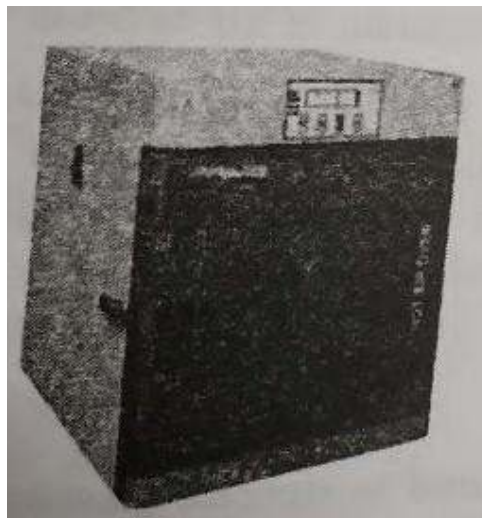
Hot air oven

Working

Before sterilization, the glassware are dried properly and wrapped in brown paper and then exposed to hot air inside the oven.

After loading of glassware, the oven is switched on, the temperature will increase slowly up to the desired point (160°C) where it will remain steady.

Then at 160°C, the oven is kept for an hour. This is the appropriate temperature for sterilization of glassware. Then, gradually the temperature is brought down and thereafter sterilization is complete.



Hot air oven

Advantages

- This treatment kills the bacterial endotoxin, not all treatments can do this.
- Dry heat sterilization by hot air oven does not leave any chemical residue.
- Eliminates 'wet pack' problems in humid climates.

Disadvantages

- Plastic and rubber items cannot be dry-heat sterilized because temperatures used (160°C - 170°C) are too high for these materials.
- Dry heat penetrates materials slowly and unevenly.
- It requires a continuous source of electricity.

Safety Guidelines

- Before placing in Hot Air Oven
 - (i) Dry glasswares completely.
 - (ii) Plug test tubes with cotton wools.
 - (iii) Wrap glasswares in kraft papers. Do not overload the oven. Overloading alters heat convection and increases the time required to sterilize
- Allow free circulation of air between the materials
- The material used for wrapping instruments and other items must be porous enough to let steam through but tightly woven enough to protect against dust particles and microorganisms.

Applications

- It is widely used to sterilize glassware in pharmaceutical industries such as petri dishes, pipettes, bottles, test tubes, flasks, pestle, etc.

Solar Power

The energy obtained from the sun is called solar energy. Sun is the source of all energy. Sunlight contains infrared radiations in large proportion, and these infrared rays heats all objects on which they fall.

Every square metre of earth's upper atmosphere receives 1.36 kJ of energy per second. However, of this only 47%, i.e., 0.64 kJ of solar energy reaches every square metre of earth's surface per second.

Harnessing solar energy

The solar energy falling on the earth is very much diffused, and scattered. In order to use solar energy for practical purposes, we have to collect and concentrate it.

The solar energy is harnessed by using the following two ways:

1. Direct harnessing
2. Indirect harnessing

1. Direct harnessing

The solar energy is either directly collected as heat or converted directly into electricity.

2. Indirect harnessing

Indirect harnessing is carried out by

- Converting solar energy into chemical energy in plants
- Harnessing the energy of wind
- Utilizing the energy of sea waves and
- Utilizing the energy due to the temperature difference of the water at different levels in oceans.

Conversion of Solar energy

Solar energy is converted into more useful forms of energies by two ways.

1. Thermal conversion
2. Photo-conversion

Thermal conversion

In thermal conversion, the heat from the direct rays of Sun is absorbed in the form of infrared radiation by surfaces, air or water and put to many uses.

Photo conversion

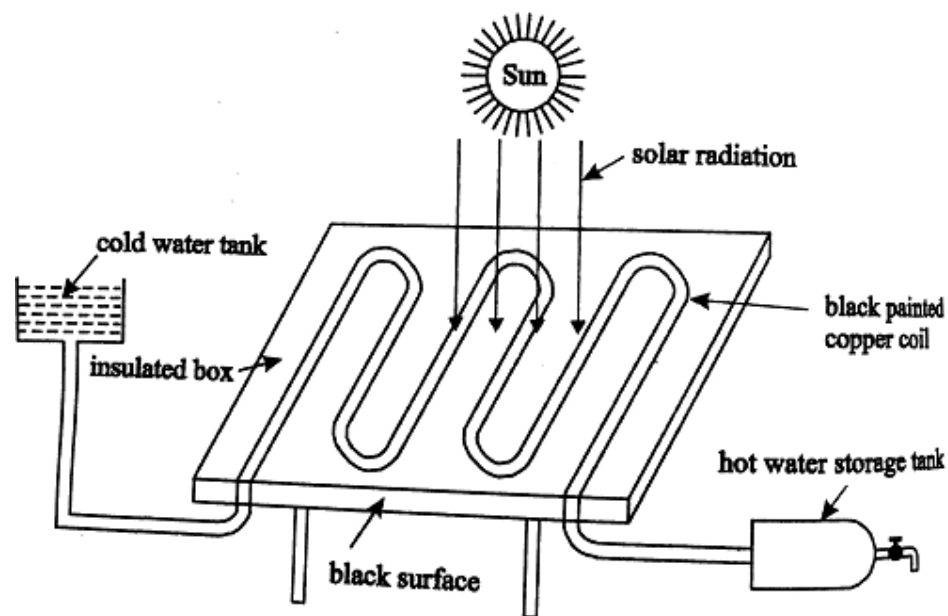
In photo conversion, when sunlight of short wavelength falls on the surface of a metal, it is absorbed and it is used to excite and eject electrons from the surface. This is known as photoelectric effect.

The ejected electrons move in a circuit in a direction opposite to that of conventional current and generate electric current. The photovoltaic or solar cells are designed based on this principle.

Solar Water Heater

It consists of an insulated box painted black from inside having a glass lid to receive and store solar heat as shown in figure. Inside the box it has black painted copper tube coil through which cold water is made to flow in.

Water gets heated and flows out into a storage tank. The hot water from the storage tank fitted on roof top is then supplied through pipes into buildings.



Solar water heater

Applications of Solar Power

- Traditional uses of solar energy
Drying clothes, drying fruits and vegetables, reducing moisture content in grains, and making salt from sea-water.
- Solar cells

Used in calculators, electronic watches, street lights, water pumps, radios and TVs.

- **Solar Battery**

When a large number of solar cells are connected in series it forms a solar battery. It produces more electricity which is enough to run water pump.

- **Solar heat collector**

It is generally used in cold places, where houses are kept in hot condition using solar heat collectors.

- It is also helpful in cooking using solar cookers or to warm up water using solar water heaters.

Advantages of Solar Power

- Solar energy systems are maintenance free and will last for decades.
- Once installed, there are no recurring costs.

Environment friendly

- Its not affected by the supply and demand of fuel. Therefore, it is not subjected to the ever-increasing price of gasoline.
- Solar energy is clean, renewable.
- It does not pollute air
- It does not contribute to the cost and problems of the recovery and transportation of fuel or the storage of radioactive waste.

Disadvantages of Solar Power

- The initial cost of purchasing and installing solar panels always is high.
- Solar energy is not available round the clock.
- Available solar energy is diffused.
- Energy has not be stored in batteries
- Air pollution and whether can affect the production of electricity.
- They need large area of land to produce more efficient power supply.

UNIT - IV

QUANTUM PHYSICS

Introduction:

We know classical theory is a macroscopic theory. It fails to explain the micro-concepts like stability of atoms, black body radiation, photo-electric effect, Compton effect. Also, according to classical theory, the hydrogen spectrum is a continuous spectrum, but the actual experimentally observed spectrum is found to be discrete. Thus to avoid these discrepancies a new microscopic theory called Quantum theory was introduced by Max Planck and the same was developed by others.

BLACK BODY RADIATION

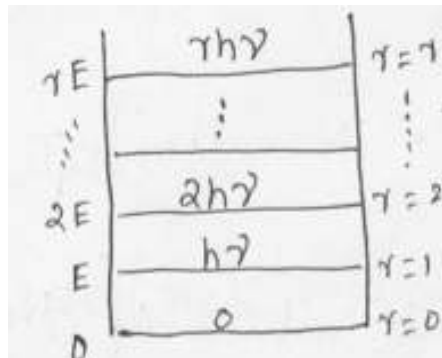
Perfect black body: A perfect black body is the one which absorbs and emits all the radiation, (corresponding to all wavelengths) that fall on it. The radiation, given out by a perfect black body is called black body radiation.

Planck's quantum theory

Assumption:

- Black Body contains electrons or so called simple harmonic oscillators, capable of vibrating with all possible frequencies.
- Frequency of emitted radiation is same as that of the frequency of its vibration.
- Electrons radiate energy in discrete manner and not continuous.
- Oscillator exchanges energy in the form of absorption or emission in terms of quanta of $h\nu$.

$$E = nh\nu$$



Planck's Radiation Law:

- Consider 'N' of oscillators with total energy as E_T

$$\text{Average Energy } E = (E_T/N) \text{ -----(1)}$$

Total no. of oscillations:

$$N = N_0 + N_1 + N_2 + \dots + N_r \text{-----(2)}$$

Total energy of oscillators

$$E_T = 0N_0 + EN_1 + 2EN_2 + \dots + rEN_r \text{-----(3)}$$

➤ According to Maxwell's distribution,

$$N_r = N_0 e^{-rE/K_B T} \text{-----(4)}$$

Example:-

$$y=0; N=N_0$$

$$y=1; N_1=N_0 e^{-E/K_B T}$$

$$y=2; N_2=N_0 e^{-2E/K_B T}$$

$$y=y; N_r=N_0 e^{-rE/K_B T}$$

By substituting these values in (2)

$$N = N_0 e^0 + N_0 e^{-E/K_B T} + N_0 e^{-2E/K_B T} + \dots + N_0 e^{-rE/K_B T}$$

$$N = N_0 [1 + e^{-E/K_B T} + e^{-2E/K_B T} + \dots + e^{-rE/K_B T}] \text{-----(5)}$$

The summation series

$$1 + x + x^2 + x^3 + \dots = \frac{1}{1-x}$$

Therefore

$$N = N_0 \left[\frac{1}{1 - e^{-E/K_B T}} \right] \text{-----(6)}$$

Substitute values of $N_0, N_1, N_2, \dots, N_r$ in equation (3)

$$E_T = 0N_0 e^{-E/K_B T} + EN_0 e^{-E/K_B T} + 2EN_0 e^{-2E/K_B T} + \dots + yEN_0 e^{-rE/K_B T}$$

$$E_T = N_0 E e^{-E/K_B T} [0 + 1 + 2e^{-E/K_B T} + 3e^{-2E/K_B T} + \dots + ye^{-(r-1)E/K_B T}] \text{-----(7)}$$

The summation series

$$1+2x+3x^2+\dots+yx^{r-1} = \frac{1}{(1-s)^2}$$

$$E_T = N_0 e^{-E/K_B T} \left[\frac{1}{(1-e^{-E/K_B T})^2} \right] \quad \text{----- (8)}$$

Substitute both the values in E

$$E = \frac{E_T}{N} = \left\{ \frac{N_0 e^{-E/K_B T}}{(1-e^{-E/K_B T})^2} \right\} \times \left\{ \frac{1}{N_0 \left(\frac{1}{1-e^{-E/K_B T}} \right)} \right\}$$

$$E = \frac{E e^{-E/K_B T}}{1-e^{-E/K_B T}}$$

(or)

$$E = \frac{E}{\frac{1-e^{-E/K_B T}}{e^{-E/K_B T}}} \Rightarrow E = \frac{hy}{e^{E/K_B T} - 1} \quad \text{----- (9)}$$

Equation (9) represents Average Energy of Oscillator.

➤ The number of oscillations per unit volume within range of frequency γ to $\gamma + d\gamma$ is given by

$$N = \frac{8\pi\nu^2}{c^3} d\nu \quad \text{----- (9a)}$$

Energy Density or (Number of oscillations x Average Energy of an oscillator)
 Total Energy per unit = per unit volume)
 volume

$$E_y d\nu = NE \quad \text{----- (10)}$$

Substituting (9) and (9a) in (10)

$$E_y d\nu = \frac{8\pi\nu^2}{c^3} d\nu \frac{hy}{e^{hy/K_B T}} \quad \text{----- (11)}$$

(or)

$$E_y = \frac{8\pi h^3 \nu^3}{c^3 (e^{hy/K_B T} - 1)} \quad \text{----- (12)}$$

Equation (12) represents Planck's Radiation Law in terms of Frequency.

➤ Same can be written in terms of wavelength by substituting $\nu = \frac{hc}{\lambda}$

Wien's Displacement Law and Rayleigh Law from Planck's Theory

1) Wien's Displacement Law :-

➤ Holds good only for shorter wavelength.

➤ $h \ll \lambda$; $\lambda \gg 1$ Therefore $\frac{hc}{\lambda k_B T} \gg 1$

So,

$$e^{hc/\lambda k_B T} - 1 \approx e^{hc/\lambda k_B T}$$

$$\text{Therefore } E_h = \frac{8\pi hc}{h^5 (e^{hc/\lambda k_B T})}$$

(or)

$$E_h = 8\pi hc h^{-5} e^{-hc/\lambda k_B T}$$

(or)

$$E_h = C_1 h^{-5} e^{-c_2/hT}$$

$$C_1 = 8\pi hc \quad \text{and } C_2 = \frac{hc}{k_B}$$

$$\text{As } T \propto \frac{1}{\lambda} \quad \text{and } E_h \propto h^{-5}$$

$$E_h \propto T^5$$

Rayleigh Jeans law:

- Rayleigh Jeans law holds good for longer wavelength
- If λ is greater, $(1/\lambda)$ will be lesser

$$e^{\frac{hc}{\lambda k_B T}} = 1 + \frac{hc}{\lambda k_B T} + \left(\frac{hc}{\lambda k_B T}\right)^2 + \dots$$

For large values of λ the value of $\left(\frac{hc}{\lambda k_B T}\right)^2 + \dots$ will be very small and hence the higher terms can be neglected

$$e^{\frac{hc}{\lambda k_B T}} = 1 + \frac{hc}{\lambda k_B T}$$

3.2 COMPTON EFFECT:

When a beam of monochromatic light radiation of high frequency is allowed to fall on particle, beam is scattered into

- (1) One having same frequency that of incident radiation called unmodified.
- (2) other having lower frequency or higher λ compared to incident radiation called modified

This effect is called Compton Effect. The shift in λ is called Compton shift.

Theory:

Principle:

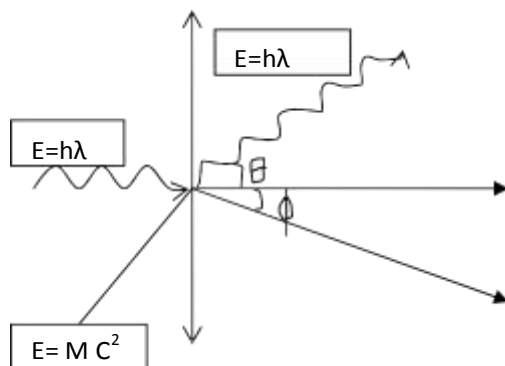
- Collision between photon and electron is considered.
- Laws of conservation of energy and momentum is applied to get Compton λ .

Assumptions:

- Collision occurs between photon and electron in scattering material.
- The electron is free and is at rest before collision.

Derivation of Compton λ :

- During collision photon and electron part of energy is given to electron.
- There is \uparrow in kinetic energy of electron and recoils at an angle ϕ .
- Scattering photons move s with an energy $h\lambda'$ at an angle θ with respect to original direction.



Energy:

(1) Before collision:

(a) e of incident photon = $h\gamma$

(b) rest energy of electron = m_0c^2

Total energy before collision = $h\gamma + m_0c^2$ (1)

(2) after collision:

(a) energy of scattered photon = $h\gamma'$

(b) Energy of scattered photon = $h\gamma' + mc^2$ (2)

By law of conservation of energy,

(1) = (2)

$$h\gamma + m_0c^2 = h\gamma' + mc^2$$
 (3)

(i) MOMENTUM :-

(a) X component : (before collision)

Incident photon = $\frac{h\gamma}{c}$

$e^- = 0$

Total X component of Momentum before collision = $\frac{h\gamma}{c}$ (4)

(i) After collision:-

$\cos \theta (\Delta OAB) = \frac{M_x}{h\gamma'/c}$ (or)

X component of Scattered photon = $\frac{h\gamma'}{c} \cos \theta$.

(ii) Recoil e^- :-

$\Delta OBC \quad \cos \phi = \frac{M_x}{mv}$

X component of recoil $e^- = mv \cos \phi$.

Total X component of = $\frac{h\gamma'}{c} \cos \theta + mv \cos \phi$

Momentum after collision ----- (5)

By law of conservation of Momentum,

(4) = (5)

$$\frac{h\gamma}{c} = \frac{h\gamma'}{c} \cos \theta + mv \cos \phi$$
 (6)

(b) Y component:-

(i) Before collision

Incident photon 0
 e^- at rest : 0.

Total Y component before collision= 0. ----- (7)

(ii) After collision:-

- Scattered Photon : $\Delta OAE \sin \theta = \frac{M_y}{h\gamma'/c}$

$$M_y = h\gamma'/c \sin \theta.$$

- Recoil e^- : $\Delta OCD \sin \Phi = \frac{-M_y}{mv}$

$$M_y = -mv \sin \Phi.$$

Total Y component after collision: $\frac{h\gamma'}{c} \sin \theta - mv \sin \Phi$ ----- (8)

Again (7) = (8) :

$$\frac{h\gamma'}{c} \sin \theta - mv \sin \Phi = 0 \text{----- (9)}$$

From (6) :

$$\frac{h\gamma}{c} - \frac{h\gamma'}{c} \cos \theta = mv \cos \Phi$$

$$m c v \cos \Phi = h(\gamma - \gamma' \cos \theta) \text{----- (10)}$$

From (9) :

$$m c v \sin \Phi = h\gamma' \sin \theta \text{----- (11)}$$

Squaring & adding both (10) & (11) ;

$$M^2 c^2 v^2 = h^2[\gamma^2 + \gamma'^2 \cos^2 \theta - 2\gamma\gamma' \cos \theta] + h^2 \gamma'^2 \sin^2 \theta .$$

$$m^2 c^2 v^2 = h^2[\gamma^2 - \gamma'^2 - 2\gamma\gamma' \cos \theta] \text{----- (12)}$$

From (3)

$$m c^2 = m_0 c^2 + h(\gamma - \gamma') .$$

Squaring both sides...

$$m^2 c^4 = m_0^2 c^4 + h^2(\gamma^2 + \gamma'^2 - 2\gamma\gamma' + 2hm_0 c^2(\gamma - \gamma')) \text{----- (13)}$$

Sub (12) from (13) [hint: open brackets with h^2 in (12) & (13)]

$$m^2 c^2 [c^2 - v^2] = m_0^2 c^4 + 2hm_0 c^2(\gamma - \gamma') - 2h^2 \gamma\gamma' (1 - \cos \theta) \text{----- (14)}$$

From theory of relativity:

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} = m^2 = m_0^2 / (1 - v^2/c^2)$$

$$m^2 c^2 (c^2 - v^2) = m_0^2 c^4 \quad \text{---(15)}$$

Equating R.H.S. of (15) & (14);

$$m^2 c^4 = m_0^2 c^4 + 2hm_0 c^2 (\gamma - \gamma') - 2h^2 \gamma \gamma' (1 - \cos\theta)$$

$$\gamma - \gamma' / \gamma \gamma' = h / m_0 c^2 (1 - \cos\theta)$$

(or)

$$1/\gamma' - 1/\gamma = h / m_0 c^2 (1 - \cos\theta) \quad \text{---(16)}$$

(or)

$$c/\gamma' - c/\gamma = h / m_0 c (1 - \cos\theta)$$

$$\lambda' - \lambda = h / m_0 c (1 - \cos\theta) \quad \text{---(17)}$$

$$\Delta\lambda = h / m_0 c (1 - \cos\theta) \quad \text{---(18)}$$

(18) represents Compton shift....

Special Cases :-

(i) $\Theta = 0$; $\cos\theta = 1$.

$$\Delta\lambda = 0$$

- No scattering at all, out coming radiation is same as that of incoming.
- A single peak is noted in experiment

Case 2:

$$\Theta = 90$$
 ; $\cos\theta = 0$

$$\Delta\lambda = h / m_0 c \quad \text{(or)} \quad \Delta\lambda = 0.02424 \text{ \AA} .$$

- This is called COMPTON WAVELENGTH.
- Has good agreement with experimental results.

Case 3:

$$\Theta = 180 \quad \cos\theta = -1.$$

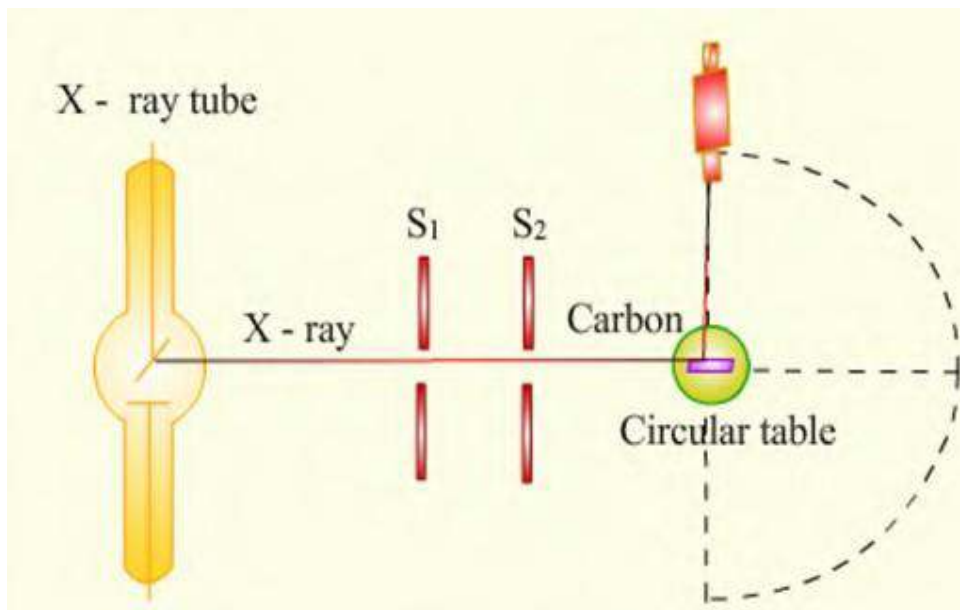
$$\Delta\lambda = 2h / m_0 c .$$

- This shift in wavelength is maximum.

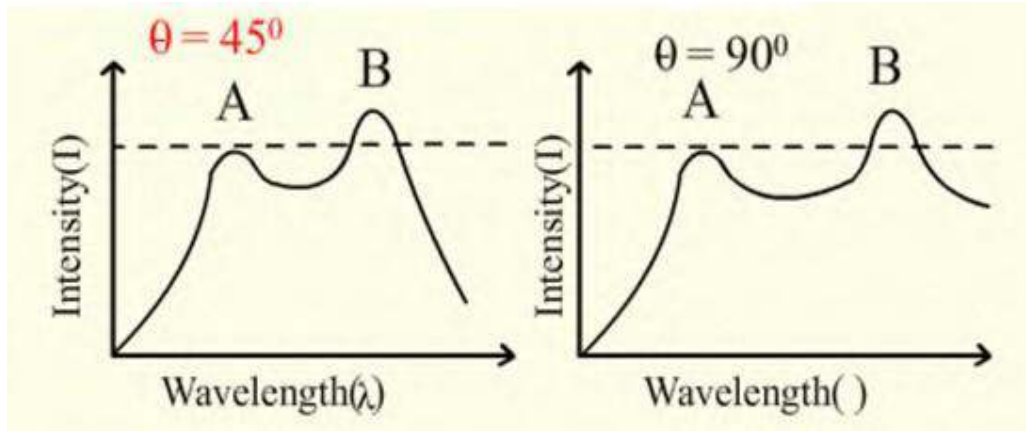
$$\Delta\lambda = 0.04848 \text{ \AA} .$$

EXPERIMENTAL VERIFICATION:-**PRINCIPLE: Compton shift.****CONSTRUCTION:-**

- it consists of X ray tube for producing X rays.
- small block of carbon C is mounted on circular table.
- brag spectrometer (Bs)freely swings in arc about scattering Element C.
- to catch scattered e^- .
- slits S_1 & S_2 helps to focus X rays to scattering element.

**WORKING:-**

- X rays are produced by X ray tube.
- passes through S_1 & S_2 scattering element.
- falls on the scattering element
- scattered X rays are received by Bragg spectrometer.
- λ is calculated & results are plotted.



- matches with theory results.
 1. $\theta = 0^\circ \Rightarrow$ single peaks
 2. $\Delta\lambda \uparrow$ as $\theta \uparrow$.
 3. At $\theta = 90^\circ \Rightarrow \Delta\lambda = 0.0236$
 ≈ 0.02424 (theoretical)
- proves Compton Effect.

DUAL NATURE OF LIGHT:-

- light exhibits dual nature namely particle & wave nature.
- wavelength associated with wave nature is called DE-BROGLIE wavelength.
- wavelength associated with material is called matter waves.

De-Broglie Wavelength:

- (i) $E = mc^2$ -----(1) [particle nature]
- (ii) $E = h\nu$ -----(2) [wave nature]

$$E = mc^2 = h\nu \text{ -----(3)}$$

- (iii) Momentum $p = mc$

$$pc = mc^2$$

Equating to (3)

$$h\nu = pc$$

$$p = \frac{h\nu}{c} = \frac{h}{\lambda}$$

(Or) De-Broglie Wavelength------(5)

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Other forms of de-Broglie wavelength:

In terms of Energy:

$$E = \frac{1}{2} mv^2$$

$$Em = \frac{1}{2} m^2 v^2$$

$$mv = \sqrt{2En}$$

(ii) Interms

$$\lambda = \frac{h}{\sqrt{2mE}}$$

K.E = $\frac{1}{2} mv^2$

$$P.E = eV$$

$$\frac{1}{2} mv^2 = eV \text{ (or) } m^2 v^2 = 2meV$$

$$mV = \sqrt{2neV}$$

$$\lambda = \frac{h}{\sqrt{2meV}}$$

(iii) Interms of temperature:

$$E = \frac{1}{2} mv^2 = \frac{3}{2} K_B T \text{ (by thermodynamics)}$$

$$\frac{1}{2} m^2 v^2 = \frac{3}{2} m K_B T$$

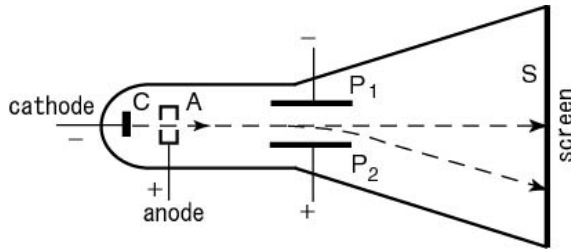
$$mv = \sqrt{3m K_B T}$$

$$\lambda = \frac{h}{mv}$$

$$\Rightarrow \lambda = \frac{h}{\sqrt{3m K_B T}}$$

.THOMSON EXPERIMENT

- G.P.Thomson studied electron diffraction effects using accelerated electrons.
- Found the diffraction pattern exactly analogous to X-ray patterns.

**Experimental arrangement**

- Shown in the figure.
- Consists of a discharge tube in which the electrons are produced from the cathode C.
- The electron beam is excited with potential up to 50,000 volts.
- The electron rays are passed through a slit S to obtain a fine beam of electrons.
- They are allowed to fall upon a very thin metallic film G of gold foil.
- The thickness of the film is of the order of 10^{-6} cm.
- The whole apparatus is exhausted to a high vacuum so that the electrons may not lose their energy in collision with the molecules of the gas.

Working

- The photograph of the incoming beam from the foil is recorded by the photographic plate
- Systematic pattern consisting of concentric rings about the central spot is obtained.
- Diffraction pattern can only be produced by waves and not particles
- So it is concluded that electrons behave like waves

SCHROEDINGER WAVE EQUATIONS:**(i) Time Dependent:**

- Particle behaves like wave only when it moves.

-To move, it has to be accelerated by a potential field.

$$\text{Total Energy } E = \text{K.E} + \text{P.E}$$

$$E = V + \frac{1}{2} mv^2$$

$$E = V + \frac{1}{2} \frac{p^2}{m} \Rightarrow$$

$$E = V + \frac{p^2}{2m}$$

Operating on wave function,

$$E \psi = V \psi + \frac{p^2}{2m} \psi \text{----- (1)}$$

According to classical mechanics, displacement is

$$Y = Ae^{-i\omega(t - \frac{x}{v})}$$

Similarly in quantum mechanics,

$\psi(x, y, z, t)$ = position

$$\psi(x, y, z, t) = Ae^{-i\omega(t - \frac{x}{v})} \text{----- (2)}$$

$$m = 2\pi\gamma$$

$$\Psi = Ae^{-2\pi i(\gamma t - \frac{v\gamma x}{v})} \text{----- (3)}$$

$$E = h\nu \text{ (or) } \gamma = \frac{E}{h} \quad \text{and} \quad \frac{v}{v} = \frac{1}{h}$$

(4) Becomes,

$$\psi = Ae^{-2\pi i \left(\frac{Et}{h} - \frac{px}{h} \right)}$$

$$\lambda = \frac{h}{p}$$

$$\psi = Ae^{-2\pi i \left(\frac{Et}{h} - \frac{px}{h} \right)}$$

$$\hbar = \frac{h}{2\pi} \Rightarrow \psi = Ae^{-i/\hbar (Et - px)} \text{----- (5)}$$

Diff (5) w.r.t. x

$$\frac{\partial \psi}{\partial x} = Ae^{-i/\hbar (Et - px)} \left(\frac{ip}{\hbar} \right)$$

$$\text{(Or) } \frac{\partial^2 \psi}{\partial x^2} = \psi \left(-\frac{p^2}{\hbar^2} \right)$$

$$\text{(Or) } p^2 \psi = -\hbar^2 \frac{\partial^2 \psi}{\partial x^2} \text{----- (6)}$$

Diff (5) w.r.t time

$$\frac{\partial \psi}{\partial t} = Ae^{-i/\hbar (Et - px)} \left(\frac{-iE}{\hbar} \right)$$

$$-i\hbar \frac{\partial \psi}{\partial t} = E \psi \text{ (or) } E \psi = i\hbar \frac{\partial \psi}{\partial t} \text{----- (7)}$$

Sub (6) & (7) in (1)

$$i\hbar \frac{\partial \psi}{\partial t} = \left[V - \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \right] \psi \text{----- (8)}$$

Equ. (8) Represents one dimensional Schrodinger Time Dependant Wave Equation:

Turning (8) to 3D

Turning (8) to 3D

$$i\hbar \frac{\partial \Psi}{\partial t} = \left[V - \frac{\hbar^2}{2m} \nabla^2 \right] \Psi$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

(Or)

$$E \Psi = H \Psi$$

Where,

$$E = i\hbar \frac{\partial}{\partial t}$$

$$H = \left[V - \frac{\hbar^2}{2m} \nabla^2 \right]$$

SCHROEDINGER TIME INDEPENDANT WAVE EQUATION:

$$\Psi = A e^{-i/\hbar (Et-px)} \quad \text{(or)} \quad \Psi = A e^{-\frac{iEt}{\hbar} + \frac{ipx}{\hbar}} \dots \dots \dots (1a)$$

Can be represented as,

$$\Psi = A \Psi' e^{-\frac{iEt}{\hbar}} \dots \dots \dots (1) [\Psi' - \text{time independent factor}]$$

Diff (1) w.r.t 't'

$$\frac{\partial \Psi}{\partial t} = A \Psi' e^{-\frac{iEt}{\hbar}} \left(-\frac{iE}{\hbar} \right) \dots \dots \dots (2)$$

Diff (1) w.r.t 'x'

$$\frac{\partial \Psi}{\partial x} = A e^{-\frac{iEt}{\hbar}} \left(\frac{\partial \Psi'}{\partial x} \right)$$

$$\frac{\partial^2 \Psi}{\partial x^2} = A e^{-\frac{iEt}{\hbar}} \left(\frac{\partial^2 \Psi'}{\partial x^2} \right) \dots \dots \dots (3)$$

Schrodinger time dep. wave equ. :

$$i\hbar \frac{\partial \Psi}{\partial t} = V \Psi - \frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} \dots \dots \dots (4)$$

Sub (1), (2) & (3) in (4)

$$i \hbar \left(\frac{-i E}{\hbar} \right) \Psi = V \Psi - \frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2}$$

$$E \psi - V \psi = -\frac{\hbar^2}{2m} \nabla^2 \psi$$

(or)

$$\frac{\hbar^2}{2m} \nabla^2 \psi + (E - V) \psi = 0 \text{ -----5}$$

Taking $\psi = \psi$ just symbols

equation 5 is 1D schrodinger time independent wave equation.

Writing in 3D,

$$\nabla^2 \psi + \frac{2m}{\hbar^2} (E - V) \psi = 0 \text{ -----6}$$

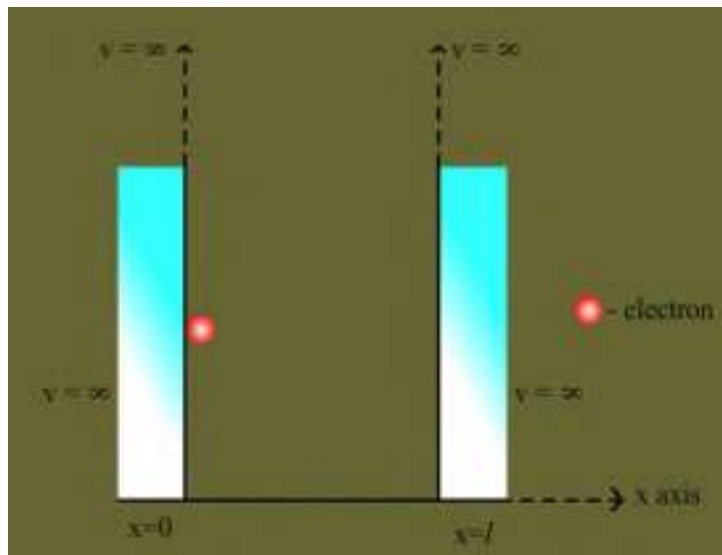
The above equation is time independent Schrodinger wave equation.

Physical significance of ψ :

Gives info about particle behavior $|\psi|^2 = \psi \psi^*$ – probability density probability of finding the particle.

PARTICLE IN A 1D POTENTIAL BOX:

- Consider a e^- with mass m moving along x axis enclosed in 1D box.
- As walls are of infinite potential, particle cannot possible.
- Length of box : l
- Outside on the wall : $v = \infty$
- Inside the wall : $v = 0$



Boundary conditions:

$$V(x) = 0, \quad 0 < x < 1$$

$$V(x) = \alpha, \quad 0 \geq x \geq 1$$

1D SWE:

$$\frac{d^2 \mathbf{f}}{d s^2} + \frac{2n}{\hbar^2} (E - V) \mathbf{f} = 0$$

For a free particle, $V = 0$

$$\frac{d^2 \mathbf{f}}{d s^2} + \frac{2n}{\hbar^2} E \mathbf{f} = 0$$

$$\frac{d^2 \mathbf{f}}{d s^2} + k^2 \mathbf{f} = 0 \text{ ----- 1}$$

Solution to 1:

$$\Psi(x) = A \sin(kx) + b \cos(kx) \text{ ----- 2}$$

Applying boundary conditions:

I. At $x=0$; $\Psi=0$

Particle cannot exist $\Psi=0$

Eqn 3 becomes

$$0 = A \sin 0 + B \cos 0$$

$$B = 0$$

II. At $x=1$, $\Psi = \alpha$ ($\Psi=0$)

$$A \sin k_1 = 0$$

A not equal to zero then $\sin k_1 = 0$

$$k_1 = n\pi \text{ (or) } k = \frac{n\pi}{1}$$

$$\Psi(x) = A \sin_1 \frac{n\pi x}{1}$$

ENERGY OF THE PARTICLE

$$\text{From, } K^2 = \frac{2nE}{\hbar^2}$$

$$n^2 \pi^2 = \frac{2nE_4 n^2}{\hbar^2}$$

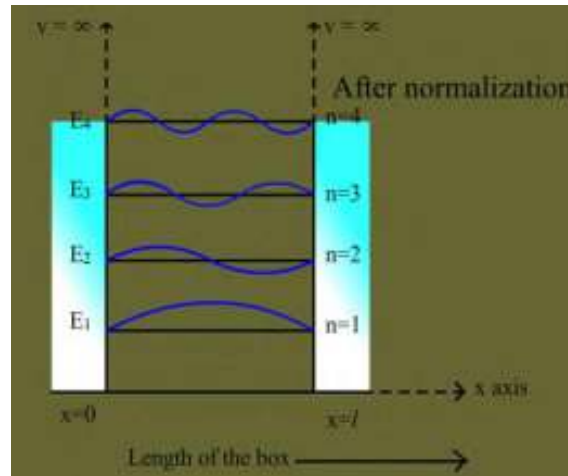
(OR)

$$E_n = \frac{n^2 h^2}{8 n l^2}$$

For each n – energy level and wave function

E_n = eigen value and Ψ_n = eigen function

Energy levels of an e^- :



$n = 1$ is the lowest energy

$$E_1 = \frac{h^2}{8 n^2 l^2} \quad E_2 = \frac{4 h^2}{8 n^2 l^2} \quad E_3 = \frac{9 h^2}{8 n^2 l^2}$$

$$E_2 = 4E_1$$

$$\text{General } E_n = n^2 E_1$$

Energy levels are discrete – big success of quantum theory.

NORMALISATION OF WAVE FUNCTION:

Process by which probability (P) of finding the particle inside the box can be done.

For a 1D box,

$$P = \int_0^l |\psi|^2 dx = 1$$

$$P = \int_0^l A^2 \left(\frac{\sin \frac{n\pi x}{l}}{s} \right)^2 dx = 1$$

$$\text{(or) } A \int_0^l \frac{1 - \cos \frac{2n\pi x}{l}}{2} dx = 1 \quad \text{(or) } \frac{A^2}{2} [x]_0^l = 1$$

$$A = \left[\frac{2}{l} \right]^{1/2}$$

Normalized wave function can be written as

$$f_s = \left[\frac{2}{l} \right]^{1/2} \sin \frac{n_x x}{l}$$

DEGENERACY AND NON-DEGENERACY:

1. **DEGENERACY:** For various combinations of quantum number, same energy eigen value but different eigen functions. This is called degeneracy states are called degenerate states.

Eg: 3 combinations of quantum number which is same eigen function.

(1,1,2) (121) (211) called 3 fold degenerate.

2. **NON-DEGENERACY:** if same energy eigen value comes for various combinations of quantum number is called non-degeneracy and state is non-degenerate state.

$$\text{Eg: } n_x = n_y = n_z = 2 \quad E_{222} = \frac{12 h^2}{8 n a^2}$$

$$f_{222} = \left(\frac{8}{a^3} \right)^{1/2} \sin \frac{2 n_x x}{a} \sin \frac{2 n_y y}{a} \sin \frac{2 n_z z}{a}$$

SCANNING TUNNELLING MICROSCOPE

STM is the highest resolution imaging and nano fabrication technique available. It depends on quantum tunnelling of electrons from a sharp metal tip to a conducting surface.

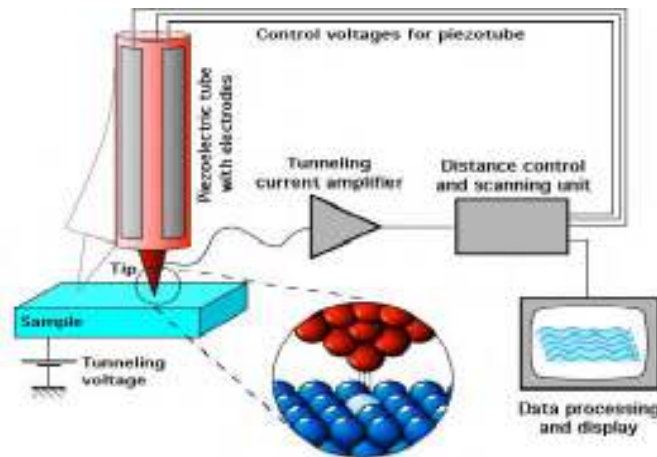
Principle

STM has a metal needle that scans a sample by moving back and forth and gathering information about the curvature of the surface. It follows the smallest changes in the contours of a sample.

The needle doesn't touch the sample, however, but stays about the width of two above it.

Instrumentation

A schematic of STM is shown in fig.



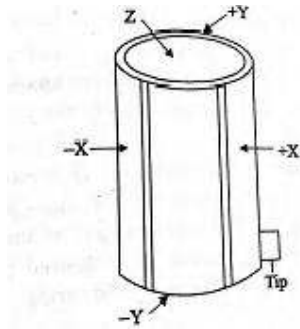
It has the following components.

- **Piezoelectric tube** with the tip and electrodes. Capable of moving in X, Y, Z direction.
- **Fine needle tip** for scanning the sample surface. A macroscale image of an etched tungsten STM tip shown in fig.



Fig. 6.17 Tungsten tip of STM

Tip is affixed to the piezoelectric tube in order to control its position and movement on an atomic scale. Piezoelectric materials exhibit an elongation or contraction along their length when an electric field is applied.



The fig 6.18 shows a tube of PZT, coated with metal both inside and outside.

Applying voltages to the appropriate regions on the tube causes the PZT to deform and move the tip. The X and Y piezos control the back and forth motion of the tip, while the Z piezo allows the tip to approach the surface.

Working

The tip is mechanically connected to the scanner, an XYZ positioning device. The sharp metal needle is brought close to the surface to be imaged. The distance is of the order of a few angstroms.

A bias voltage is applied between the sample and the tip. When the needle is at a positive potential with respect to the surface, electrons can tunnel through the gap and set up a small “tunnelling current” in the needle. This feeble tunnelling current is amplified and measured.

With the help of the tunnelling current, the feedback electronics keeps the distance between tip and sample constant. The sensitivity of the STM is so large that electronic corrugation of surface atoms and the electron distribution around them can be detected.

Application of STM

1. The STM shows the positions of atoms- or more precisely, the positions of some of the electrons.
2. Uses of STM to study metals and semiconductors surface can provide non-trial real space information especially in studying semiconductor such as SI (100) surface.
3. One innovative applications of STM recently found is manipulation of atoms. For example, Iron atoms are placed on Cu surface and dragged by the STM tip and moves across the surface to a desired position. Then, the tip was withdrawn by lowering the tunneling current.
4. To analyze the electronic structures of the active sites at catalyst surfaces.
STM is used in the study of structure, growth, morphology, electronic structure of surface, thin films and nano structures.

Disadvantages of STM

- A small vibration, even a sound, could smash the tip and the sample together.
- A single dust particle, for example, could damage the needle

UNIT – V**Crystal Physics****INTRODUCTION**

The matter in this wonderful world is found to exist in three states viz, solids, liquids and gases.

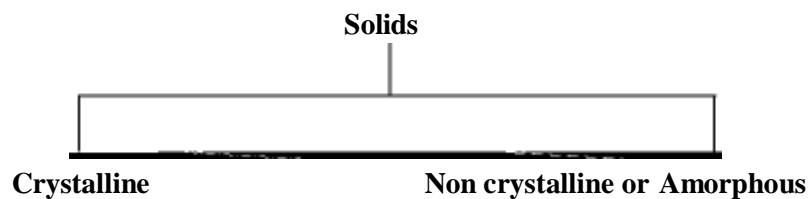
We know that some solids are brittle, some are ductile, some are malleable, some are strong, some are weak, some are good conductors of heat and electricity, some are bad conductors of heat and electricity, some are magnetic, some are non – magnetic and so on, but all these materials are composed of atoms and molecules.

The constituent particles. i.e., atoms (or) molecules of matter are held together by the forces of attraction. The attractive forces which hold the constituent particles of a substance together are called bonds.

The reason for these different properties of the solids are due to their structure. i.e., the behaviour of the solid materials is closely related to the structure of the material. In this chapter we are going to discuss about the different crystal structures and their properties.

BASIC DEFINITIONS Matter

- Ductile
- Malleable
- Good or bad conductors
- All are composed of atoms & molecules
- Attractive forces which hold the constituent particles : BONDS



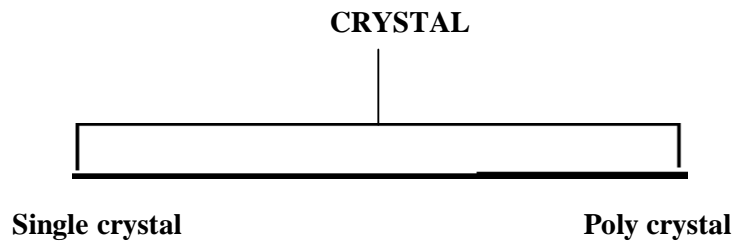
Crystalline

- Atoms are arranged in orderly fashion
- Have directional Properties called ANISOTROPIC

Non crystalline or Amorphous

- Not in orderly fashion
- No directional properties called ISOTROPIC
- do not have sharp melting point

eg: plastics



Single crystal

- Contains only one Crystal
- Produced artificially from their vapour or liquid state

Poly crystal

- contains aggregate of many small crystals
- separated well by grain boundaries
- have a sharp melting point

eg: Diamond, copper

CRYSTALLOGRAPHY:

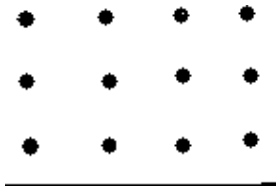
Study of geometric form & other physical properties by X-rays.

FUNDAMENTALS:

- **CRYSTAL:**
 1. Regular polyhedral bounded by smooth surfaces.
 2. Formed by chemical compound under action of its inter atomic forces.
- **CRYSTALLIZATION:** Phase change from liquid or gas to solid.

- **X-RAY CRYSTALLOGRAPHY:** Determination of crystal structure with the help of X-Ray.
- **LATTICE:** An array of points imaginarily kept to represent the position of atoms in the crystal.
- **SPACE LATTICE OR CRYSTAL LATTICE:**
 - 3 dimensional collections of points in space.
 - Environment about any particular point is same everywhere.
 - Geometrical representation of crystal structure.

Example: 2D space lattice



LATTICE POINTS: Position of atoms or molecules in the crystal.

BASIS OR MOTIF: A unit of atoms or molecules which are identified with respect to the position of lattice points.

Lattice + basis \longrightarrow crystal structure

Note: no of atoms in basis may be 1 or 2 or even 100

Example: Al & Ba : 1 NaCl, KCl \longrightarrow 2

CaF₂ 3

CRYSTAL STRUCTURE:

Structure formed by addition of basis to every lattice point of lattice.

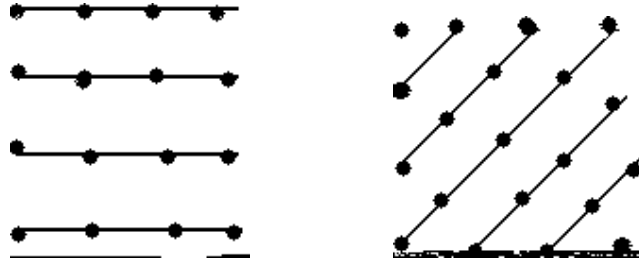
Space lattice + Basis ---- >Crystal structure

LATTICE PLANES:

-Set of parallel & equally spaced planes in space lattice.

- Formed with respect to lattice planes for an array of lattice points

Example:

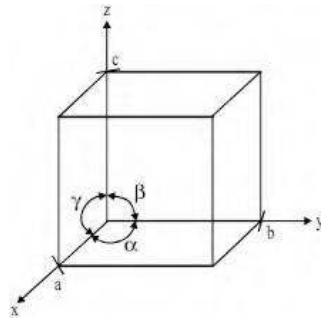


UNIT CELL:

The unit cell is defined as the smallest geometric figure which is repeated to derive the actual crystal structure. The unit cell fully represents the characteristics of the entire crystal.

LATTICE PARAMETERS: Lines drawn parallel to lines of intersection of any three faces of unit cell which do not lie in same plane are called Crystallographic axes.

- Intercepts (a, b, c) which defines the dimensions of a unit cell are called **PRIMITIVE** or **CHARACTERISTIC** Intercepts.
- a, b, c: Fundamentals translational vectors or axial lengths.
- The angles between (a, b), (b, c), & (c, a) are represented as α , β , γ which are interfacial angles.



- Intercepts (a, b, c) and Interfacial angles (α , β , γ) are called as cell parameters.

PRIMITIVE CELL: Simplest type of unit cell which contains only one lattice point per unit cell.
(Contains lattice points at its corner only)

Example: Simple cubic

NON PRIMITIVE CELL: More than one lattice point in a unit cell.

Example: BCC, FCC

CRYSTAL SYSTEMS:

Classified into 7 crystal systems based on lattice parameters. [(a, b, c) & (α , β , γ)]

7 BASIC CRYSTAL SYSTEMS:**1. TRICLINIC:**

- Axis is not perpendicular.
- Axis lengths not equal.

$$a \neq b \neq c$$

$$\alpha \neq \beta \neq \gamma \neq 90^\circ$$

Forms: only simple

2. MONOCLINIC:

- Axes perpendicular to each other (two).
- Third axis is obliquely inclined.
- Axial lengths not equal.

$$a \neq b \neq c$$

$$\alpha = \gamma \neq 90^\circ \neq \beta$$

Forms: simple or BCC

3. ORTHORHOMIC:

- Axes (3) perpendicular to each other.
- Axial lengths are unequal.

$$a \neq b \neq c$$

$$\alpha = \beta = \gamma = 90^\circ$$

Forms: simple, base centered, BCC, FCC

4. TETRAGONAL:

- Axes (3) perpendicular to each other.
- Two of axial length are equal, third is different.

$$a \neq b = c$$

$$\alpha = \beta = \gamma = 90^\circ$$

Forms: simple or BCC.

5. HEXAGONAL:

- Two of crystal axes are 120° apart.
- Third is perpendicular to both of them.

- Axial length along 120° are equal, third is different.

$$a = b = c$$

$$\alpha = \beta = 90^\circ, \gamma = 120^\circ$$

Form: simple

6. TRIGONAL OR RHOMBOHEDRAL

- (i). 3 Axes are inclined to each other at angle other than 90° . Axial lengths are equal

$$a = b = c$$

$$\alpha = \beta = \gamma \neq 90^\circ$$

Forms: Simple

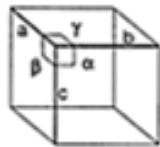
7. CUBIC:

- (i). 3 crystal axes perpendicular to each other

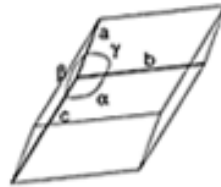
- ii. Axial lengths are same

$$a = b = c$$

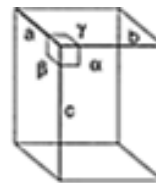
$$\alpha = \beta = \gamma = 90^\circ$$



Cubic
 $a = b = c$
 $\alpha = \beta = \gamma = 90^\circ$



Trigonal or
 $a = b = c$
 $\alpha = \beta = \gamma = 90^\circ$ $\gamma = 120^\circ, \alpha = \beta = 90^\circ$

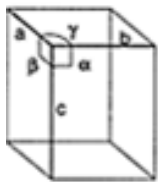


Tetragonal
 $a = b \neq c$
 $\alpha = \beta = \gamma = 90^\circ$

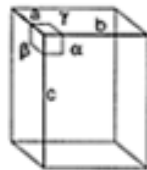
Forms:
Simple
Cubic, BCC,
FCC

Fig

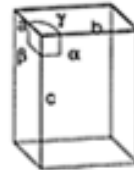
Bravais Lattice



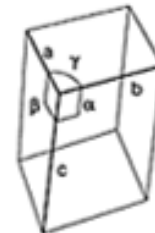
Hexagonal
 $a = b \neq c$
 $\gamma = 120^\circ, \alpha = \beta = 90^\circ$



Orthorhombic
 $a \neq b \neq c$
 $\alpha = \beta = \gamma = 90^\circ$



Monoclinic
 $a \neq b \neq c$
 $\gamma \neq \alpha = \beta = 90^\circ$



Triclinic
 $a \neq b \neq c$
 $\gamma \neq \alpha \neq \beta = 90^\circ$

- 14 Possible type of Space lattices with 7 crystal Systems
- Together Called Bravais Lattices

CUBIC SYSTEM – 3 axes perpendicular to each other axial lengths equal

FORMS: SC



BCC



FCC

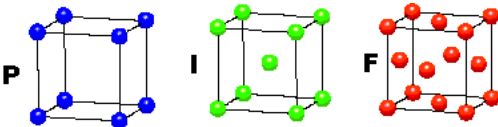
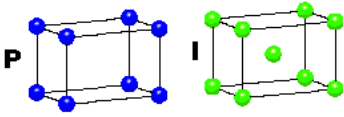
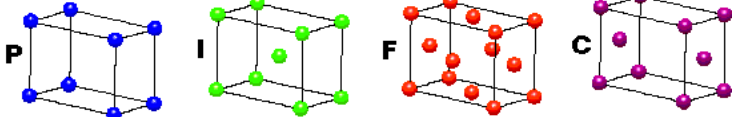
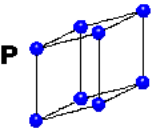
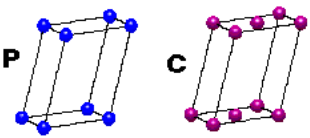
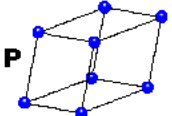


One Atom/Corner of Unit cell (8 corners) nit cell	One Atom/Corner of Unit cell +one in centre of unit cell	One Atoms/Corner of Unit cell + Atoms in each face f
Eg: Polonium	Unit cell eg: Chromium	Unit cell eg: Copper

ii. Hexagonal:

Atoms are located at each of 12 corners of hexagon + 2 atoms (One at centre of top hexagonal face & one at centre of bottom hexagonal face)

PARAMETERS DETERMINING CRYSTAL STRUCTURE OF MATERIALS:

<p>CUBIC $a = b = c$ $\alpha = \beta = \gamma = 90^\circ$</p>		<p><u>No of Atoms /Unit cell or Effective Number</u></p> <p>Total no of Atoms present in or shared by a Unit cell.</p> <p><u>Atomic Radius:</u></p> <p>Half of distance between any two nearest neighbour atoms which each other. Usually</p>
<p>TETRAGONAL $a = b \neq c$ $\alpha = \beta = \gamma = 90^\circ$</p>		
<p>ORTHORHOMBIC $a \neq b \neq c$ $\alpha = \beta = \gamma = 90^\circ$</p>		
<p>HEXAGONAL $a = b \neq c$ $\alpha = \beta = 90^\circ$ $\gamma = 120^\circ$</p>	<p>TRIGONAL $a = b = c$ $\alpha = \beta = \gamma \neq 90^\circ$</p> 	
<p>MONOCLINIC $a \neq b \neq c$ $\alpha = \gamma = 90^\circ$ $\beta \neq 120^\circ$</p>		
<p>TRICLINIC $a \neq b \neq c$ $\alpha \neq \beta \neq \gamma \neq 90^\circ$</p>		

4 Types of Unit Cell
 P = Primitive
 I = Body-Centred
 F = Face-Centred
 C = Side-Centred
 +
 7 Crystal Classes
 → 14 Bravais Lattices

expressed in terms of cube edge 'a'.

Co-ordination Number

No of nearest neighboring atoms to a particular atom.

Atomic packing factor or packing density of Density of packing

Ratio between volumes occupied by total no of atoms/Unit cell to total volume of unit cell.

$$\text{APF} = \frac{\text{Volume occupied by total no of atoms/Unit cell}}{\text{Total volume of unit cell}} = \frac{v}{V}$$

OR

$$\text{APF} = \frac{\text{Total no of atoms/ Unit cell} \times \text{volume of one atom}}{\text{Total volume of unit cell}}$$

SIMPLE CUBIC (SC) STRUCTURE

The simple cubic structure is the simplest and easiest crystal structure. In this structure, there is one atom at each of the '8' corners of unit cell. These atoms touch each other along cube edges.

NUMBER OF ATOMS / UNIT CELL :

Simple Cubic:

Corners Shared by 8 atoms

∴ Each Corner atom contribution = $1/8$

∴ No of Atoms / unit cell = $1/8 \times 8 = 1$

Simple Cubic is Primitive cell.

F

Effective No = 1



Fig

ATOMIC RADIUS:

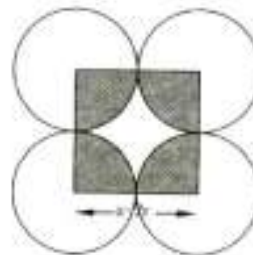
Simple Cubic:

Corner Atoms touch each other along edges:

Nearest neighbour distance is lattice constant a .

$$a = 2r$$

$r = a/2$



Fig

ATOMIC PACKING FACTOR**Definitions**

a) SC

$$\text{APF} = \frac{\text{No of atoms/unit cell} \times \text{volume of one atom}}{\text{Total volume of unit cell}}$$

$$= \frac{Nv}{V} \quad \longrightarrow \quad (1)$$

For SC

$$(i) \quad N=1, (ii) \quad v = \frac{4}{3} \pi r^3 \left\{ r = \frac{a}{2} \right\}$$

$$v = \frac{4\pi a^3}{3 \cdot 8}$$

$$= \frac{\pi a^3}{6}$$

$$(ii) \quad V = a^3$$

Substitute in (1)

$$\text{APF} = \frac{\frac{\pi a^3}{6}}{a^3} = \frac{\pi}{6}$$

$$\text{APF} = \frac{\pi}{6} = 0.52$$

- Only 52% of volume of unit cell is occupied where as 48% is vacant

BODY-CENTRED CUBIC (BCC) STRUCTURE

In this crystal structure, the unit cell has one atom at each corner of the cube and one atom at the body centre of the cube.

NUMBER OF ATOMS / UNIT CELL :-> From Corner atoms = $1/8 \times 8 = 1$

-> Body Centre atom is not shared = 1

Effective No = 2

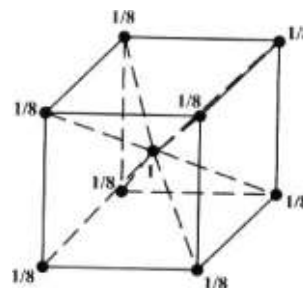
Fig

ATOMIC RADIUS:

Corner atoms do not touch each other

Each corner atom touches body

Centered atom along 'BODY DIAGONAL'



: . 2 atoms (A& D) are joined by diagonals

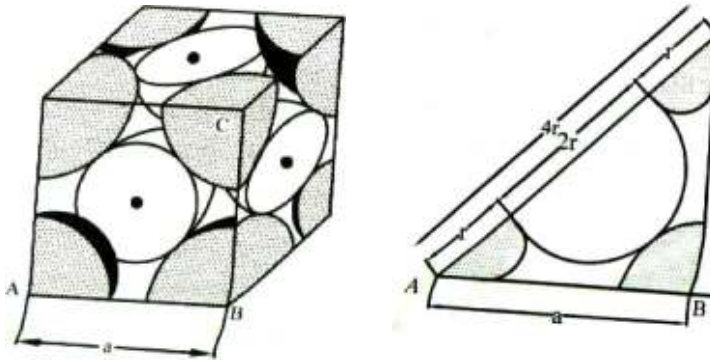


Fig.

$$AD^2 = AC^2 + CD^2$$

$$AC^2 = AB^2 + BC^2$$

SUB

$$AD^2 = AB^2 + BC^2 + CD^2$$

$$AD = 4r; \quad AD = \sqrt{16r^2}$$

$$\Rightarrow 16r^2 = 3a^2$$

$$r = \frac{a\sqrt{3}}{4}$$

ATOMIC PACKING FACTOR

$$APF = \frac{V_{atoms}}{V_{unit\ cell}}, \quad \text{for BCC}$$

$$(i) N=2 \quad (ii) \quad v = \frac{4\pi r^3}{3} \left\{ \frac{r = \frac{a\sqrt{3}}{4}} \right\}$$

$$v = \frac{4}{3} \pi \frac{a^3 \sqrt{3}}{16} = \frac{na^3 \sqrt{3}}{16}$$

$$(iii) V = a^3$$

$$\text{Substitute } APF = 2 \times \frac{\frac{na^3 \sqrt{3}}{16}}{a^3} = \frac{n\sqrt{3}}{8}$$

$$APF = \frac{n\sqrt{3}}{8} = 0.68$$

- 68% of volume of unit cell is occupied where as 32% is vacant
- Tightly packed compared to SC as their APF is more

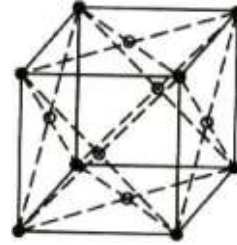
FACE – CENTRED CUBIC (FCC) STRUCTURE

In this type of crystal structure, the unit cell has one atom at each corner of the cube and one atom at the centre of each face.

NUMBER OF ATOMS / UNIT CELL :

$$\begin{aligned} \text{From Corner Atoms } 1/8 \times 8 &= 1 \\ \text{Face Atom is shared by 2 Atoms } 1/2 \times 6 &= 3 \end{aligned}$$

$$\text{Effective No} = 4$$



Corner atoms do not touch each other. Each corner atom touches the face centered atom. Corner A & C are joined by face diagonal.

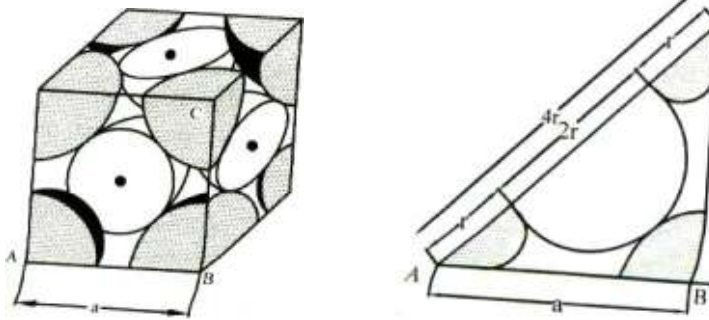
ATOMIC RADIUS

Fig.

$$AC^2 = AB^2 + BC^2, \quad (4r)^2 = a^2 + a^2$$

$$r = (a\sqrt{2})/4$$

ATOMIC PACKING FACTOR

$$APF = \frac{V}{V}$$

$$\text{For FCC, (i) } N = 4 \text{ (ii) } v = \frac{4\pi r^3}{3} \left\{ r = \frac{a\sqrt{2}}{4} \right\}$$

$$v = \frac{4}{3} \pi \frac{a^3 \sqrt{2}}{16 \times 4}$$

$$= \frac{na^3 \sqrt{2}}{3 \times 8}$$

$$(iii) V = a^3$$

$$\text{Substitute } APF = \frac{4 \times \frac{4}{3} \pi r^3 \sqrt{2}}{a^3}$$

$$APF = \frac{n\sqrt{2}}{6} = 0.74$$

- 74% of volume of unit cell is occupied and 26% is vacant
- FCC is most tightly packed compared to BCC & SC

HEXAGONAL CLOSELY PACKED STRUCTURE (HCP)

A unit cell of a close – packed hexagonal structure.

In this type of crystal structure, the unit cell has one atom at each of the 12 corners of the hexagonal face, one atom each at the centre of the two hexagonal faces. 3 atoms symmetrically arranged inside the unit cell.

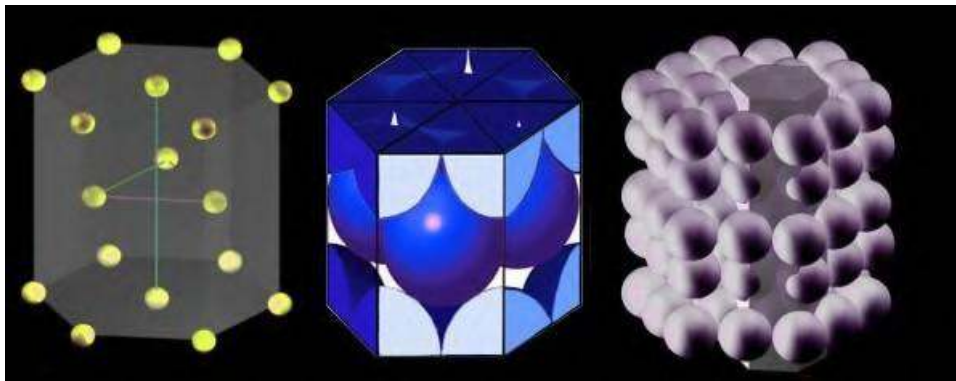
There are 3 layers of atoms in it. At the bottom layer (A), the central atom has ‘6’ nearest neighbouring atoms in the same plane. The second layer (B) which is at a distance $c/2$ from the bottom layer has 3 atoms.

(HCP)

There are three types of atoms

- 12 corner atoms, one at each corner of hexagon
- 2 base centered, one at top and bottom of hexagon
- 3 atoms in between top and bottom faces

Example: Mg, Zn, Cd etc



1) CO-ORDINATION NUMBER:

HCP structure has 3 layers:

→ Bottom layer B_L

→ Middle layer M_L

→ Top layer T_L

Taking reference atom 'x' of BL_1 6 atoms of its own plane + 3 atoms of ML_1 + 3 atoms of ML_2 (of neighbouring atoms)

So, co-ordination number = $6+3+3 = 12$

2) NUMBER OF ATOMS PER UNIT CELL:

Corner atoms: shared by 6 unit cells

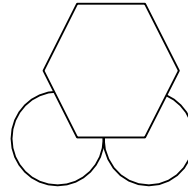
Top and bottom face: shared by 2 unit cells

Centre atoms: no sharing

$$\frac{1}{6} \times 12 + 2 \times \frac{1}{2} + 3 = 6$$

3) ATOMIC RADIUS:

$$a = 2r$$



4) RELATION BETWEEN 'C' AND 'A':

c – height of unit cell

a – distance between two neighbouring

JKLMNI – corner atoms

PQT – middle layer atoms

→ Draw a perpendicular from O to IN at a point R

→ Triangle IRO

$$\angle IOS = 30^\circ$$

$$\cos 30^\circ = \frac{OR}{OI}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \cos 30^\circ$$

$$a = OI$$

$$\text{so, } OR = \frac{a\sqrt{3}}{2}$$

→ 'S' is the orthocenter for INO,

$$OS = \frac{2}{3}OR$$

$$OS = \frac{2a\sqrt{3}}{6}$$

$$OS = \frac{a}{\sqrt{3}}$$

→ $PS = \frac{c}{2}$ {half of unit cell}

→ Triangle SOP, $OP^2 = OS^2 + SP^2$

$$a^2 = \left(\frac{a}{\sqrt{3}}\right)^2 + \left(\frac{c}{2}\right)^2$$

$$a^2 - \frac{a^2}{3} = \frac{c^2}{4} \implies \frac{2a^2}{3} = \frac{c^2}{4}$$

$$\implies \frac{c}{a} = \left(\frac{8}{3}\right)^{1/2}$$

$$\implies \text{Axial ratio} = \frac{c}{a} = 1.639$$

ATOMIC PACKING FACTOR (APF)

$$\text{APF} = \frac{\text{No of atoms/unit cell} \times \text{volume of one atom}}{\text{Volume of unit cell}}$$

$$\text{APF} = \frac{6 \times \frac{4}{3} \pi r^3}{\text{Vol of unit Cell}}$$

Vol of unit cell : Area of base x height

Area of base = 6 x Area of one Triangle (eg: ONI)

$$= 6 \times \frac{1}{2} (\text{NI}) (\text{OR}) (\text{Base} \times \text{ht})$$

$$\text{OR} = a \sqrt{3} = 6 \times \frac{1}{2} a - \frac{a\sqrt{3}}{2}$$

$$= 3\sqrt{3} a^2/2$$

$$\text{Volume of unit cell} = 6 \times \frac{3\sqrt{3}a^2}{2} \times C$$

$$\text{Therefore APF} = \frac{6 \times \frac{4}{3} \times n \times (a/2)^3}{6 \times \frac{3\sqrt{3}a^2}{2} \times C}$$

$$\frac{6 \times \frac{4}{3} \times n \times (a/2)^3}{6 \times \frac{3\sqrt{3}a^2}{2} \times C}$$

$$= \frac{24 n a^3 2\pi}{6 \times \frac{3\sqrt{3}a^2}{2} \times C} \quad (a/c) \quad (c/a = (\sqrt{8/3}))$$

$$\text{APF} = \frac{2\pi}{\sqrt{3/8}}$$

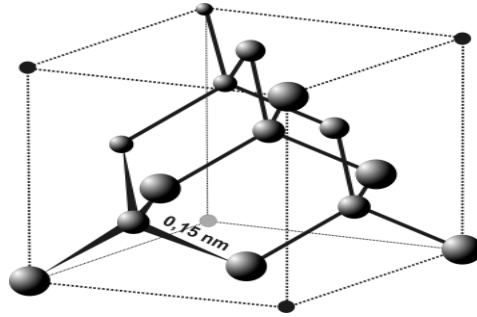
$$\text{APF} = 0.74$$

Packing Density 74% and remaining 26% is vacant.

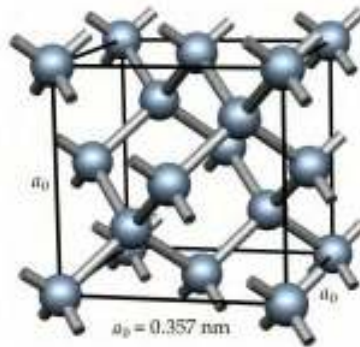
HCP is also tightly or closely packed.

DIAMOND

- Diamond has Face Centered Cubic (FCC) structure with the basis of two carbon atoms at X and Y. The 'X' atom is located with an origin of (0,0,0) and the Y atom is located with an origin of (a/4,a/4,a/4) (i.e.) one quarter of way along the body diagonal.



- It is clearly seen from the figure. The diamond structure is formed due to interpenetration of two FCC sub lattices, having origin(0,0,0) and $(1/4,1/4,1/4)$ along the body diagonal
- The important parameters of the structure are discussed below:



1) NUMBER OF ATOMS PER UNIT CELL :

In diamond there are three types of atoms named as A, B ,C

- CORNER ATOMS: represented by C
- FACE CENTERED ATOMS represented as F
- FOUR ATOMS present inside the unit cell represented as 1,2,3,4

- a. Number of corner atoms per unit cell: Each corner atoms are shared by 8 unit cells. There are 8 corner atoms. Therefore number of corner atoms per unit cell

$$= (1 / 8) * 8$$

$$= 1$$

- b. Number of Face centered atoms per unit cell: Each face centered atoms are shared by 2 unit cells. There are 6 face centered atoms. Therefore number of face centered atoms per unit cell

$$= (1 / 2) * 6$$

$$= 3$$

- c. Number of atoms inside the unit cell: Inside the unit cell there are 4 atoms. These atoms are not shared with other unit cell. Therefore number of atoms per unit cell

$$= (1) * 4$$

$$= 4$$

NUMBER OF ATOMS PER UNIT CELL = 1 + 3 + 4 = 8

2) ATOMIC RADIUS:

Atomic radius is defined as half the distance between any two nearest neighbours, which have direct contact with each other.

- The corner and face centered atoms do not have contact with each other. But the corner and face atoms have contact with inside 4 atoms
- From figure the nearest neighbours are X and Y
- Drop a perpendicular to Y meeting the unit cell at Z at a distance $a/4$

$$\begin{aligned} (XY)^2 &= (XZ)^2 + (ZY)^2 \\ &= (XT)^2 + (TZ)^2 + (ZY)^2 \\ &= (a/4)^2 + (a/4)^2 + (a/4)^2 \\ &= 3a^2/16 \end{aligned}$$

XY can be written equal to $2r$.so,

$$4r^2 = 3a^2/16$$

$$r^2 = 3a^2/64$$

$$r = \frac{a\sqrt{3}}{4}$$

$$\text{ATOMIC RADIUS} = r = \frac{a\sqrt{3}}{4}$$

3) COORDINATION NUMBER:

The number of nearest neighbours to atoms to a particular atom. The number of nearest neighbours to Y atom is 4

$$\text{COORDINATION NUMBER} = 4$$

4) ATOMIC PACKING FACTOR:

$$\text{Atomic packing factor} = \frac{\text{Volume occupied by the atoms per unit cell}}{\text{Volume occupied by the unit cell}}$$

$$\text{Volume occupied by one atom} = \frac{4}{3} \pi r^3$$

In diamond there are 8 such atoms. Therefore volume occupied by 8 atoms

$$= 8 \times \frac{4}{3} \pi r^3$$

The relation between atomic radius and lattice constant is $r = \frac{a\sqrt{3}}{8}$

Volume occupied by atoms per unit cell = $v = 8 \times \frac{4}{3} \pi \frac{n}{16} \frac{a^3}{8^3}$

Volume occupied by atoms per unit cell = $v = \frac{n a^3 \sqrt{3}}{16}$

Volume of the unit cell = a^3

$$\text{Therefore APF} = \frac{\frac{n a^3 \sqrt{3}}{16}}{a^3} = \frac{n \sqrt{3}}{16} = 0.34$$

ATOMIC PACKING FACTOR = 0.34

It can be said that 34% volume of the unit cell in diamond cubic structure is occupied by atoms and remaining 66% volume is vacant. As the packing density is low, it is termed as very loosely packed structure.

MILLER INDICES

- Crystals are made up of large number of parallel equidistant planes through lattice points called lattice planes.
- Perpendicular distance between any two adjacent planes is called interplanar distance(d).
- Set of three numbers to designate a plane in a crystal.

Definition :

Reciprocal of intercepts made by plane on crystallographic axes reduced to smallest numbers.

3 steps:

- Intercept made by plane ABC along three axes is found and written in multiples of axial length.
- Example:

$$OA:OB:OC=PA:QB:RC$$

$$\{P=Q=3=3\}$$

Find the co-efficient of intercepts
- Find the reciprocal of these number
- Convert the reciprocal into the whole number by multiplying their LCM.
- Enclose three numbers within brackets.

Important features of miller indices

- If the plane is parallel to any axis, then its reciprocal is ∞
Miller index=0
All equal spaced lines will have same miller indices

PROCEDURE TO SKETCH PLANE FROM MILLER INDICES:-

STEP1:Take reciprocal of given indices

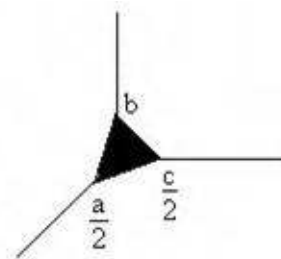
STEP2:represent reciprocal in terms of intercepts along axial length by multiplying by LCM.

Eg: (4 2 4)

h k l

1) LCM=4

$$4 \cdot \frac{1}{4} : \frac{1}{2} \cdot 4 : \frac{1}{4} \cdot 4 = 1 : 2 : 1 \text{ or } \frac{1}{2} : 1 : \frac{1}{2} \text{ or } \frac{a}{2} : b : \frac{c}{2}$$



d' SPACING IN CUBIC LATTICE (Derivation):

Consider a plane ABC in cubic crystal with a as length of the cube edge as given below. This plane is one of the plane of the family of planes whose Miller indices are $(h k l)$. A normal ON is drawn to the plane ABC from the origin of the cube. Let ON be interplanar spacing (d) of this of plane.

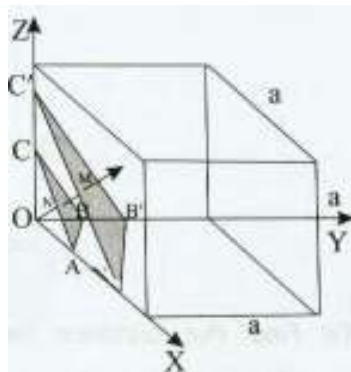


Fig.

This plane ABC makes the intercepts OA , OB and OC on the reference axes OX , OY and OZ α , β and γ are the angles between reference axes and the normal ON respectively.

$$OA = \frac{a}{h}; OB = \frac{a}{k}; OC = \frac{a}{l}$$

From ONA:

$$\cos\alpha = \frac{ON}{OA} = \frac{d_1}{a/h} = \frac{d_1 h}{a}$$

From ONB:

$$\cos\beta = \frac{ON}{OB} = \frac{d_1 k}{a}$$

From ONC:

$$\cos\gamma = \frac{ON}{OC} = \frac{d_1 l}{a}$$

From law of Cosines

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

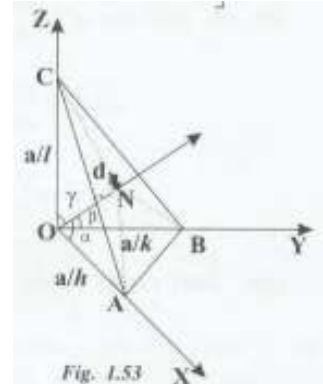
$$\frac{(hd)^2}{(a)^2} + \frac{(kd)^2}{(a)^2} + \frac{(ld)^2}{(a)^2} = 1$$

$$\frac{d^2}{a^2}(h^2 + k^2 + l^2) = 1$$

$$d^2(h^2 + k^2 + l^2) = a^2$$

$$d^2 = a^2 / (h^2 + k^2 + l^2)$$

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$



CRYSTAL GROWTH TECHNIQUES

Types

- Ranges from small inexpensive to complicated methods. They are broadly classified as
- ❖ from solution
- ❖ from melt
- ❖ from vapour

Solution growth

- used to grown crystals with high solubility
- there are 3 basic steps
- ❖ achievement of super saturation

- ❖ formation of crystal nuclei
- ❖ successive growth of crystals

Types of solution growth

- low temperature SG
- high temperature SG

Gel growth:

- used when crystal decomposes before melting point
- solution is created
- gel(semi solid) is formed
- gel is treated to form crystal
- hydrothermal growth
- crystals of materials which do not have sufficient solubility in water or other solvents at normal pressure or temperature

Flux growth

- It is similar to hydrothermal growth

Melt growth

- growth from melt is used for growing large single crystal
- used for material which melt congruently and show the absence of any phase transition below transition

Types

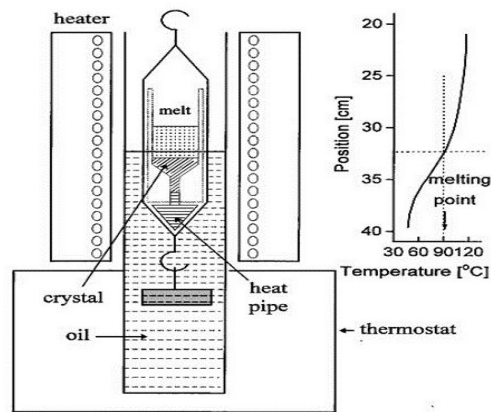
- normal freezing
- crystal pulling
- zone melting
- flame fusion
- Bridgman technique

Bridgeman Technique:

Principle:

1. boat with molten charge is moved across a temperature gradient

- so molten charge solidifies to crystal



Experimental setup:

- contains of a vertical cylindrical container in which the crystal to be grown is kept as shown in fig
- the container is tapered conically with a point bottom surrounded by 2 furnace
- furnace1 – hot zone
- furnace2 – cold zone
- the container is moved up/down to move from hot/cold zone

Working

- the material to be crystallized is kept in boat and kept inside the container
- furnace1 is on
- Material is heated to high temp.
- material goes to molten state
- lowered from hot zone to cold zone
- movement speed: 1 – 30 mm/hour
- crystallization begins at the tip and starts solidifying

Advantages

- relatively cheaper
- melt composition can be controlled

Disadvantages

- growth rate is low
- more dislocation in crystal
- polycrystal might grow

Czochralski Technique

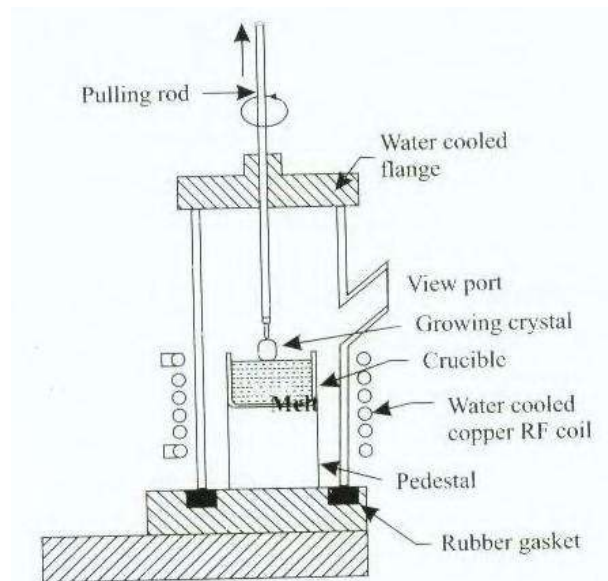


Fig.

- one of the basic method to grow large single crystal
- material is taken in suitable crystal
- heated by radio frequency heater
- clear melt is got with liquid surface on top
- monocrystalline seed is attached to the rod and mounted on melt
- the current is fed to heater coil so that the portion of dipped seed is melted and a meniscus is formed
- the rod is rotated upwards and as melt freezes on the crystal on the same orientation of the seed

- otherwise called as 'necking procedure'

Growth from vapour

- material to be grown is supplied in the form of vapour
- the powders of material are atomized
- seed crystal is brought near the melting point so that atoms arrive on it
- thin film deposition

Types

1. vacuum evaporation
2. sputtering
3. chemical deposition

Epitaxy

- growth of single crystal semiconductor layers
- crystal is grown on a substrate of the same material it is homoepitaxy
- crystal is grown on a substrate of the different material it is heteroepitaxy

Types of epitaxy

- liquid phase epitaxy
- liquid phase electron epitaxy
- vapour phase epitaxy
- molecular beam epitaxy
- chemical beam epitaxy

CRYSTAL DEFECTS

The real crystals are of finite size they have a surface as their boundary. At the boundary atomic bonds terminate and hence the surface itself is an imperfection. In addition, there are other occasional disruptions in periodicity within a crystal. Many of the properties of crystalline materials are structure sensitive properties. They mainly depend on the presence or absence of imperfections. Crystalline imperfections can be classified on the basis of their geometry as follows

- Point imperfections
- Line imperfections and
- Surface imperfections
- volume Imperfections

Point imperfections

They are imperfect point like regions, one or two atomic diameters in size and hence referred to as zero dimensional imperfections. There are different kinds of point imperfections. They are

- A vacancy refers to an atomic site from where the atom is missing as shown in figure

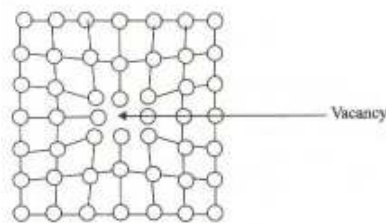


Figure 5.43:
Vacancy

- A Substitutional impurity or solute is a point imperfection. It refers to a foreign atom that substitutes for or replaces a parent atom in the crystal as in figure

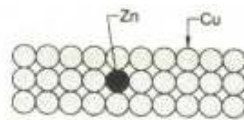


Figure 5.44: Substitute Defect

- A small sized atom occupying the void space in the parent crystal without disturbing the parent atoms from their regular sites is an Interstitial impurity as shown figure

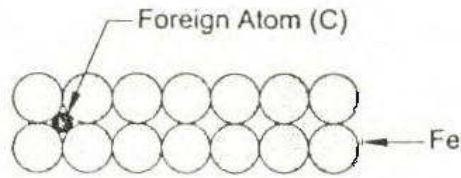


Figure 5.45: Interstitial Impurity

- In ionic crystals, an ion displaced from a regular site to an interstitial site is called Frenkel imperfections as shown in figure. As cations are generally the smaller ones, it is possible for them to get displaced into the void space.
- A pair of one cation and one anion can be missing from an ionic crystal and such a pair of vacant ion site is called Schottky imperfections. This type of defect is dominant in alkali halides

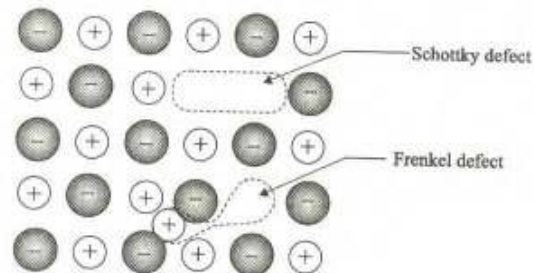


Figure 5.46: Schottky Frenkel imperfections

Line Imperfections

These are defects in crystalline materials which extend for large distances through a crystal. They are vitally important since they govern the bulk mechanical properties of the material. These line defects are known as dislocations. There are two main types of dislocations which can occur in crystalline materials. They are as follows

- edge dislocation :
- screw dislocation

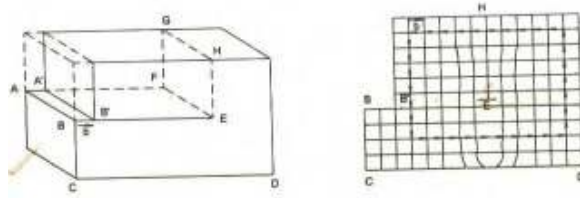


Figure 5.47: Edge Dislocation

Because of dislocation, just above the discontinuity, the atoms are squeezed and are in a state of compression. Just below the discontinuity, the atoms are pulled apart and are in a state of tension. The distorted configuration extends all along the edge into the crystal. Thus as the region of maximum distortion is centred around the edge of the incomplete plane, this distortion represents a line imperfection called an edge dislocation. Edge dislocations are symbolically represented by \rightarrow and \leftarrow depending on whether the incomplete plane starts from the top or from bottom of the crystal. These two configurations are referred to as positive and negative edge dislocations. The magnitude and the direction of the displacement are defined by a vector called the Burgers vector.

Screw dislocation results from a displacement of the atoms in one part of a crystal relative to the rest of crystal, forming a spiral ramp around the dislocation line as illustrated in fig. 5.28. It shows a method of determining the Burgers vector applied to an edge location. It is first necessary to choose arbitrary a positive direction for the dislocation and then to find out the vector which closes a circuit. In the Figure, Burgers circuit is drawn, Burgers vector b is required to close the circuit, which is parallel to the dislocation line.

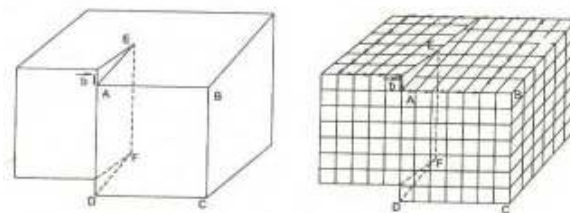


Figure 5.48: Screw Dislocation

Surface Imperfection

Surface Defects (or) plane Defects (or) Two Dimensional Defects The defects, which take place on the surface of a material, are known as surface defects or plane defects. The surface defects takes place either due to imperfect packing of atoms during crystallization or defective orientation of the surface.

The following are the different types of surface defects.

(d) Stacking Fault Whenever the Stacking of atoms is not in proper sequence throughout the crystal, the defect caused is called Stacking fault. Explanation Fig. shows the proper sequence of atomic planes if we read from bottom to top as A-B-C-A- B-C-A-B-C



Figure 5.52: Proper arrangement

Fig. shows the improper sequence of atomic planes as A-B-C-A-B- A-B-A-B-C

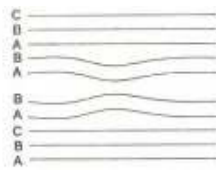


Figure 5.53: Stacking Fault

These are the crystal impurities.